Power Flow III

Text: 10

ECEGR 451

Power Systems

Topics

- Examples

N-R Method

- Use N-R to solve

$$0 = -\cos(x_1) + x_2 - x_1$$

$$0 = \frac{1}{2}x_1 - x_2$$

- Use initial guess of:

$$x_1^0 = 1.5$$

$$x_2^0 = 1.125$$

- Stop when $||\Delta \boldsymbol{x}|| < 0.01$

> Note: $||\boldsymbol{y}|| = (y_1^2 + y_2^2 + \cdots + y_n^2)^{1/2}$
> superscript indicate exponents

N-R Method

- Write the equations in the form:

$$f_1(x_1, x_2) = 0 = -\cos(x_1) + x_2 - x_1$$

$$f_2(x_1, x_2) = 0 = \frac{1}{2}x_1 - x_2$$

- Write the Taylor Series (ignore h.o.t)

$$f(x,y) \approx f(x^m, y^m) + \frac{\partial f(x^m, y^m)}{\partial x}(x - x^m) + \frac{\partial f(x^m, y^m)}{\partial y}(y - y^m)$$

$$g(x,y) \approx g(x^m, y^m) + \frac{\partial g(x^m, y^m)}{\partial x}(x - x^m) + \frac{\partial g(x^m, y^m)}{\partial y}(y - y^m)$$

$$f_1(x_1, x_2) \approx \left(-\cos(x_1^0) + x_2^0 - x_1^0\right) + \left(\sin(x_1^0) - 1\right)\Delta x_1 + 1\Delta x_2$$

$$f_2(x_1, x_2) \approx \left(\frac{1}{2}x_1^0 - x_2^0\right) + \frac{1}{2}\Delta x_1^0 - \Delta x_2^0$$

N-R Method

- These equations can be interpreted as follows:

$$f_1(x_1, x_2) \approx \left(-\cos(x_1^0) + x_2^0 - x_1^0\right) + \left(\sin(x_1^0) - 1\right)\Delta x_1 + 1\Delta x_2$$

$$f_2(x_1, x_2) \approx \left(\frac{1}{2}x_1^0 - x_2^0\right) + \frac{1}{2}\Delta x_1^0 - \Delta x_2^0$$

- In the x_1, x_2 plane at point (x_1, x_2), $f_1(x_1,x_2)$ and $f_2(x_1,x_2)$ equal zero
- We want to find x_1, x_2, but we only know x_1^0 and x_2^0 (our initial guesses) and the corresponding f_1 and f_2 values
- We can substitute 0 into the left-hand-side of each equation and solve for Δx_1 and Δx_2. This tells us how far to move to get closer to (x_1, x_2)

N-R Method

- Express the truncated Taylor Series in matrix form

$$f_1(x_1, x_2) = 0 \approx \left(-\cos(x_1^0) + x_2^0 - x_1^0\right) + \left(\sin(x_1^0) - 1\right)\Delta x_1 + 1\Delta x_2$$

$$f_2(x_1, x_2) = 0 \approx \left(\frac{1}{2}x_1^0 - x_2^0\right) + \frac{1}{2}\Delta x_1^0 - \Delta x_2^0$$

$$-\underbrace{\begin{bmatrix} -\cos(x_1^0) + x_2^0 - x_1^0 \\ \frac{1}{2}x_1^0 - x_2^0 \end{bmatrix}}_{\text{Mismatches, error of the current guess}} = \underbrace{\begin{bmatrix} \sin(x_1^0) - 1 & 1 \\ \frac{1}{2} & -1 \end{bmatrix}}_{\text{Jacobian}} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

N-R Method

- We can solve this for Δx_1 and Δx_2, but first we need to plug in some values

$$-\begin{bmatrix} -\cos(x_1^0)+x_2^0-x_1^0 \\ \frac{1}{2}x_1^0-x_2^0 \end{bmatrix} = \begin{bmatrix} \sin(x_1^0)-1 & 1 \\ \frac{1}{2} & -1 \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

