35-Synchronous Motors

Text: 8.1 to 8.7

ECEGR 450
Electromechanical Energy Conversion

Overview

- Principle of Operation
- Equivalent Circuit-Round Rotor
- Equivalent Circuit-Salient Pole
- Power Expressions
- Excitation Effects

Introduction

Important characteristics of synchronous motors:

- rotate at synchronous speed
- not self-starting
- can be operated at leading, unity or lagging PF
- physically identical to synchronous generator

Introduction

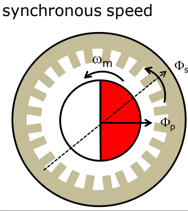
- Synchronous motors rotate at synchronous speed
  - Constant speed-torque characteristic
    - $N_m = N_s$
  - Synchronous speed dependent on number of poles and frequency of applied voltage:

$$N_s = N_m = \frac{120f}{P}$$

- Applied load torque must be within the capability of the motor, otherwise the motor stops

Principle of Operation

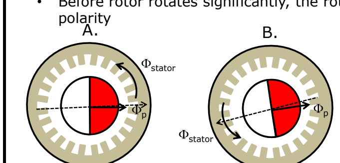
- Rotor is similar to synchronous generator
  - contains field winding or PM
  - portions of the rotor act like north and south poles
- Applied AC source acts like rotating magnet that "pulls" the rotor at synchronous speed



Rotor flux attracted to (tries to align with) stator flux. Torque on rotor will be in direction that aligns rotor flux to stator flux with shortest amount of rotation.

Principle of Operation (Starting)

- Synchronous motors do not self-start
- Synchronous speed of rotating field is usually very fast (e.g. 3600 rpm)
- Rotor has a large inertia J
- Before rotor rotates significantly, the rotating field has switched polarity



t = 0s
torque on rotor CCW

t = 0.0083s
torque on rotor CW

2-pole synchronous motor

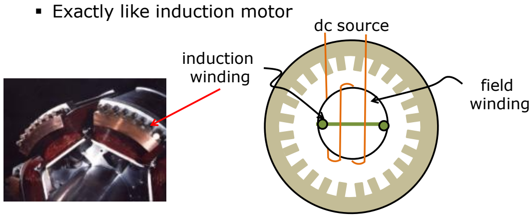
A. Rotor is at standstill
- Stator flux rotates CCW
- Torque on rotor CCW

B. Rotor has started rotating CCW
- Stator flux advances ~180° in ~1/120 s
- Torque on rotor is now CW

Rotor cannot rotate far enough before torque switches direction.

Principle of Operation (Starting)

- Add induction (armortisseur) winding to rotor
- Induced current will cause rotor to revolve with stator flux
  - Exactly like induction motor



Principle of Operation

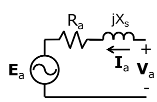
- Synchronous motors are started as induction motors
- Near synchronous speed the field winding "locks" the rotor in at synchronous speed
  - Current no longer induced in induction winding
- Field winding should not be disconnected during start-up
  - Very large voltage can be induced
- Under load, induction winding prevents "hunting"
  - Hunting: small variations in motor speed due to sudden changes in load

Equivalent Circuit (Round Rotor)

- Per-phase equivalent circuit is similar to the synchronous generator
  - Phase current in opposite direction
- Circuit equations

$$\mathbf{V}_a = \mathbf{E}_a + \mathbf{I}_a R_a + j\mathbf{I}_a X_s$$

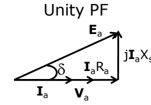
$$\mathbf{I}_a = \frac{\mathbf{V}_a - \mathbf{E}_a}{R_a + jX_s}$$



Equivalent Circuit (Round Rotor)

- Synchronous Motors (all PFs):
  - $\mathbf{E}_a$ lags $\mathbf{V}_a$
  - Power angle δ is negative

Unity PF

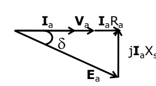


$$\mathbf{E}_a = \mathbf{V}_a + \mathbf{I}_a R_a + j\mathbf{I}_a X_s$$

Synchronous Generator

$\mathbf{I}_a$ defined as leaving $\mathbf{E}_a$

Unity PF



$$\mathbf{V}_a = \mathbf{E}_a + \mathbf{I}_a R_a + j\mathbf{I}_a X_s$$

Synchronous Motor

$\mathbf{I}_a$ defined as entering $\mathbf{E}_a$

Equivalent Circuit (Round-Rotor)

