33-Synchronous Generators Part 2

text: 7.12

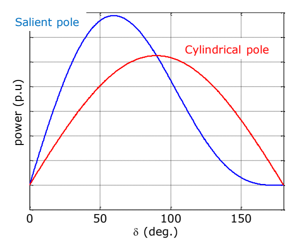
ECEGR 450

Electromechanical Energy Conversion

Overview

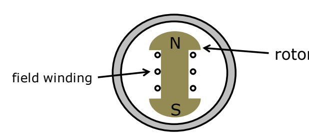
- Salient Pole Generator
- Equivalent Circuit
- Power Relationship
- External Characteristic

Power Relationship



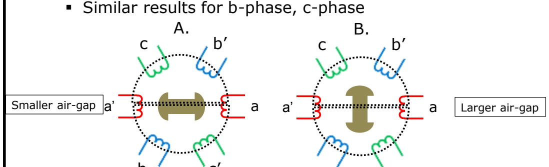
Salient Pole Generator

- Previous lecture assumed a round (cylindrical rotor)
  - Air gap remains uniform
  - Constant reluctance around periphery
- Salient pole generators
  - Significantly larger air gap between rotor poles



Salient Pole Generator

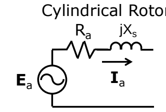
- Recall: $L = \frac{N^2}{\Re}$
- Reluctance of a-phase coil smaller in position A than in position B
  - Leakage reactance greater in position A
  - Cannot model as single reactance
  - Similar results for b-phase, c-phase



Equivalent Circuit

- Recall that X_s models leakage and armature reactance in cylindrical rotor generators
- Cylindrical rotor model will not suffice (X_s is not position dependent)
- $\mathbf{I}_a$ will experience varying reactances as the rotor rotates

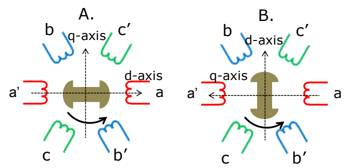
Cylindrical Rotor



$$\mathbf{E}_a = \mathbf{I}_a R_a + jX_s \mathbf{I}_a + \mathbf{V}_a$$

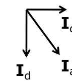
Equivalent Circuit

- Split the reactance into two components
  - Direct axis (d-axis) synchronous reactance: X_d
  - Quadrature axis (q-axis) synchronous reactance: X_q
- q-axis leads d-axis by 90 degrees



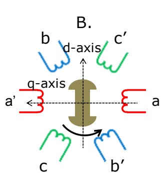
Equivalent Circuit

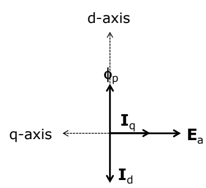
- Armature current is resolved into two components: $\mathbf{I}_a = \mathbf{I}_d + \mathbf{I}_q$
  - $\mathbf{I}_d$: direct armature current (A)
  - $\mathbf{I}_q$: quadrature armature current (A)
- $\mathbf{I}_d$ experiences X_d
  - Causes voltage drop: $j\mathbf{I}_d X_d$
- $\mathbf{I}_q$ experiences X_q
  - Causes voltage drop: $j\mathbf{I}_q X_q$



Phasor Relationships

- ϕ_p in phase with d-axis
- $\mathbf{E}_a$ is induced by ϕ_p and lags it by 90 degrees: $e = -N\frac{d\phi}{dt}$
- $\mathbf{I}_d$ lags $\mathbf{E}_a$ by 90 degrees





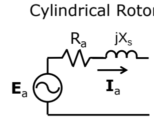
Equivalent Circuit

- We can now write the circuit equation:

$$\mathbf{E}_a = \mathbf{V}_a + \mathbf{I}_a R_a + \underbrace{j\mathbf{I}_d X_d + j\mathbf{I}_q X_q}$$

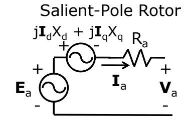
Voltage drop due to d-, q-axis leakage reactance. Model as voltage source.

Cylindrical Rotor



$$\mathbf{E}_a = \mathbf{I}_a R_a + jX_s\mathbf{I}_a + \mathbf{V}_a$$

Salient-Pole Rotor



$$\mathbf{E}_a = \mathbf{I}_a R_a + jX_d\mathbf{I}_d + jX_q\mathbf{I}_q + \mathbf{V}_a$$

Equivalent Circuit

- We want to express the relationship using armature current $\mathbf{I}_a$

$$\mathbf{E}_a = \mathbf{V}_a + \mathbf{I}_a R_a + j\mathbf{I}_d X_d + j\mathbf{I}_q X_q$$

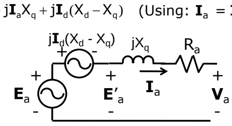
- We can express:

$$j\mathbf{I}_d X_d = j\mathbf{I}_d X_q + j\mathbf{I}_d (X_d - X_q)$$

- Through substitution

$$\mathbf{E}_a = \mathbf{V}_a + \mathbf{I}_a R_a + j\mathbf{I}_d X_q + j\mathbf{I}_d (X_d - X_q) + j\mathbf{I}_q X_q$$

$$\mathbf{E}_a = \mathbf{V}_a + \mathbf{I}_a R_a + j\mathbf{I}_a X_q + j\mathbf{I}_d (X_d - X_q) \quad \text{(Using: } \mathbf{I}_a = \mathbf{I}_d + \mathbf{I}_q\text{)}$$



