32-Synchronous Generators Part 1

text: 7.7 to 7.9

ECEGR 450
Electromechanical Energy Conversion

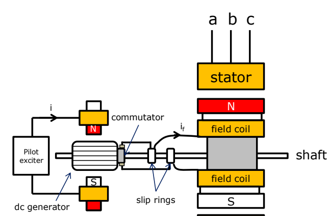
Overview

- Excitation
- Induced frequency
- Induced EMF
- Equivalent Circuit
- Armature Reaction
- Power Relationship
- Approximate Power Relationship

Excitation

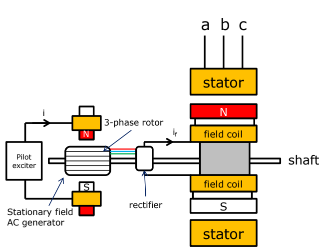
- Synchronous generators (motors) require revolving magnetic field
  - Permanent magnet
  - Field winding (dc)
- Exciter: supplies current to field winding (i_f)
  - DC generator
  - Brushless generator
  - Power rating: <3% of generator rating
- Field current is related to ϕ_p by k_f

Excitation



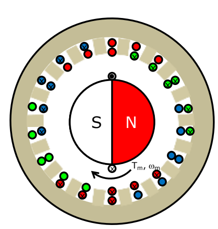
Voltage: 125 to 600VDC
Automatically controlled
(terminal voltage magnitude, reactive power)

Brushless Excitation



Induced Frequency

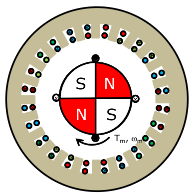
- 2-pole synchronous generator
- Balanced three-phase voltage induced



Two-pole machine:
electrical frequency = mechanical frequency

Induced Frequency

- 4-pole synchronous generator
- Each coil "sees" two Norths and two Souths per rotation
  - Two electrical sinewaves for each mechanical rotation
- In general: $f = \frac{f_m P}{2} = \frac{N_m P}{120}$



Induced EMF

- We next examine the induced emf in a synchronous generator
- Flux linking a single stator coil (ϕ_c):

$\phi_c = \phi_p k_p \cos(\omega t)$

- Induced voltage in an N_c turn coil is:

$e_c = N_c \phi_p \omega k_p \sin(\omega t)$

- Maximum induced voltage:

$E_m = N_c k_p \phi_p \omega$

Induced EMF

- RMS value of the induced emf:

$E_m = N_c k_p \phi_p \omega$

$|\mathbf{E}_c| = \frac{1}{\sqrt{2}} E_m = 4.44 f N_c k_p \phi_p$

$f = \frac{N_m \times P}{120}$

- Induced voltage in a phase group, accounting for the number of coils in series, pitch factor and the distribution factor, is:

$|\mathbf{E}_{pg}| = n k_d E_c = 4.44 n N_c k_p k_d f \phi_p$

$|\mathbf{E}_{pg}| = 4.44 n N_c k_w f \phi_p$

$k_w \triangleq k_p k_d$ (winding factor)

Induced EMF

- If a generator has "a" parallel paths and P poles, then the emf per phase is:

$|\mathbf{E}_a| = \frac{P}{a} 4.44 n N_c k_w f \phi_p$

$N_e \triangleq \frac{P n N_c k_w}{a}$

  - N_e: effective turns per phase
- We can then write:

$|\mathbf{E}_a| = 4.44 N_e f \phi_p$

Example

Consider a 16-pole, 144-slot, three phase Y-connected synchronous generator. Each coil has 10 turns, and the flux per pole is 0.025Wb. The rotor rotates at 375 rpm. The pitch factor is 0.94, and the distribution factor is 0.96. There are two parallel paths.

Compute:

frequency of the induced voltage

RMS value of the per-phase voltage

RMS value of the line-line voltage

Example

Consider a 16-pole, 144-slot, three phase Y-connected synchronous generator. Each coil has 10 turns, and the flux per pole is 0.025Wb. The rotor rotates at 375 rpm. The pitch factor is 0.94, and the distribution factor is 0.96. There are two parallel paths.

Induced frequency:

$f = \frac{375 \times 16}{120} = 50$ Hz

Example

Consider a 16-pole, 144-slot, three phase Y-connected synchronous generator. Each coil has 10 turns, and the flux per pole is 0.025Wb. The rotor rotates at 375 rpm. The pitch factor is 0.94, and the distribution factor is 0.96. There are two parallel paths.