- Like synchronous generators, synchronous motors are doubly fed
  - stator and rotor are connected to an external circuit
- Input power:

$$P_{in} = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} + v_f i_f$$

- Copper losses

$$P_{cu} = 3|\mathbf{I}_a|^2 R_a + v_f i_f$$

- Power developed:

$$P_d = P_{in} - P_{cu} = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi - 3|\mathbf{I}_a|^2 R_a$$

Equivalent Circuit (Round-Rotor)

- Torque developed by the motor is:

$$T_d = \frac{P_d}{\omega_s}$$

- Power out is:

$$P_o = P_d - P_r - P_{sl}$$

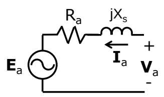
- Output power is mechanical, so it is common to express in horsepower
  - 1hp = 745.7 W

Power Expressions (Round Rotor)

- First consider the round (cylindrical) rotor
- Circuit equations

$$\mathbf{V}_a = \mathbf{E}_a + \mathbf{I}_a R_a + j\mathbf{I}_a X_s$$

$$\mathbf{I}_a = \frac{\mathbf{V}_a - \mathbf{E}_a}{R_a + jX_s}$$



Power Expressions (Round Rotor)

- From the equivalent circuit:

$$\mathbf{I}_a = \frac{\mathbf{V}_a - \mathbf{E}_a}{R_a + jX_s} = \frac{\mathbf{V}_a - \mathbf{E}_a}{\mathbf{Z}_s}$$

- Power developed:

$$P_d = 3\mathrm{Re}\{\mathbf{E}_a \mathbf{I}_a^*\} = 3\mathrm{Re}\{\frac{\mathbf{E}_a \mathbf{V}_a^* - |\mathbf{E}_a|^2}{\mathbf{Z}_s^*}\}$$

$$= 3\mathrm{Re}\{\frac{\mathbf{E}_a \mathbf{V}_a^* \mathbf{Z}_s}{|\mathbf{Z}_s|^2} - \frac{|\mathbf{E}_a|^2 R_a}{|\mathbf{Z}_s|^2} - j\frac{|\mathbf{E}_a|^2 X_s}{|\mathbf{Z}_s|^2}\}$$

Note that:
$$P_d = 3\mathrm{Re}\{\mathbf{E}_a \mathbf{I}_a^*\} = 3|\mathbf{E}_a||\mathbf{I}_a|\cos(\delta + \phi_{PF})$$

- using:

$$\mathbf{Z}_s = |\mathbf{Z}_s|\angle\theta_z$$

$$\mathbf{Z}_s^* = |\mathbf{Z}_s|\angle -\theta_z$$

$$\frac{1}{\mathbf{Z}_s^*} = \frac{1}{\mathbf{Z}_s^*}\frac{\mathbf{Z}_s^*}{\mathbf{Z}_s^*} = \frac{|\mathbf{Z}_s|\angle -\theta_z}{|\mathbf{Z}_s|^2 \angle -2\theta_z} = \frac{|\mathbf{Z}_s|\angle\theta_z}{|\mathbf{Z}_s|^2} = \frac{\mathbf{Z}_s}{|\mathbf{Z}_s|^2}$$

Recall that dividing by a phasor means dividing by the magnitude and subtracting the angle

Power Expressions (Round Rotor)

- Let the applied voltage be the reference:

$$\mathbf{V}_a = |\mathbf{V}_a|\angle 0°$$

- Then:

$$\mathbf{E}_a = |\mathbf{E}_a|\angle\delta$$

$$|\mathbf{E}_a|\cos(\delta) + j|\mathbf{E}_a|\sin(\delta)$$

Power Expressions (Round Rotor)

