30-Induction Machine Models

ECEGR 450
Electromechanical Energy Conversion

Overview

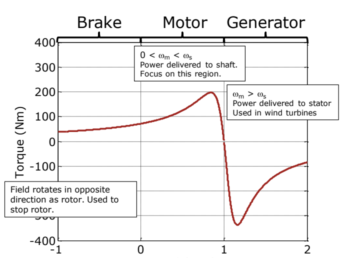
Questions

- Can an induction motor be used as an induction generator?
- What is the speed versus torque characteristic of an induction motor?
- What are the losses in induction motors?

Introduction

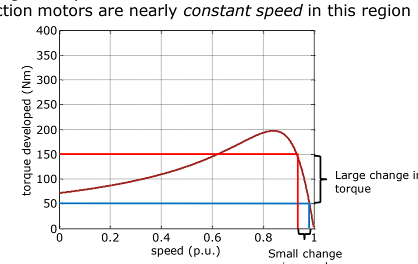
- We now analyze balanced three phase induction machines
- Focus is on Induction Motors (models are same for induction generators)

General Characteristics



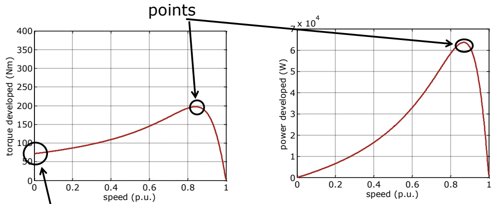
General Characteristics

Slip is generally low < 5%
Induction motors are nearly *constant speed* in this region



General Characteristics

Unique maximum torque and power operating points

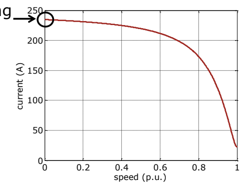


Non-zero starting torque

Recall:
power = torque x speed

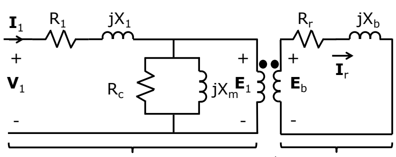
General Characteristics

High starting current



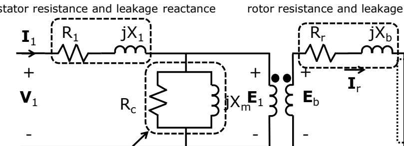
Blocked Rotor Model

- Consider an induction motor with a blocked rotor (rotor is mechanically prevented from moving)
  - s = 1
- Identical to a transformer
- Equivalent per-phase circuit (assuming Y rotor, Y stator)



Blocked Rotor Model

$R_r << X_b$

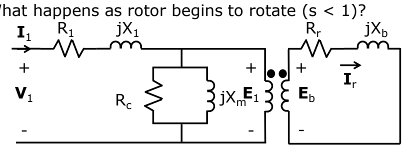


Blocked Rotor Model

- Secondary (rotor) circuit is shorted (unless wound rotor)
  - Large stator current will flow under blocked rotor or start-up conditions

$$\mathbf{I}_r = \frac{\mathbf{E}_b}{R_r + jX_b}$$

- What happens as rotor begins to rotate (s < 1)?



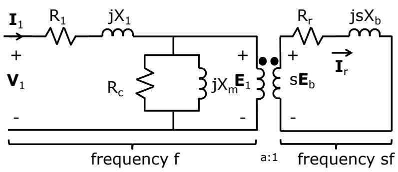
Induction Motor Model

- Behavior is now like a rotating transformer
  - Primary operates at frequency f
  - Secondary operates at frequency sf
- Secondary induced voltage is reduced
  - $\mathbf{E}_r = \mathbf{E}_b s$
- Secondary leakage reactance decreases
  - $jX_r = jX_b s$
- Real power is delivered to the mechanical load
  - Function of slip and rotor impedance
  - Desirable to model in circuit

Induction Motor Model

- Note: $\mathbf{V}_1$, $\mathbf{E}_1$ $\mathbf{I}_1$, and $\mathbf{E}_b$, $\mathbf{I}_r$ have different frequencies (f, sf)
- Solving secondary (rotor) circuit:

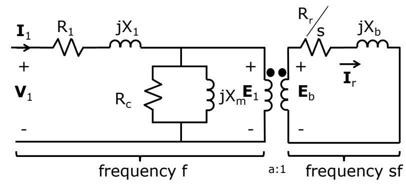
$$\mathbf{I}_r = \frac{s\mathbf{E}_b}{R_r + jsX_b}$$



Induction Motor Model

- Customary to write rotor current as:

$$\mathbf{I}_r = \frac{s\mathbf{E}_b}{R_r + jsX_b} = \frac{\mathbf{E}_b}{R_r/s + jX_b}$$ (divide numerator and denominator by s)

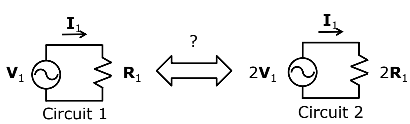


An Observation

- Examine the previous operation:

