21-Torque and Back EMF

ECEGR 450
Electromechanical Energy Conversion

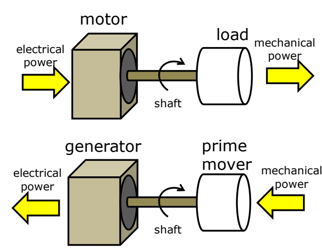
Overview

- Torque
- Power
- Angular Acceleration
- Rotational Dynamics
- Torque vs Speed
- Torque and Machine Size
- Back EMF

Questions

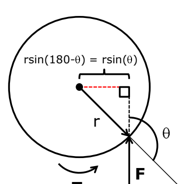
- What determines the speed an object will rotate at?
- Why do bicycles and automobiles have different gears?

Introduction



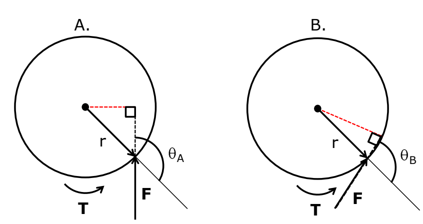
Torque

- Torque: tangential force times radial distance at which it is applied measured from axis of rotation
- T = Frsin(θ) (in Nm)



Torque

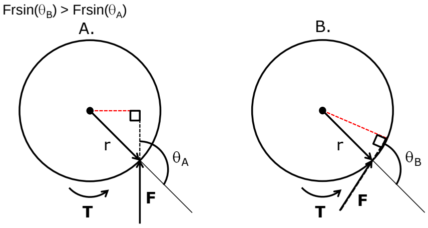
Which experiences greater torque?



Torque

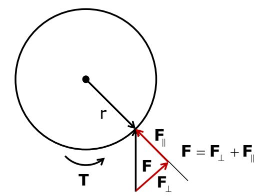
B. experiences greater torque

$F\sin(\theta_B) > F\sin(\theta_A)$



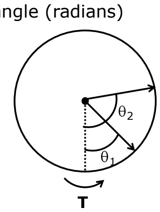
Alternative Forms

- **T** = **r** x **F** (cross product, use right hand rule for direction)
  - Thumb out of the paper CCW
  - Thumb into paper: CW
- Torque: tangential force multiplied by r: $T = rF_{\perp}$



Work

- Torque through an angle is work
- For constant torque: $W = T(\theta_2 - \theta_1)$ (Joules)
  - θ_1: starting angle (radians)
  - θ_2: ending angle (radians)



Power

- Recall that power is rate of work

$$P = \frac{dW}{dt} = T\frac{d\theta}{dt} = T\omega_m \quad \text{(watts)}$$

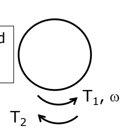
$$T = \frac{P}{\omega_m}$$

- where:
  - ω_m: angular velocity (rad/sec)

Angular Acceleration

- A shaft experiencing torque will accelerate according to:
  $\alpha = (T_1 - T_2)J$
- Where
  - α: angular acceleration (rad/s^2)
  - J: mass moment of inertia (kgm^2)

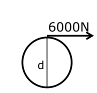
If $T_1 = T_2$: speed unchanged
If $T_1 < T_2$: slows down
If $T_1 > T_2$: speeds up



Example

A large reel of paper installed at the end of a paper machine has a diameter of 1.8m and a moment of inertia of 4500 kgm^2. It is coupled to a variable speed dc motor revolving at 120 rpm. The paper is kept under constant tension of 6000 N.

Compute:
the torque exerted on the reel
the power output by the motor



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Compute:

the torque exerted on the reel

$T = Fr = 6000 \times 0.9 = 5400$ Nm

the power output by the motor

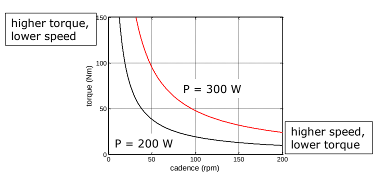
$P = 5400 \times 120 \times 2\pi/60 = 67.85$ kW

Torque vs Speed

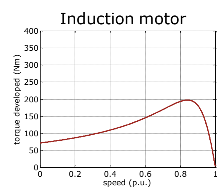
- The power, torque and speed relationship is of interest to electric motors and generators
- Governs rotational dynamics of system
- Different machines have different torque vs speed relationships for a given power level

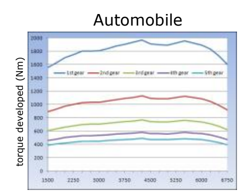
Torque vs Speed

- Recall: $P = T\omega$
  - $T = P/\omega$
- For a <u>hypothetical</u> constant power machine:



Torque vs Speed

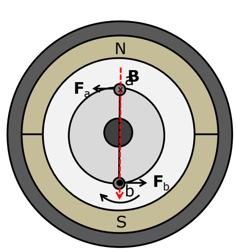




Source: http://www.gusongames.com/content/car_physics_2.html

Generator Torque

- Consider a generator whose shaft is rotating with torque T_m (supplied by prime mover) in CW direction at speed ω
- Conductors experience Lorentz Force
  - Torque T_e in CCW direction

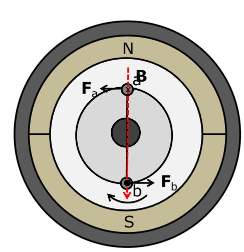


Generator Torque

- Magnitude of the force experienced by the coil is: $\mathbf{F} = 2iLB$
  - L: length of conductor
- electrical torque is

  $T_e = F_e r = 2i\mathbf{B}Lr$
  - r: radius where the conductors are located (m)

Same force and torque equations can be applied to motors



Generator Torque

In generators, applied mechanical torque is opposed by electromagnetic torque

- $T_e > T_m$: shaft decelerates
- $T_e < T_m$: shaft accelerates
- $T_e = T_m$: shaft continues to rotate at ω

Example

Consider a generator supplying a load of 1MW at a constant voltage at 3600 rpm. The load suddenly increases to 1.1MW. Which of the following statements are true?

A. The generator will begin decelerating.
B. The generator will continue to operate at 3600 rpm.
C. The generator will begin accelerating.
D. The electric torque is higher at 1.1 MW than 1MW.
E. The electric torque is lower at 1.1 MW than 1MW.

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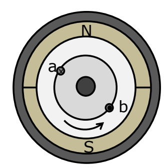
If voltage is constant and power increases, current must increase. Increased current will lead to increased electric torque. Unless mechanical torque also increases, the generator will decelerate.

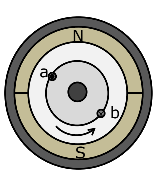
Back EMF

Recall that a motor with shown current polarity will rotate in CCW direction



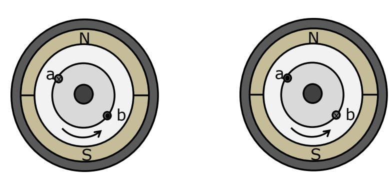
Back EMF

Recall that a generator rotating CCW will have induced current with the shown polarity



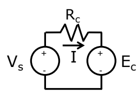
Back EMF

- Applied current (voltage) and induced current (voltage) oppose each other
- Induced voltage is known as *back emf*
- Analogous to T_e opposing T_m in generators



Back EMF

- Consider the following scenario:
  - Applied voltage to stator is increased
  - I increases (Ohm's Law)
  - T_e increases (Lorentz Force equation) $T_e = 2\mathbf{IB}Lr$
  - ω increases $(\alpha = (T_1 - T_2)J)$
  - Back emf increases $E_c = \frac{P}{\pi}\Phi_p\omega_m$
  - Current decreases



Torque and Back EMF

- Counter torque and back emf exist in rotating electric machines
- Act to oppose change in operating state
- Account for torque, back emf in mechanical and electrical models of machines

Torque and Machine Size

Which generator do you expect to be physically larger?

- Machine A Ratings: 100 kW, 250V, 2000 rpm
- Machine B Ratings: 100 kW, 250V, 1000 rpm

Torque and Machine Size

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- Machine A Ratings: 100 kW, 250V, 2000 rpm
- Machine B Ratings: 100 kW, 250V, 1000 rpm

According to $\mathbf{e} = N\Phi_p \sin\theta \frac{d\theta}{dt}$ if speed is lower, either Φ_p or N must increase to maintain the same voltage. This means using larger magnets (or electro-magnets) and/or increasing number of turns. Both options increase the size.

Torque and Machine Size

- For equal power output, lower speed machines are more expensive than higher speed machines
  - If low rpm is needed, consider using a gear box
- Generally, size of the machine is dependent on the torque required
  - 100 kW, 2000 rpm machine is similar in size to a 10 kW, 200 rpm machine

Summary

- Torque = radius x force
- Power = torque x speed
- Machines exhibit different torque vs speed characteristics and are important in determining the application of the machine
- Generators producing current will experience a counter torque in opposite direction to the applied mechanical torque
- Motors drawing current will experience an induced voltage that opposes the applied voltage