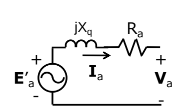
Equivalent Circuit

- Let:

  $\mathbf{E}'_a \triangleq \mathbf{E}_a - j\mathbf{I}_d(X_d - X_q)$ (effective induced voltage)
- Circuit equation:

$$\mathbf{E}'_a = \mathbf{V}_a + \mathbf{I}_a R_a + j\mathbf{I}_a X_q$$

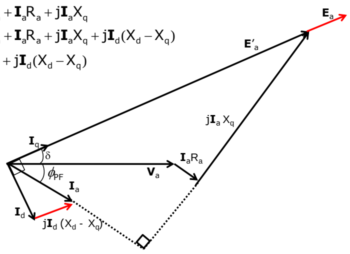
- Note:
  - $\mathbf{E}'_a$ and $\mathbf{E}_a$ are in phase
  - Phase difference between $\mathbf{E}'_a$ and $\mathbf{E}_a$ with $\mathbf{V}_a$ is δ



Equivalent Circuit

- Phasor diagram with lagging power factor

$$\mathbf{E}_a' = \mathbf{V}_a + \mathbf{I}_a R_a + j\mathbf{I}_a X_q$$
$$\mathbf{E}_a = \mathbf{V}_a + \mathbf{I}_a R_a + j\mathbf{I}_a X_q + j\mathbf{I}_d (X_d - X_q)$$
$$\mathbf{E}_a = \mathbf{E}_a' + j\mathbf{I}_d (X_d - X_q)$$



Salient Pole Generator

- Power angle can be computed from:

$$\tan\delta = \frac{|\mathbf{I}_a| X_q \cos\phi_{PF} - |\mathbf{I}_a| R_a \sin\phi_{PF}}{|\mathbf{V}_a| + |\mathbf{I}_a|(R_a \cos\phi_{PF} + X_q \sin\phi_{PF})}$$ (Inspecting the phasor diagram)

- $\mathbf{I}_d$ and $\mathbf{I}_q$ are computed from $\mathbf{I}_a$:

$$\mathbf{I}_d = |\mathbf{I}_a| \sin(\delta + \phi_{PF}) \angle \delta - 90^\circ$$
$$\mathbf{I}_q = |\mathbf{I}_a| \cos(\delta + \phi_{PF}) \angle \delta$$

- Power output:

$$P_o = 3\mathrm{Re}\{\mathbf{V}_a \mathbf{I}_a^*\} = 3\mathrm{Re}\{\mathbf{V}_a(\mathbf{I}_d^* + \mathbf{I}_q^*)\} = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF}$$

Approximate Power Relationship

- Ignoring armature resistance:

$$|\mathbf{I}_q| = \frac{|\mathbf{V}_a|\sin\delta}{X_q}$$
$$|\mathbf{I}_d| = \frac{|\mathbf{E}_a| - |\mathbf{V}_a|\cos\delta}{X_d}$$
$$\tan\delta = \frac{|\mathbf{I}_a| X_q \cos\phi_{PF}}{|\mathbf{V}_a| + |\mathbf{I}_a| X_q \sin\phi_{PF}}$$

Example

A salient-pole generator supplies 100MW at a power factor of 0.90 leading. The generator's line-line terminal voltage is 12kV and its parameters are X_d = 1Ω, X_q = 0.75Ω.

Compute the power angle and the per-phase armature current.

Example

A salient-pole generator supplies 100MW at a power factor of 0.90 leading. The generator's line-line terminal voltage is 12kV and its parameters are X_d = 1Ω, X_q = 0.75Ω.

$$\mathbf{V}_a = \frac{12000}{\sqrt{3}} = 6928\angle 0^\circ \text{V}$$
$$\phi_{PF} = -\cos^{-1}(0.90) = -25.8^\circ$$
$$\mathbf{S}_a = \frac{100}{3\times 0.9}\angle -25.8^\circ = 37.04\angle -25.8^\circ \text{MVA}$$
$$\mathbf{I}_a = \left(\frac{\mathbf{S}_a}{\mathbf{V}_a}\right)^* = 534.6\angle 25.8^\circ \text{A}$$
$$\tan\delta = \frac{|\mathbf{I}_a| X_q \cos\phi_{PF}}{|\mathbf{V}_a| + |\mathbf{I}_a| X_q \sin\phi_{PF}} = 0.389$$
$$\delta = 21.29^\circ$$

Approximate Power Relationship

- Approximate power output:

$$P_o = 3\mathrm{Re}\{\mathbf{V}_a \mathbf{I}_a^*\} = 3|\mathbf{V}_a|\mathrm{Re}\{(\mathbf{I}_d^* + \mathbf{I}_q^*)\}$$