Computing generator parameters:

$$k_w = k_p k_d = 0.902$$

$$n = \frac{144}{16 \times 3} = 3 \text{ coils/pole/phase}$$

Example

Consider a 16-pole, 144-slot, three phase Y-connected synchronous generator. Each coil has 10 turns, and the flux per pole is 0.025Wb. The rotor rotates at 375 rpm. The pitch factor is 0.94, and the distribution factor is 0.96.

Induced RMS phase voltage:

$$N_e = \frac{PnN_c k_w}{a} = \frac{16 \times 3 \times 10 \times 0.902}{2} = 216.48$$

$$|\mathbf{E}_a| = 4.44 f N_e \phi_p = 1201.5\text{V}$$

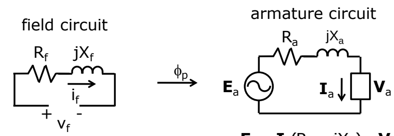
$$|\mathbf{E}_L| = 1201.5 \times \sqrt{3} = 2081\text{V}$$

Equivalent Circuit

- Generator terminal voltage ($\mathbf{V}_a$) of a synchronous generator depends upon the load
  - Terminal voltage may be greater or smaller than induced emf
  - Will be higher when the power factor is leading
  - Assumes generator is not grid-connected
- Terminal voltage is affected by:
  - Armature resistance voltage drop
  - Armature leakage reactance voltage drop
  - Armature reaction

Equivalent Circuit

- Equivalent circuit ignoring armature reaction
  - R_a: per-phase armature resistance (Ohm)
  - X_a: armature leakage reactance (Ohm)

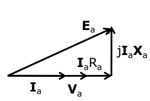


$$\mathbf{E}_a = \mathbf{I}_a(R_a + jX_a) + \mathbf{V}_a$$

Equivalent Circuit

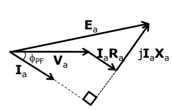
Phasor diagrams (compare magnitude of $\mathbf{E}_a$, $\mathbf{V}_a$)

$$\mathbf{E}_a = \mathbf{I}_a(R_a + jX_a) + \mathbf{V}_a$$



Unity power factor

$\mathbf{V}_a$: reference
$\mathbf{I}_a$: in phase $\mathbf{V}_a$ (unity PF)
$\mathbf{I}_a R_a$: in phase with $\mathbf{I}_a$
$j\mathbf{I}_a X_a$: 90° out of phase from $\mathbf{I}_a$
$\mathbf{E}_a > \mathbf{V}_a$

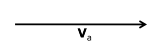


Lagging power factor

$\mathbf{V}_a$: reference
$\mathbf{I}_a$: lags $\mathbf{V}_a$ (by ϕ_{PF})
$\mathbf{I}_a R_a$: in phase with $\mathbf{I}_a$
$j\mathbf{I}_a X_a$: 90° out of phase from $\mathbf{I}_a$
$E_a > V_a$

Equivalent Circuit

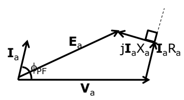
Draw the phasor diagram for a synchronous generator with a leading PF



Leading power factor

Equivalent Circuit

Draw the equivalent circuit for a leading PF



Leading power factor

$\mathbf{V}_a$: reference
$\mathbf{I}_a$: leads $\mathbf{V}_a$ (by ϕ_{PF})
$\mathbf{I}_a R_a$: in phase with $\mathbf{I}_a$
$j\mathbf{I}_a X_a$: 90° out of phase from $\mathbf{I}_a$
$\mathbf{E}_a < \mathbf{V}_a$

Possible for induced voltage to be larger than terminal voltage

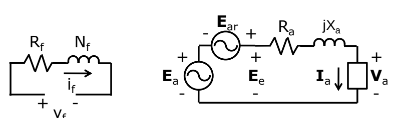
Armature Reaction

Consider a load with unity power factor

- $\mathbf{E}_a$ lags ϕ_p by 90 degrees
- $\mathbf{I}_a$ is in phase with $\mathbf{V}_a$
- $\phi_e = \phi_p + \phi_{ar}$
  - ϕ_e: effective flux per pole
  - ϕ_{ar}: armature reaction flux per pole
- ϕ_e induces an emf the armature winding
  - $\mathbf{E}_{ar}$ armature reaction emf
  - $\mathbf{E}_{ar}$ lags ϕ_{ar} by 90 degrees
- $\mathbf{E}_e = \mathbf{E}_a + \mathbf{E}_{ar}$
  - Effective per-phase emf
- $\mathbf{E}_e = \mathbf{V}_a + \mathbf{I}_a(R_a + jX_a)$