Power can be written as:

$$P_d = 3\mathrm{Re}\{\frac{\mathbf{E}_a \mathbf{V}_a^* \mathbf{Z}_s}{|\mathbf{Z}_s|^2} - \frac{|\mathbf{E}_a|^2 R_a}{|\mathbf{Z}_s|^2} - j\frac{|\mathbf{E}_a|^2 X_s}{|\mathbf{Z}_s|^2}\}$$

$$= 3\mathrm{Re}\{\frac{\mathbf{E}_a \mathbf{V}_a^* \mathbf{Z}_s}{|\mathbf{Z}_s|^2}\} - \frac{|\mathbf{E}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$= 3\mathrm{Re}\{\frac{\mathbf{E}_a \mathbf{V}_a^* (R_a + jX_s)}{|\mathbf{Z}_s|^2}\} - \frac{|\mathbf{E}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$= 3\mathrm{Re}\{\frac{|\mathbf{E}|_a(\cos\delta + j\sin\delta)|\mathbf{V}|_a^*(R_a + jX_s)}{|\mathbf{Z}_s|^2}\} - \frac{|\mathbf{E}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$= \frac{3|\mathbf{E}_a||\mathbf{V}_a|}{|\mathbf{Z}_s|^2}(R_a\cos\delta - X_s\sin\delta) - \frac{3|\mathbf{E}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

Important result

How does this compare to the power output of a synchronous generator?

Power Expressions (Round Rotor)

- Ignoring armature resistance:

$$P_d = \frac{-3|\mathbf{E}_a||\mathbf{V}_a|\sin\delta}{X_s}$$

Similar expression as synchronous generator power output. Negative sign is a consequence of assumed current direction.

- Torque ignoring armature resistance:

$$T_d = \frac{-3|\mathbf{E}_a||\mathbf{V}_a|\sin\delta}{X_s\omega_s}$$

- What happens with δ = 0 degrees?

Power Expressions (Round Rotor)

- Ignoring armature resistance:

$$P_d = \frac{-3|\mathbf{E}_a||\mathbf{V}_a|\sin\delta}{X_s}$$

- And the torque:

$$T_d = \frac{-3|\mathbf{E}_a||\mathbf{V}_a|\sin\delta}{X_s\omega_s}$$

- What happens with δ = 0 degrees?
  - Torque equals 0
  - Motor stalls

Power Relationship Summary

$P_{in} = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} + v_f i_f$ (input electrical power)

→ $v_f i_f$ (field winding loss)

→ $P_{cu} = 3|\mathbf{I}_a^2|R_a$ (copper loss in armature)

$$P_d = \frac{3|\mathbf{E}_a||\mathbf{V}_a|}{|\mathbf{Z}_s|^2}(R_a\cos\delta - X_s\sin\delta) - \frac{3|\mathbf{E}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$= 3\mathrm{Re}\{\mathbf{E}_a\mathbf{I}_a^*\} = 3|\mathbf{E}_a||\mathbf{I}_a|\cos(\delta + \phi_{PF})$$

developed electrical power

→ $P_r + P_{sl}$ (rotational and stray load loss)

$P_o = T_s\omega_s$ (output mechanical power)

Power Relationship Example

$P_{in} = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} + v_f i_f = 8928.53\text{W}$

Let:
v_f=14V
i_f = 5A
$P_r + P_{sl}$= 230W
$\mathbf{Z}_s$ = 0.1 + j5Ω
δ = 30°
$\mathbf{V}_a$ = 130V
$|\mathbf{E}_a|$ = 230V

→ $V_f i_f$ = 70W (field winding loss)

→ $P_{cu} = 3|\mathbf{I}_a^2|R_a = 261.05\text{W}$

$$P_d = \frac{3|\mathbf{E}_a||\mathbf{V}_a|}{|\mathbf{Z}_s|^2}(R_a\cos\delta - X_s\sin\delta) - \frac{3|\mathbf{E}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$= 8642.47\text{W}$

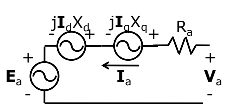
→ $P_r + P_{sl}$ = 230W

P_o= 8412.47W

Equivalent Circuit (Salient Pole)

- Similar equivalent circuit as salient pole generators
  - Phase current is in opposite direction