- substitute initial values for x_1, x_2

$$-\begin{bmatrix} -\cos(1.5)+1.125-1.5 \\ 0.75-1.125 \end{bmatrix} = \begin{bmatrix} \sin(1.5)-1 & 1 \\ \frac{1}{2} & -1 \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

N-R Method

- Solve for Δx_1 and Δx_2

$$-\begin{bmatrix} -\cos(1.5)+1.125-1.5 \\ 0.75-1.125 \end{bmatrix} = \begin{bmatrix} \sin(1.5)-1 & 1 \\ \frac{1}{2} & -1 \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$$\begin{bmatrix} 0.4457 \\ 0.375 \end{bmatrix} = \begin{bmatrix} -0.0025 & 1 \\ \frac{1}{2} & -1 \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

Multiply both sides by inverse Jacobian

$$\underbrace{\begin{bmatrix} 2.0101 & 2.0101 \\ 1.005 & 0.005 \end{bmatrix}}_{\text{inverse Jacobian}}\begin{bmatrix} 0.4457 \\ 0.375 \end{bmatrix} = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} == \begin{bmatrix} 1.6497 \\ 0.4499 \end{bmatrix}$$

N-R Method

- Update x_1 and x_2

$$\begin{bmatrix} x_1^1 \\ x_2^1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1.125 \end{bmatrix} + \begin{bmatrix} 1.6497 \\ 0.4499 \end{bmatrix} = \begin{bmatrix} 3.1497 \\ 1.5749 \end{bmatrix}$$

- Should we stop?

$$\|\Delta \boldsymbol{x}\| = 1.71 > 0.01$$

- Calculate the mismatch for the next iteration

$$\begin{bmatrix} -\cos(x_1^1)+x_2^1-x_1^1 \\ \frac{1}{2}x_1^1-x_2^1 \end{bmatrix} = \begin{bmatrix} -0.5749 \\ 0 \end{bmatrix}$$

Common to use the norm of the mismatch to evaluate determine stopping condition rather than norm of $\Delta \boldsymbol{x}$

N-R Method

- Everything looks great so far. We can continue our iterations until we stop, but…
- This problem does not converge with the N-R method. Why?
- It turns out we have a bad starting point.
- We can write this problem in terms of x_1 only, via substitution:

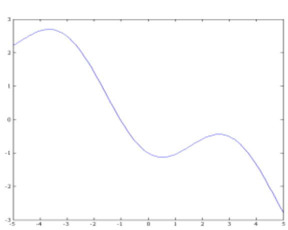
$$x_2 = \frac{1}{2}x_1$$

$$f(x_1) = -\cos(x_1) + \frac{1}{2}x_1 - x_1$$

N-R Method

We can easily plot this function to see the solution space:

f(x_1)



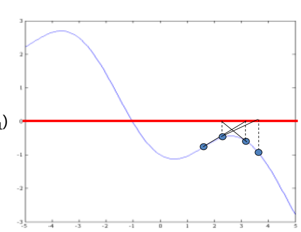
x_1

$$f(x_1) = -\cos(x_1) + \frac{1}{2}x_1 - x_1$$

N-R Method

- Recall, we want to find x_1, such that the f(x_1) equals zero

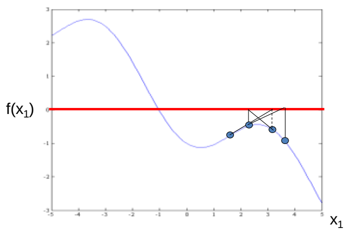
f(x_1)



x_1

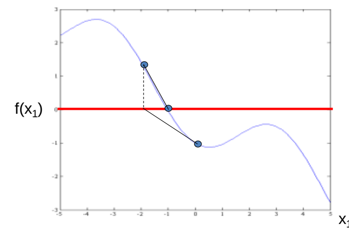
N-R Method

- We bounce back and forth and never get to the solution!