Armature Reaction

- Armature reaction can be modeled by placing a voltage source in series with the induced emf
- Equivalent circuit

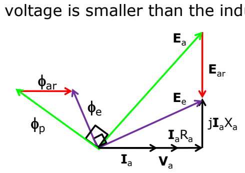


$$\mathbf{E}_a = -\mathbf{E}_{ar} + \mathbf{I}_a(R_a + jX_a) + \mathbf{V}_a$$
$$\mathbf{E}_e = \mathbf{E}_a + \mathbf{E}_{ar}$$

Armature Reaction

- Phasor Diagram (unity PF load)
  - Armature reaction reduces the effective flux per pole
  - Terminal voltage is smaller than the induced voltage

$\phi_e = \phi_p + \phi_{ar}$



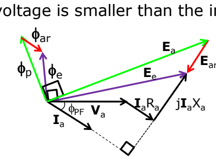
Unity power factor

$$\mathbf{E}_a = -\mathbf{E}_{ar} + \mathbf{E}_e = -\mathbf{E}_{ar} + \mathbf{I}_a(R_a + jX_a) + \mathbf{V}_a$$
$$\mathbf{E}_e = \mathbf{E}_a + \mathbf{E}_{ar}$$

Armature Reaction

- Phasor Diagram (lagging PF load)
  - Armature reaction reduces the effective flux per pole
  - Terminal voltage is smaller than the induced voltage

$\phi_e = \phi_p + \phi_{ar}$



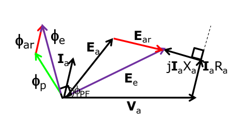
Lagging power factor

$$\mathbf{E}_a = -\mathbf{E}_{ar} + \mathbf{E}_e = -\mathbf{E}_{ar} + \mathbf{I}_a(R_a + jX_a) + \mathbf{V}_a$$
$$\mathbf{E}_e = \mathbf{E}_a + \mathbf{E}_{ar}$$

Armature Reaction

- Phasor Diagram (leading PF load)
  - Armature reaction <u>increases</u> the effective flux per pole
  - Terminal voltage is <u>greater</u> than the induced voltage

$\phi_e = \phi_p + \phi_{ar}$



Leading power factor

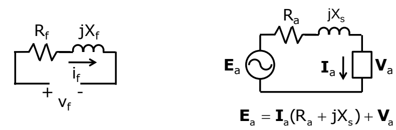
$$\mathbf{E}_a = -\mathbf{E}_{ar} + \mathbf{E}_e = -\mathbf{E}_{ar} + \mathbf{I}_a(R_a + jX_a) + \mathbf{V}_a$$
$$\mathbf{E}_e = \mathbf{E}_a + \mathbf{E}_{ar}$$

Armature Reaction

- Note: i_f must be adjusted with changing load to keep terminal voltage constant
- Armature reaction can be expressed as a magnetizing reactance
  - $\mathbf{E}_{ar} = -j\mathbf{I}_a X_m$ (emf uses active sign convetion)
- X_m and X_a can be combined together as the synchronous reactance, X_s
  - $X_s = X_m + X_a$
- Synchronous impedance of the generator is
  - $\mathbf{Z}_s = R_a + jX_s$

Equivalent Circuit

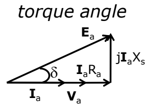
- Per phase equivalent circuit:
  - X_s is used instead of $\mathbf{E}_{ar}$, X_a



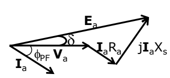
$$\mathbf{E}_a = \mathbf{I}_a(R_a + jX_s) + \mathbf{V}_a$$

Equivalent Circuit

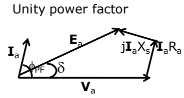
- Phasor diagrams of new per-phase circuit
  - δ: angle between $\mathbf{E}_a$ and $\mathbf{V}_a$ (induced voltage and terminal voltage), known as the *power angle* or *torque angle*



Unity power factor



Lagging power factor

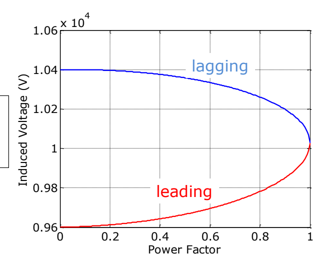


Leading power factor

δ is measured from $\mathbf{V}_a$ to $\mathbf{E}_a$
δ is positive for generators

Equivalent Circuit

$\mathbf{E}_a$ as a function of power factor. Terminal voltage held constant at 10kV.