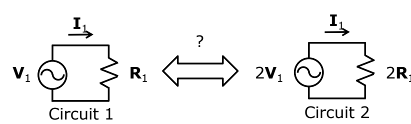
$$\mathbf{I}_r = \frac{s\mathbf{E}_b}{R_r + jsX_b} = \frac{\mathbf{E}_2}{R_r/s + jX_r}$$ (divide numerator and denominator by s)

- Current value is preserved, but are the circuits equivalent?
- Example:



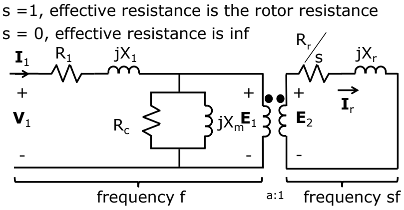
An Observation

- Current is the same, but power is not
  - Circuit 1: $P_1 = I_1^2 R_1$
  - Circuit 2: $P_2 = I_1^2 2R_1 \neq P_1$
- Division by s does not preserve power
- Since s < 1, the new equivalent circuit is consuming power in R_r/s <u>in addition to rotor copper loss</u>



Effective Resistance

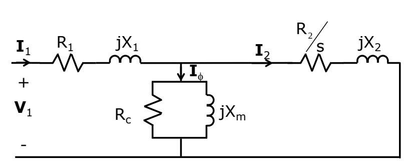
- R_r/s is the effective resistance
- Hypothetical, <u>accounts for rotor copper losses and mechanical power delivered to load</u>
- If s =1, effective resistance is the rotor resistance
- If s = 0, effective resistance is inf



Induction Motor Model

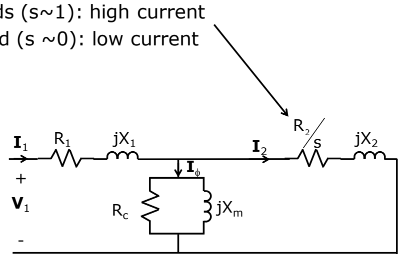
- More convenient to eliminate ideal xfmr in circuit model
- Refer secondary side to primary (stator)
- Transformation ratio: $a = \frac{N_1 k_{\omega 1}}{N_2 k_{\omega 2}}$ (assume $k_{\omega 1}$, $k_{\omega 2}$ are equal for now)
- Quantities become:

$$R_2 = a^2 R_r$$
$$X_2 = a^2 X_b$$
$$\mathbf{I}_2 = \frac{\mathbf{I}_r}{a}$$



Induction Motor Model

- Per-phase stator current: $\mathbf{I}_1 = \mathbf{I}_\phi + \mathbf{I}_2$
- Low speeds (s~1): high current
- High speed (s ~0): low current



Induction Motor Power

- Per-phase power delivered to the rotor: $P_{r,\phi} = \frac{|\mathbf{I}_2|^2 R_2}{s}$
  - Copper loss
  - Power developed (available to shaft)
- Per-phase copper loss: $P_{r,cu,\phi} = |\mathbf{I}_2|^2 R_2$
- Per-phase power developed:

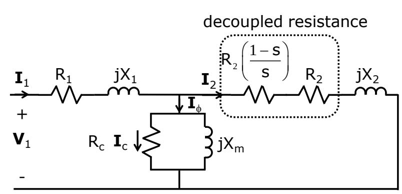
$$P_{d,\phi} = \underbrace{\frac{|\mathbf{I}_2|^2 R_2}{s}}_{P_{r,\phi}} - \underbrace{|\mathbf{I}_2|^2 R_2}_{P_{r,cu,\phi}} = |\mathbf{I}_2|^2 R_2 \left(\frac{1-s}{s}\right)$$

Effective Resistance

- Effective resistance can be decoupled into copper loss resistance, and resistance <u>representing</u> mechanical power

$$\frac{R_2}{s} = \underbrace{R_2}_{\text{rotor copper}} + \underbrace{R_2\left(\frac{1-s}{s}\right)}_{\text{load resistance}}$$

Effective Resistance



Induction Motor Power

- Total input power to induction motor: $P_{in} = 3\mathbf{V}_1\mathbf{I}_1 \cos\phi$
- Total stator copper loss: $P_{s,cu} = 3|\mathbf{I}_1|^2 R_1$
- Total stator core loss: $P_m = 3|\mathbf{I}_c|^2 R_c$
- Remaining power must be delivered to rotor (air-gap power):

$$P_{ag} = P_{in} - P_m - P_c = \frac{3|\mathbf{I}_2|^2 R_2}{s} = 3P_{r\phi}$$

Induction Motor Power

- Total rotor copper loss: $P_{r,cu} = 3P_{r,cu,\phi} = 3|\mathbf{I}_2|^2 R_2 = sP_{ag}$
  - Percentage of air gap power consumed by electrical losses is s
- Mechanical <u>power developed</u> by the motor:

$$P_d = P_{ag} - P_{r,cu} = \frac{3|\mathbf{I}_2|^2(1-s)R_2}{s} = (1-s)P_{ag}$$

- Subtracting rotational losses yields total <u>power output</u> by the motor: $P_{out} = P_d - P_{rot}$