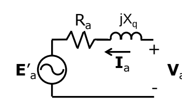
$$\mathbf{E}_a = \mathbf{V}_a - \mathbf{I}_a R_a - j\mathbf{I}_d X_d - j\mathbf{I}_q X_q$$
$$\mathbf{E}_a = \mathbf{V}_a - \mathbf{I}_a R_a - j\mathbf{I}_a X_q - j\mathbf{I}_d (X_d - X_q)$$



Equivalent Circuit (Salient Pole)

- Continuing:

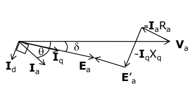
$$\mathbf{E}_a = \mathbf{V}_a - \mathbf{I}_a R_a - j\mathbf{I}_a X_q - j\mathbf{I}_d (X_d - X_q)$$
$$\mathbf{E}_a' = \mathbf{V}_a - \mathbf{I}_a R_a - j\mathbf{I}_a X_q$$
$$\mathbf{E}_a = \mathbf{E}_a' - j\mathbf{I}_d (X_d - X_q)$$

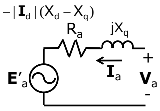


- $\mathbf{E}'_a$ is the effective excitation voltage
- $\mathbf{E}'_a$, $\mathbf{E}_a$ have the same phase angle (δ)
- Note that $\mathbf{E}'_a = \mathbf{E}_a$ when $X_d = X_q$ (round rotor)

Equivalent Circuit (Salient Pole)

- Example phasor diagram of lagging power factor
  - For lagging: $|\mathbf{E}_a'| = |\mathbf{E}_a| + |\mathbf{I}_d|(X_d - X_q)$
  - For leading/unity: $|\mathbf{E}_a'| = |\mathbf{E}_a| - |\mathbf{I}_d|(X_d - X_q)$



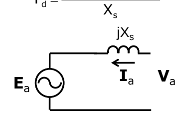


Note: $|\mathbf{I}_d| = \dfrac{|\mathbf{V}_a|\cos(\delta) - |\mathbf{E}_a|}{X_d}$

Salient Pole Power Expressions

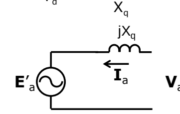
- The salient pole power expressions are derived from the round rotor by inspection

$$P_d = \frac{-3|\mathbf{E}_a||\mathbf{V}_a|\sin\delta}{X_s}$$



round rotor

$$P_d = \frac{-3|\mathbf{V}_a||\mathbf{E}_a'|\sin\delta}{X_q}$$



salient pole

Power Expressions (Salient Pole)

- Recall that: $|\mathbf{E}_a'| = |\mathbf{E}_a| \pm |\mathbf{I}_d|(X_d - X_q)$
  - + for lagging PF
  - - for leading or unity PF
- Through substitution:

$$P_d = \frac{-3|\mathbf{V}_a||\mathbf{E}_a'|\sin\delta}{X_q}$$

$$= \frac{-3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_q} \pm 3|\mathbf{V}_a||\mathbf{I}_d|\sin\delta[\frac{X_d - X_q}{X_q}]$$

$$= \frac{-3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_d} \pm 3|\mathbf{V}_a|\sin\delta[\frac{X_d - X_q}{X_d X_q}](|\mathbf{E}_a| \pm |\mathbf{I}_d|X_d)$$

using $|\mathbf{E}_a| \pm |\mathbf{I}_d|X_d = \mathbf{V}_a\cos(\delta)$

$$P_d = \frac{-3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_d} - 3|\mathbf{V}_a|^2\sin(2\delta)[\frac{X_d - X_q}{2X_d X_q}]$$ Important result

Power Expressions (Salient Pole)

- Power developed by salient pole motor:

$$P_d = \frac{-3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_d} - 3|\mathbf{V}_a|^2\sin(2\delta)[\frac{X_d - X_q}{2X_d X_q}]$$

- Torque developed:

$$T_d = \frac{P_d}{\omega_m} = \frac{-3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_d\omega_m} - 3|\mathbf{V}_a|^2\sin(2\delta)[\frac{X_d - X_q}{2X_d X_q\omega_m}]$$

How do these expressions compare to power and torque expressions for synchronous generators?