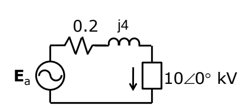
Example

A synchronous generator has a per-phase synchronous impedance of 0.2 + j4. The generator supplies a per-phase load current of 100A at a lagging power factor of 0.866 lagging. The per-phase terminal voltage is 10kV.

Compute the per-phase induced voltage.
Compute the power angle.

Example

- Per phase armature current:
  $\mathbf{I}_a = 100\angle -30° \text{ A}$
- Solving the circuit:
  $\mathbf{E}_a = 10.2\angle 1.8° \text{ kV}$
- Power angle: 1.8 degrees



$$\mathbf{E}_a = \mathbf{I}_a(R_a + jX_s) + \mathbf{V}_a$$

Voltage Regulation

- Similar to dc generators, the voltage regulation of a synchronous generator is:

$$VR = \frac{|\mathbf{E}_a| - |\mathbf{V}_a|}{|\mathbf{V}_a|} \times 100$$

  - $\mathbf{E}_a$: induced emf, also the no-load terminal voltage
  - $\mathbf{V}_a$: terminal voltage at full load (V)

Power Relationships

- Mechanical power supplied to the shaft of a synchronous generator by the prime mover
  - steam turbine
  - combustion turbine
  - dc motor
  - others

Power Relationships

- Mechanical power in:

$$P_{inm} = T_s \omega_s$$

  - T_s: shaft torque (Nm)
  - ω_s: shaft speed (rad/s)
- Total power in:

$$P_{in} = T_s \omega_s + v_f i_f$$

- Electrical power out:

$$P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = 3\,\mathrm{Re}\{\mathbf{V}_a \mathbf{I}_a^*\}$$

- Copper losses:

$$P_{cu} = 3|\mathbf{I}_a|^2 R_a$$

Power Relationships

- Power output: $P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = 3\,\mathrm{Re}\{\mathbf{V}_a \mathbf{I}_a^*\}$
  - Requires knowledge (usually computation) of armature current
- Desired to have an equivalent expression of generator power output without having to compute armature current

Power Expressions

- From the equivalent circuit:

$$\mathbf{I}_a = \frac{\mathbf{E}_a - \mathbf{V}_a}{R_a + jX_s} = \frac{\mathbf{E}_a - \mathbf{V}_a}{\mathbf{Z}_s}$$

- Power developed:

$$P_o = 3\,\mathrm{Re}\{\mathbf{V}_a \mathbf{I}_a^*\} = 3\,\mathrm{Re}\{\frac{\mathbf{V}_a \mathbf{E}_a^* - |\mathbf{V}_a|^2}{\mathbf{Z}_s^*}\}$$

$$= 3\,\mathrm{Re}\{\frac{\mathbf{V}_a \mathbf{E}_a^* \mathbf{Z}_s}{|\mathbf{Z}_s|^2} - \frac{|\mathbf{V}_a|^2 \mathbf{Z}_s}{|\mathbf{Z}_s|^2}\} = 3\,\mathrm{Re}\{\frac{\mathbf{V}_a \mathbf{E}_a^* \mathbf{Z}_s}{|\mathbf{Z}_s|^2} - \frac{|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2} - j\frac{|\mathbf{V}_a|^2 X_s}{|\mathbf{Z}_s|^2}\}$$

  - Above expansion uses:

$$\mathbf{Z}_s = |\mathbf{Z}_s| \angle \theta_z$$

$$\mathbf{Z}_s^* = |\mathbf{Z}_s| \angle -\theta_z$$

$$\frac{1}{\mathbf{Z}_s^*} = \frac{1}{\mathbf{Z}_s^*}\frac{\mathbf{Z}_s^*}{\mathbf{Z}_s^*} = \frac{|\mathbf{Z}_s|\angle -\theta_z}{|\mathbf{Z}_s|^2 \angle -2\theta_z} = \frac{|\mathbf{Z}_s|\angle \theta_z}{|\mathbf{Z}_s|^2} = \frac{\mathbf{Z}_s}{|\mathbf{Z}_s|^2}$$