Induction Motor Torque

- The total developed torque is:

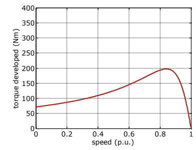
$$T_d = \frac{P_d}{\omega_m} = \frac{P_{ag}}{\omega_s} = 3\,|\mathbf{I}_2|^2 \frac{R_2}{s\omega_s}$$

- The total output torque is:

$$T_{out} = \frac{P_{out}}{\omega_m}$$

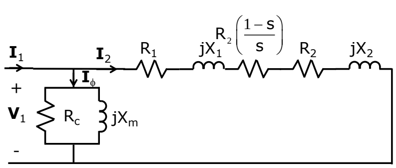
Induction Motor Torque

- Torque is non-zero at starting (s =1)
- Torque is zero at synchronous speed (s = 0)
  - $\mathbf{I}_2$ is zero



Approximate Circuit

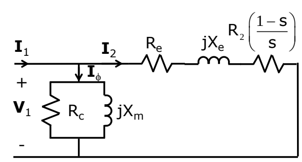
- R_1, X_1 are designed to be minimized
- R_c, X_m designed to be maximized
- Per-phase circuit can be simplified (approximated)



Approximate Circuit

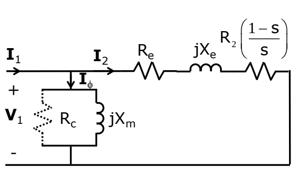
Move R_1, X_1 past shunt branch and combine with R_2, X_2

- $R_e = R_1 + R_2$
- $X_e = X_1 + X_2$



Approximate Circuit

- Further simplifications:
  - replace core resistance with constant power loss
  - Assume rotational losses are constant



Example

- A 60 Hz, 480-V, wye-connected induction motor has the following constants (referred to the stator)
  - R_1 = 0.322 Ω; R_2 = 0.196 Ω; X_1 =0.675 Ω; X_2 =0.510 Ω; X_m =12.5 Ω
  - Rotational and core losses: 1850 W
- The motor operates at a slip of 3% (3492 RPM). Compute the stator current and the power factor of the motor using the approximate model.

Example

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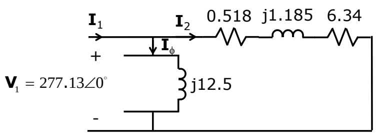


Example

- Compute stator current by KCL: $\mathbf{I}_1 = \mathbf{I}_\phi + \mathbf{I}_2$

$$\mathbf{I}_1 = \frac{277.13\angle 0^\circ}{12.5\angle 90^\circ} + \frac{277.13\angle 0^\circ}{6.858 + j1.185} = 22.17\angle -90^\circ + 39.83\angle -9.8^\circ$$

$$= 48.78\angle -36.42^\circ \text{ A}$$

- Compute the power factor:
  - PF: cos(-36.42^0) = 0.805 (lagging)

How does this compare if the exact circuit model is used?

Example

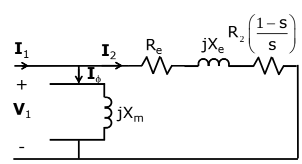
Consider the motor in the previous example.
Compute the input power and stator copper losses.

Example

- The stator and copper losses are:

$$P_{input} = 3|\mathbf{V}_1||\mathbf{I}_1|\cos\phi = 32,634 \text{ W}$$

$$P_{s,cu} = 3|\mathbf{I}_2|^2 R_1 = 1,533 \text{ W}$$



Example

Consider the motor in the previous example.
Compute the air gap power, power developed and power output of the motor.

Example

Consider the motor in the previous example.

$$P_{ag} = P_{in} - P_{s,cu} = 31,101 \text{ W}$$

$$P_d = (1-s)P_{ag} = 30,168 \text{ W}$$

Note: rotor copper losses are 933 W

$$P_{out} = P_d - P_{rot} = 28,318 \text{ W}$$

Efficiency: P_{out}/P_{in} = 86.77%

Example

Consider the motor in the previous example.
Compute the torque developed and the output torque.

Example

Consider the motor in the previous example.
Compute the torque developed and the output torque.

$$\omega_m = \frac{3492 \times 2\pi}{60} = 365.8 \text{ (rad/s)}$$

$$T_d = \frac{P_d}{\omega_m} = 82.49 \text{ Nm}$$

$$T_{out} = \frac{P_{out}}{\omega_m} = 77.43 \text{ Nm}$$

Summary

- Induction motors are nearly constant speed
- Non-linear torque, power and current vs speed relationship
- Induction machine model is based on transformer model
- $R_2\left(\frac{1-s}{s}\right)$ term is the load resistance, which represents mechanical output