Substitue for current using:

$$\mathbf{I}_d = |\mathbf{I}_d| \angle \delta - 90^\circ = |\mathbf{I}_d|\sin\delta - j|\mathbf{I}_d|\cos\delta$$
$$\mathbf{I}_d^* = |\mathbf{I}_d|\sin\delta - j|\mathbf{I}_d|\cos\delta$$
$$\mathbf{I}_q = |\mathbf{I}_q|\angle\delta = |\mathbf{I}_q|\cos\delta + j\mathbf{I}_q|\sin\delta$$
$$\mathbf{I}_q^* = |\mathbf{I}_q|\cos\delta - j|\mathbf{I}_q|\sin\delta$$

Recall:

$$|\mathbf{I}_q| = \frac{|\mathbf{V}_a|\sin\delta}{X_q}$$ (in phase with $\mathbf{E}_a$)

$$|\mathbf{I}_d| = \frac{|\mathbf{E}_a| - |\mathbf{V}_a|\cos\delta}{X_d}$$ (lags $\mathbf{E}_a$ by 90°)

Therefore:

$$P_o = 3|\mathbf{V}_a|(|\mathbf{I}_q|\cos(\delta) + |\mathbf{I}_d|\sin(\delta))$$
$$= 3|\mathbf{V}_a|\left(\frac{|\mathbf{V}_a|\sin(\delta)\cos(\delta)}{X_q} + \frac{|\mathbf{E}_a|\sin(\delta)}{X_d} - \frac{|\mathbf{V}_a|\cos(\delta)\sin(\delta)}{X_d}\right)$$

Approximate Power Relationship

- Continuing:

$$P_o = 3|\mathbf{V}_a|\left(\frac{|\mathbf{V}_a|\sin(\delta)\cos(\delta)}{X_q} + \frac{|\mathbf{E}_a|\sin(\delta)}{X_d} - \frac{|\mathbf{V}_a|\cos(\delta)\sin(\delta)}{X_d}\right)$$

$$= \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin(\delta)}{X_d} + 3|\mathbf{V}_a|\left(\frac{|\mathbf{V}_a|\sin(2\delta)}{2X_q} - \frac{|\mathbf{V}_a|\sin(2\delta)}{2X_d}\right)$$

$$= \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin(\delta)}{X_d} + 3|\mathbf{V}_a|\left(\frac{X_d|\mathbf{V}_a|\sin(2\delta) - X_q|\mathbf{V}_a|\sin(2\delta)}{2X_dX_q}\right)$$

$$= \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin(\delta)}{X_d} + \frac{3(X_d - X_q)|\mathbf{V}_a|^2\sin(2\delta)}{2X_dX_q}$$

Important result

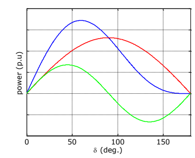
Approximate Power Relationship

- Approximate power developed:

$$P_d = \underbrace{\frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_d}}_{\text{power developed by cylindrical-rotor}} + \underbrace{\frac{3(X_d - X_q)}{2X_qX_d}|\mathbf{V}_a|^2\sin(2\delta)}_{\text{effect of saliency}}$$

- Approximate torque developed:

$$T_d = \underbrace{\frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_d\omega_s}}_{\text{torque developed by cylindrical-rotor}} + \underbrace{\frac{3(X_d - X_q)}{2X_qX_d\omega_s}|\mathbf{V}_a|^2\sin(2\delta)}_{\text{effect of saliency}}$$



Salient pole: greater power and torque for δ less than 90 deg.

Note: since R_a is ignored, power developed = power output

Power Relationship

If the field current is zero, but the generator is synchronized to the grid (terminal voltage is constant), does a salient-pole generator rotate?

$$P_d = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_d} + \frac{3(X_d - X_q)}{2X_qX_d}|\mathbf{V}_a|^2\sin(2\delta)$$

$$T_d = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_d\omega_s} + \frac{3(X_d - X_q)}{2X_qX_d\omega_s}|\mathbf{V}_a|^2\sin(2\delta)$$

Power Relationship

If the field current is zero, but the generator is synchronized to the grid (terminal voltage is constant), does a salient-pole generator rotate?

Yes. $\mathbf{I}_a$ is negative (from grid into generator). δ will be negative since $|\mathbf{V}_a| > |\mathbf{I}_a|X_q\sin\phi_{PF}$. Negative δ implies developed power is negative, hence the generator acts as a motor, even without excitation.

$$\tan\delta = \frac{|\mathbf{I}_a|X_q\cos\phi_{PF}}{|\mathbf{V}_a| + |\mathbf{I}_a|X_q\sin\phi_{PF}}$$

$$P_d = \frac{3|\mathbf{V}_a||0|\sin\delta}{X_d} + \frac{3(X_d - X_q)}{2X_qX_d}|\mathbf{V}_a|^2\sin(2\delta) \neq 0$$

Summary

- Salient pole generators provide greater power and torque at power angles less than 90 degrees
- Non-uniformity in reluctance requires decoupling of model into d-axis and q-axis
- Power and torque developed due to excitation and saliency of rotor