Recall that dividing by a phasor means dividing by the magnitude and subtracting the angle

Power Expressions

Continuing:

$$P_d = 3\,\mathrm{Re}\{\frac{\mathbf{V}_a \mathbf{E}_a^* \mathbf{Z}_s}{|\mathbf{Z}_s|^2} - \frac{|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2} - j\frac{|\mathbf{V}_a|^2 X_s}{|\mathbf{Z}_s|^2}\}$$

$$= 3\,\mathrm{Re}\{\frac{\mathbf{V}_a \mathbf{E}_a^* \mathbf{Z}_s}{|\mathbf{Z}_s|^2}\} - \frac{|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$= 3\,\mathrm{Re}\{\frac{\mathbf{V}_a \mathbf{E}_a^* (R_a + jX_s)}{|\mathbf{Z}_s|^2}\} - \frac{|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$= 3\,\mathrm{Re}\{\frac{|\mathbf{V}|_a|\mathbf{E}_a|(\cos\delta - j\sin\delta)(R_a + jX_s)}{|\mathbf{Z}_s|^2}\} - \frac{|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$= \frac{3|\mathbf{V}|_a|\mathbf{E}_a|}{|\mathbf{Z}_s|^2}(R_a\cos\delta + X_s\sin\delta) - \frac{|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

Important result

Note:

$\mathbf{E}_a = |\mathbf{E}_a| \angle \delta$

$\mathbf{V}_a = |\mathbf{V}_a| \angle 0° = |\mathbf{V}_a|$

Power Relationships

- Power balance equation:

$$P_{in} = T_s\omega_s + i_f v_f = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} + 3|\mathbf{I}_a|^2 R_a + i_f v_f + P_r + P_{sl}$$

  - P_r: rotational losses (W)
  - P_{sl}: stray load losses (W)
- Constant losses grouped as:

$$P_c = i_f v_f + P_r + P_{sl}$$

Power Relationship

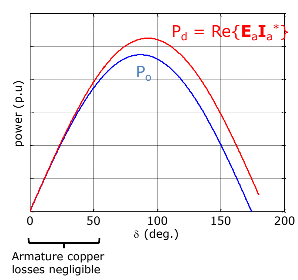
- Generator efficiency:

$$\eta = \frac{3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF}}{3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} + 3|\mathbf{I}_a|^2 R_a + P_c}$$

- For maximum efficiency:

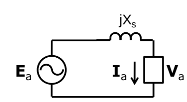
$$3|\mathbf{I}_a|^2 R_a = P_c$$

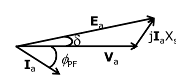
Power Relationship



Approximate Power Relationship

- Armature resistance is small
- Common to ignore it





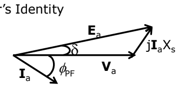
Example lagging PF load

Approximate Power Relationship

Computing the real power output:

$$\mathbf{E}_a = |\mathbf{E}_a|\angle\delta = |\mathbf{E}_a|\cos\delta + j|\mathbf{E}_a|\sin\delta$$
$$\mathbf{I}_a = |\mathbf{I}_a|\angle -\phi_{PF} = |\mathbf{I}_a|\cos\phi_{PF} - j|\mathbf{I}_a|\sin\phi_{PF}$$
$$\mathbf{V}_a = |\mathbf{V}_a|\angle 0 = |\mathbf{V}_a| + j0$$

} Euler's Identity



Example lagging PF load

$\mathbf{V}_a = \mathbf{E}_a - j\mathbf{I}_a X_s$

$$\mathbf{I}_a = \frac{\mathbf{E}_a - \mathbf{V}_a}{jX_s} = \frac{|\mathbf{E}_a|\cos\delta - |\mathbf{V}_a|}{jX_s} + \frac{j|\mathbf{E}_a|\sin\delta - 0}{jX_s}$$
$$= \frac{|\mathbf{E}_a|\sin\delta}{X_s} - j\frac{|\mathbf{E}_a|\cos\delta - |\mathbf{V}_a|}{X_s}$$

$$|\mathbf{I}_a|\cos\phi_{PF} = \frac{|\mathbf{E}_a|\sin\delta}{X_s} \text{ (equating real parts)}$$

$$P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$

Important result

Approximate Power Relationship

- Synchronous generator power output (approximate)

$$P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$

- Assumes:
  - Armature resistance is zero
  - Constant speed
  - Constant field current
  - Cylindrical rotor