N-R Method

- if we tried x_1=0, we readily converge:



N-R Example

- Consider the equation:
  $1 = (x)^2 + x$
- Use N-R to find x
- Stop when ||f(x)|| < 0.005
  - norm of the mismatch
- Use x =1 for the initial guess

N-R Example

$$1 = (x)^2 + x$$
$$f(x) = (x)^2 + x - 1$$
$$f(x^0) = 1$$
$$\frac{df(x)}{dx} = 2x + 1$$
$$\frac{df(x^0)}{dx} = 2x^0 + 1 = 3$$
$$\Delta x = \frac{-f(x^0)}{\frac{df(x^0)}{dx}} = -\frac{1}{3}$$

m	x^m	$f(x^m)$	$\frac{df}{dx}(x^m)$	Δx
0	1	1	3	-0.3333

N-R Example

m	x^m	$f(x^m)$	$\frac{df}{dx}(x^m)$	Δx
0	1	1	3	-.3333
1	0.6667	0.1111	2.333	-0.0476

$$x^1 = x^0 + \Delta x = 0.667$$
$$f(x^1) = (x^1)^2 + x^1 - 1 = 0.111 \text{ (do not stop)}$$
$$\frac{df(x^1)}{dx} = 2x^1 + 1 = 2.333$$
$$\Delta x = \frac{-f(x^1)}{\frac{df(x^1)}{dx}} = -0.0476$$

N-R Example

$$x^2 = x^1 + \Delta x = 0.619$$
$$f(x^2) = (x^2)^2 + x^2 - 1 = 0.0023 \text{ (stop)}$$

m	x^m	$f(x^m)$	$\frac{df}{dx}(x^m)$	Δx
0	1	1	3	-0.3333
1	0.6667	0.1111	2.333	-0.0476
2	0.6190	-	-	-

N-R Example

- Our final answer is x = 0.619
- Now we will review how to apply the N-R method to the power flow problem
- We will use the 2-dimensional example as a template
- In a previous N-R example, we had 2 unknowns, and 2 equations

$$0 = -\cos(x_1) + x_2 - x_1$$

$$0 = \frac{1}{2}x_1 - x_2$$

- we must now find the unknowns and equations for the power flow problem

N-R Power Flow

- Recall: the goal of solving the power flow problem is to determine the voltage magnitude and angle at each bus

N-R Power Flow

- Given a system with 10 buses, and 3 generators, find:
  - The number of unknown variables
    - 1 slack bus: (0 unknowns)
    - 2 PV: (1 unknown each)
    - 7 PQ: (2 unknowns each)
    - 0 + 2 + 14 = 16;
  - What are they?
    - PV: θ
    - PQ: $|V|$, θ

Bus Numbering

- For this example, assume that the buses are numbered as follows:
  - Slack bus: 1
  - PV buses: 2, 3
  - PQ buses: 4 -10

Load Buses

- We need to find two equations that do not introduce any new unknowns for each load bus, *i*
- These equations correspond to: $0 = -\cos(x_1) + x_2 - x_1$

$$0 = \frac{1}{2}x_1 - x_2$$

- We can use the power flow equations:

$$P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik})$$

Load Buses

- Recall that we know the P_i and Q_i for load buses

$$P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik})$$

- What about the PV buses?

Generator Bus

- We need 1 equation for each generation bus, j, that does not introduce any more unknown variables
- Which equation should we use?

$$P_j = \sum_{k=1}^{n} |V_j||V_k|\left(G_{jk}\cos\theta_{jk} + B_{jk}\sin\theta_{jk}\right)$$

$$Q_j = \sum_{k=1}^{n} |V_j||V_k|\left(G_{jk}\sin\theta_{jk} - B_{jk}\cos\theta_{jk}\right)$$

- The real power, since P_j is known, but Q_j is not, so use the real power equation