Power Expressions (Salient Pole)

- From $P_d = \frac{-3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_d} - 3|\mathbf{V}_a|^2\sin(2\delta)[\frac{X_d - X_q}{2X_d X_q}]$
- Note that if $\mathbf{E}_a = 0$ and $\delta \neq 0$

$$P_d = \frac{-3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_d} - 3|\mathbf{V}_a|^2\sin(2\delta)[\frac{X_d - X_q}{2X_d X_q}] \neq 0$$

- Salient pole motors will rotate without a field winding connection

Effect of Excitation

- Power and torque developed by the motor depends on $|\mathbf{E}_a|$

$$P_d = \frac{-3|\mathbf{E}_a||\mathbf{V}_a|\sin\delta}{X_s}$$

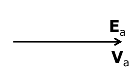
- $|\mathbf{E}_a|$ is a function of the excitation (field) current
- We can control the motor through excitation

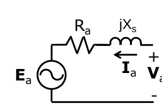
Effect of Excitation

- We next examine the effect of excitation on synchronous motor operation
- Assumptions:
  - No armature resistance
  - No rotational losses
  - Connected to infinite bus

Effect of Excitation

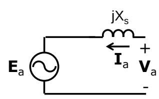
- From the equivalent circuit if an ideal motor has no load, then it draws no current
  - $\mathbf{E}_a = \mathbf{V}_a$
  - $\delta = 0°$





Effect of Excitation

- Normal excitation: When the field current is adjusted so that $|\mathbf{E}_a| = |\mathbf{V}_a|$
- Over-excitation: field current is greater than what is needed for normal excitation $|\mathbf{E}_a| > |\mathbf{V}_a|$
- Under-excitation: field current is less than what is needed for normal excitation $|\mathbf{E}_a| < |\mathbf{V}_a|$



Over-Excited Operation

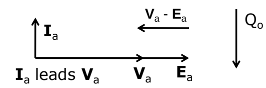
- When over-excited and under no load:
  - $|\mathbf{E}_a| > |\mathbf{V}_a|$
  - $\delta = 0°$

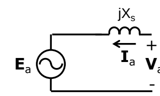
$$\mathbf{I}_a = \frac{\mathbf{V}_a - \mathbf{E}_a}{jX_s} = |\mathbf{I}_a| \angle 90° \text{A}$$

$$P_o = \mathrm{Re}\{\mathbf{V}_a \mathbf{I}_a^*\} = 0\text{W}$$

$$Q_o = \mathrm{Im}\{\mathbf{V}_a \mathbf{I}_a^*\} < 0\text{VAR} \text{ (motor supplies reactive power)}$$

$$\delta = 0°$$





Over-Excited Operation

- When a motor operates in this mode, it is also known as a <u>synchronous condenser</u>
- Improves the power factor of a facility

Under-Excited Operation

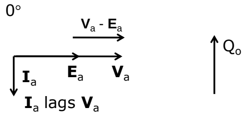
- When under-excited and under no load:
  - $|\mathbf{E}_a| < |\mathbf{V}_a|$
  - $\delta = 0°$

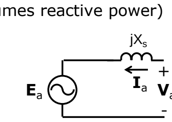
$$\mathbf{I}_a = \frac{\mathbf{V}_a - \mathbf{E}_a}{jX_s} = |\mathbf{I}_a| \angle -90° \text{A}$$

$$P_o = \mathrm{Re}\{\mathbf{V}_a \mathbf{I}_a^*\} = 0\text{W}$$

$$Q_o = \mathrm{Im}\{\mathbf{V}_a \mathbf{I}_a^*\} > 0\text{VAR} \text{ (motor consumes reactive power)}$$

$$\delta = 0°$$





Effect of Excitation

- Now consider a motor under a load
- From: $P_d = \frac{-3|\mathbf{E}_a||\mathbf{V}_a|\sin\delta}{X_s}$
  - δ cannot be zero
- Assume that $\mathbf{V}_a$ and P_d are constant
- If $\mathbf{E}_a$ changes due to a change in field current, then $|\mathbf{E}_a|\sin\delta$ must be constant to equal P_d
  - δ must decrease as $|\mathbf{E}_a|$ increases