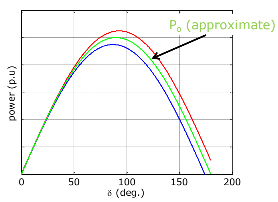
Approximate Power Relationship

- Power-angle relationship:

$$P_o = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$

- Maximum power:

$$P_{dm} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|}{X_s}$$



Power Relationship

- Torque developed (approximate):

$$T_d = \frac{P_d}{\omega_s} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$

- Maximum torque (approximate):

$$T_{dm} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|}{X_s\omega_s}$$

- Maximum power and torque occur at $\delta = 90^o$

Example

A 2-pole synchronous generator has a per-phase terminal voltage of 7.5 kV, a per-phase induced voltage of 7.9 kV and a synchronous reactance of 1Ω. If the power angle is 15 degrees, compute the total real power delivered to the load. Assume the rotational losses are 1MW.

Example

A 2-pole synchronous generator has a per-phase terminal voltage of 7.5 kV, a per-phase induced voltage of 7.9 kV and a synchronous reactance of 1Ω. If the power angle is 15 degrees, compute the total real power delivered to the load. Assume the rotational losses are 1MW.

$$P_o = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s} = 46\text{MW}$$

Rotational losses are not electric, so we do not need to subtract them.

Power Expressions

Several different forms of round-rotor power output:

$$P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF}$$
$$= 3\text{Re}\{\mathbf{V}_a\mathbf{I}_a^*\}$$
$$= \frac{3|\mathbf{E}_a||\mathbf{V}_a|}{|\mathbf{Z}_s|^2}(R_a\cos\delta + X_s\sin\delta) - \frac{3|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

$$P_o = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$ (valid only if R_a can be ignored)

Power Relationship Summary

$P_{in} = T_s\omega_s + v_f i_f$ (total input power)

→ $v_f i_f$ (field winding loss)

$P_{inm} = T_s\omega_s$ (input mechanical power)

→ $P_r + P_{sl}$ (rotational and stray load loss)

$P_d = 3\text{Re}\{\mathbf{E}_a\mathbf{I}_a^*\}$ (developed electrical power)

→ $P_{cu} = 3|\mathbf{I}_a^2|R_a$ (copper loss in armature)

$P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = 3\text{Re}\{\mathbf{V}_a\mathbf{I}_a^*\}$ (output electrical power)

$$= \frac{3|\mathbf{E}_a||\mathbf{V}_a|}{|\mathbf{Z}_s|^2}(R_a\cos\delta + X_s\sin\delta) - \frac{3|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2}$$

Power Relationship Example

Let:
$v_f = 400V$
$i_f = 250A$
$P_r + P_{sl} = 2MW$
$\mathbf{Z}_s = 0.2 + j4\Omega$
$\delta = 30°$
$\mathbf{V}_a = 10kV$
$|\mathbf{E}_a| = 11kV$

$P_{in} = T_s\omega_s + v_f i_f = 44.21MW$

→ $V_f i_f = 0.1MW$

$P_{inm} = T_s\omega_s = 44.11MW$

→ $P_r + P_{sl} = 2MW$

$P_d = 3\mathrm{Re}\{\mathbf{E}_a \mathbf{I}_a^*\} = 42.11MW$

→ $P_{cu} = 3|\mathbf{I}_a^2|R_a = 1.14MW$

$$P_o = \frac{3|\mathbf{E}_a||\mathbf{V}_a|}{|\mathbf{Z}_s|^2}(R_a\cos\delta + X_s\sin\delta) - \frac{3|\mathbf{V}_a|^2 R_a}{|\mathbf{Z}_s|^2} = 40.97MW$$

Summary

- Exciters are used to supply DC current to the rotor of synchronous generators
- Frequency of induced voltage increases with the number of poles for a fixed mechanical speed
- Leakage reactance and armature reaction can be combined into X_s, the synchronous reactance
- Approximate power delivered by a synchronous generator is:

$$P_o = 3|\mathbf{V}_a||\mathbf{I}_a|\cos\phi_{PF} = \frac{3|\mathbf{V}_a||\mathbf{E}_a|\sin\delta}{X_s}$$