Slack Bus

- We have 0 unknowns, so we don't need to find any equations

N-R Equation

- Verify that we have 16 equations for the 16 unknowns:

$$\Delta P_j = 0 = P_j - \sum_{k=1}^{n} |V_j||V_k|\left(G_{jk}\cos\theta_{jk} + B_{jk}\sin\theta_{jk}\right), \text{ for each } j = 2,\ldots,10$$

$$\Delta Q_i = 0 = Q_i - \sum_{k=1}^{n} |V_i||V_k|\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right), \text{ for each } i=4,\ldots,10$$

N-R Jacobian Comparison

- What is the dimension of the Jacobian?
  - (16 x 16)

$$\boldsymbol{J} = \begin{bmatrix} \left[\frac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}\right] & \left[\frac{\partial \mathbf{P}}{\partial |\mathbf{V}|}\right] \\ \left[\frac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}}\right] & \left[\frac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}\right] \end{bmatrix}$$

N-R Jacobian Comparison

- What is the dimension of $\left[\frac{\partial P}{\partial \theta}\right]$?
  - (9x9): 9 P eqns, 9 unknown θ
- What is the dimension of $\left[\frac{\partial P}{\partial |V|}\right]$?
  - (9x7): 9 P eqns, 7 unknown |V|
- What is the dimension of $\left[\frac{\partial Q}{\partial \theta}\right]$?
  - (7x9): 7 Q eqns, 9 unknown θ
- What is the dimension of $\left[\frac{\partial Q}{\partial |V|}\right]$?
  - (7x7): 7 Q eqns, 7 unknown |V|

$$\boldsymbol{J} = \begin{bmatrix} \left[\frac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}\right] & \left[\frac{\partial \mathbf{P}}{\partial |\mathbf{V}|}\right] \\ \left[\frac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}}\right] & \left[\frac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}\right] \end{bmatrix}$$

NR Δx Solution

- From Taylor Series

$$\begin{bmatrix} \Delta \boldsymbol{P}^0 \\ \Delta \boldsymbol{Q}^0 \end{bmatrix} = \begin{bmatrix} \left[\frac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}\right] & \left[\frac{\partial \mathbf{P}}{\partial |\mathbf{V}|}\right] \\ \left[\frac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}}\right] & \left[\frac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}\right] \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta} \\ \Delta |\mathbf{V}| \end{bmatrix}$$

- solving for voltage angles, and magnitudes

$$\boldsymbol{J}^{-1}\begin{bmatrix} \Delta \boldsymbol{P}^0 \\ \Delta \boldsymbol{Q}^0 \end{bmatrix} = \begin{bmatrix} \Delta \boldsymbol{\theta} \\ \Delta |\mathbf{V}| \end{bmatrix}$$

NR Update

- Update values:

$$\begin{bmatrix} \theta^1 \\ |\mathbf{V}|^1 \end{bmatrix} = \begin{bmatrix} \theta^0 \\ |\mathbf{V}|^0 \end{bmatrix} + \begin{bmatrix} \Delta\theta \\ \Delta|\mathbf{V}| \end{bmatrix}$$

- Check stopping conditions:

$$\left\{ \left(\Delta P_2^1\right)^2 + \ldots + \left(\Delta P_{10}^1\right)^2 + \left(\Delta Q_4^1\right)^2 + \ldots + \left(\Delta Q_{10}^1\right)^2 \right\}^{1/2} \le \varepsilon \; ?$$

Implementation Notes

- We assumed convenient bus numbering—this will not always be the case
- We used the norm of the mismatch of P & Q to determine when to stop
- After the completion of the N-R method, we have voltage magnitude and angle information (some given, some solved for). We can solve for power flows, net injections

$$P_i = \sum_{k=1}^{n} |V_i||V_k| \left(G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik} \right) \quad \{ \text{slack bus}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k| \left(G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik} \right) \quad \begin{cases} \text{slack bus} \\ \text{PV buses} \end{cases}$$

Implementation Notes

- Losses can be computed by summing the real power from all generators and subtracting the total real load
- Line flows can be computed as done previously as all voltage angles and magnitudes will be known