Effect of Excitation

- Armature current also changes due to:

$$\mathbf{I}_a = \frac{\mathbf{V}_a - \mathbf{E}_a}{R_a + jX_s}$$

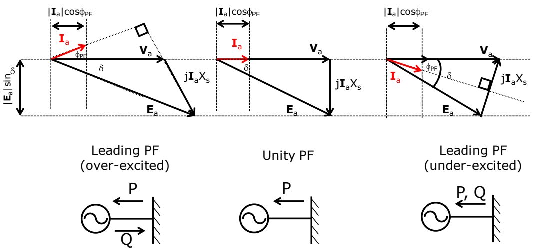
- The power into the motor should not change

$$P_{in} = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF}$$

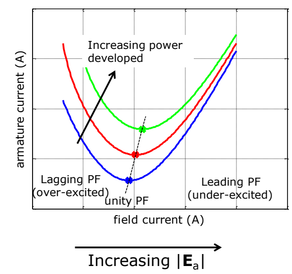
- Therefore, $|\mathbf{I}_a|\cos\phi_{PF}$ should be constant
  - As $|\mathbf{I}_a|$ changes in response to $\mathbf{E}_a$ changing, the power factor changes

Phasor Diagrams

Each diagram has same real power developed, differing excitation



V Curves



Excitation (infinite bus)

- For synchronous motors connected infinite bus, adjusting excitation:
  - Does not affect real power consumption if power angle is zero, put reactive power will be affected
  - Increases real power consumption if power angle is held constant (non-zero)
  - May not affect real power consumption if power angle is adjusted such that $|\mathbf{E}_a|\sin\delta$ is constant, but reactive power will be affected

Power Effects

- How does varying the power output effect δ, $\mathbf{I}_a$, ϕ_p for a constant excitation $|\mathbf{E}_a|$ and terminal voltage $\mathbf{V}_a$?
- To increase power:
  - $|\mathbf{E}_a|\sin\delta$ increases $\quad P_o = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$
  - $|\mathbf{I}_a|\cos\phi_{PF}$ increases $\quad P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF}$
- Increasing P_o also increases power angle δ
- Real part of $\mathbf{I}_a$ also increases

Mechanical Power

- How do we change the real power developed?
- Power described by:

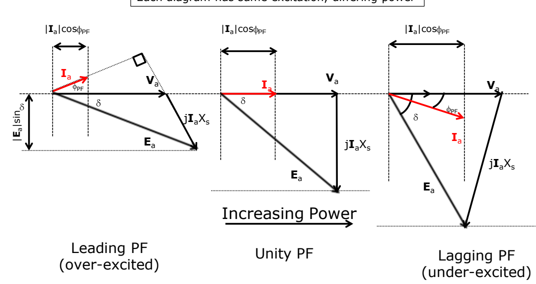
$$P_d = \frac{-3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$

$$P_{in} = 3|\mathbf{V}_a||\mathbf{I}_a|\cos(\phi_{PF}) \quad (P_d = P_{in} \text{ with } R_a \text{ ignored})$$

- If excitation is held constant, increasing the power developed by the motor results in:
  - Increase in power angle magnitude (more negative) δ
  - Increase in $|\mathbf{I}_a|\cos\phi_{PF}$ (real part of armature current)

Phasor Diagrams

Each diagram has same excitation, differing power



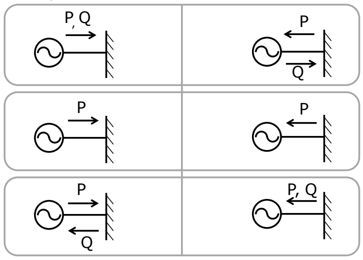
Synchronous Machines

Generator Operation
δ positive

Motor Operation
δ negative

over-excited

under-excited



Summary

- Synchronous motors are physically similar to synchronous generators
- Synchronous motors do not self-start without induction winding
- Armature current is minimum when it is in-phase with the terminal voltage
  - Unity power factor
- When under-excited ($|\mathbf{E}_a| < |\mathbf{V}_a|$) the motor behaves as an inductor (lagging power factor)
- When over-excited ($|\mathbf{E}_a| > |\mathbf{V}_a|$) the motor behaves as a capacitor (leading power factor)