Power Flow Part 1

Text: 10-10.2

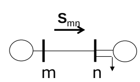
ECEGR 451
Power Systems

Topics

- Power Transmission Revisited
- Power Flow Equations
- Power Flow Problem Statement

Power Transmission

Recall that for a <u>short transmission</u> line, the power from bus m to bus n, $\mathbf{S}_{mn}$ can be written as:



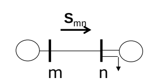
$$\mathbf{S}_{mn} = \mathbf{V}_m \mathbf{I}_m^* = \mathbf{V}_m \left(\frac{\mathbf{V}_m - \mathbf{V}_n}{\mathbf{Z}_{mn}} \right)^* = \frac{|\mathbf{V}_m|^2}{\mathbf{Z}_{mn}^*} - \frac{\mathbf{V}_m \mathbf{V}_n^*}{\mathbf{Z}_{mn}^*}$$

$$\mathbf{S}_{mn} = \frac{|\mathbf{V}_m|^2}{r_{mn} - jx_{mn}} - \frac{|\mathbf{V}_m| e^{j\theta_m} |\mathbf{V}_n| e^{-j\theta_n}}{r_{mn} - jx_{mn}}$$

$$= \frac{|\mathbf{V}_m|^2}{r_{mn} - jx_{mn}} - \frac{|\mathbf{V}_m||\mathbf{V}_n| e^{j\theta_{mn}}}{r_{mn} - jx_{mn}} = \frac{|\mathbf{V}_m|^2}{r_{mn} - jx_{mn}} - \frac{|\mathbf{V}_m||\mathbf{V}_n|(\cos\theta_{mn} + j\sin\theta_{mn})}{r_{mn} - jx_{mn}}$$

Power Transmission

Substituting and separating real and imaginary parts



$$\frac{1}{r_{mn} + jx_{mn}} \triangleq g_{mn} + jb_{mn}$$

$$\mathbf{S}_{mn} = \frac{|\mathbf{V}_m|^2}{r_{mn} - jx_{mn}} - \frac{|\mathbf{V}_m||\mathbf{V}_n|(\cos\theta_{mn} + j\sin\theta_{mn})}{r_{mn} - jx_{mn}}$$

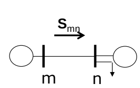
$$= |\mathbf{V}_m|^2 (g_{mn} - jb_{mn}) - |\mathbf{V}_m||\mathbf{V}_n|(\cos\theta_{mn} + j\sin\theta_{mn})(g_{mn} - jb_{mn})$$

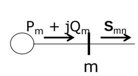
$$P_{mn} = g_{mn}|\mathbf{V}_m|^2 - g_{mn}|\mathbf{V}_n||\mathbf{V}_m|\cos\theta_{mn} - b_{mn}|\mathbf{V}_n||\mathbf{V}_m|\sin\theta_{mn}$$

$$Q_{mn} = -b_{mn}|\mathbf{V}_m|^2 - g_{mn}|\mathbf{V}_n||\mathbf{V}_m|\sin\theta_{mn} + b_{mn}|\mathbf{V}_n||\mathbf{V}_m|\cos\theta_{mn}$$

Power Transmission

Real, Imaginary Power from m to n must equal the power output by the generator



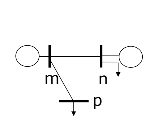


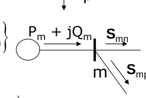
$$P_{mn} = g_{mn}|\mathbf{V}_m|^2 - g_{mn}|\mathbf{V}_n||\mathbf{V}_m|\cos\theta_{mn} - b_{mn}|\mathbf{V}_n||\mathbf{V}_m|\sin\theta_{mn} = P_m$$

$$Q_{mn} = -b_{mn}|\mathbf{V}_m|^2 - g_{mn}|\mathbf{V}_n||\mathbf{V}_m|\sin\theta_{mn} + b_{mn}|\mathbf{V}_n||\mathbf{V}_m|\cos\theta_{mn} = Q_m$$

Power Transmission

- What if another bus is added?
  $\mathbf{S}_m = \mathbf{S}_{mn} + \mathbf{S}_{mp}$
- Equations become:

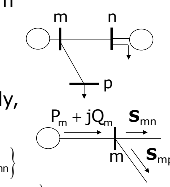




$$P_m = \left\{ g_{mn}|V_m|^2 - g_{mn}|V_n||V_m|\cos\theta_{mn} - b_{mn}|V_n||V_m|\sin\theta_{mn} \right\}$$
$$+ \left\{ g_{mp}|V_m|^2 - g_{mp}|V_p||V_m|\cos\theta_{mp} - b_{mp}|V_p||V_m|\sin\theta_{mp} \right\}$$
$$Q_m = \left\{ -b_{mn}|V_m|^2 - g_{mn}|V_n||V_m|\sin\theta_{mn} + b_{mn}|V_n||V_m|\cos\theta_{mn} \right\}$$
$$+ \left\{ -b_{mp}|V_m|^2 - g_{mp}|V_p||V_m|\sin\theta_{mp} + b_{mp}|V_p||V_m|\cos\theta_{mp} \right\}$$

Power Transmission

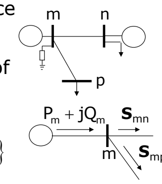
- What if the transmission line from m to p is medium or long length?
- Let the total line shunt admittance be y_{shunt}
- Note that since it is capacitance only, the real power is not affected



$$P_m = \left\{ g_{mn}|V_m|^2 - g_{mn}|V_n||V_m|\cos\theta_{mn} - b_{mn}|V_n||V_m|\sin\theta_{mn} \right\}$$
$$+ \left\{ g_{mp}|V_m|^2 - g_{mp}|V_p||V_m|\cos\theta_{mp} - b_{mp}|V_p||V_m|\sin\theta_{mp} \right\}$$
$$Q_m = -|V_m|^2 \frac{y_{shunt}}{2} + \left\{ -b_{mn}|V_m|^2 - g_{mn}|V_n||V_m|\sin\theta_{mn} + b_{mn}|V_n||V_m|\cos\theta_{mn} \right\}$$
$$+ \left\{ -b_{mp}|V_m|^2 - g_{mp}|V_p||V_m|\sin\theta_{mp} + b_{mp}|V_p||V_m|\cos\theta_{mp} \right\}$$

Power Transmission

- What if there was a shunt admittance added to bus m?
- Note this is difference from a load, since the load is modeled in terms of power
- Let the admittance be $\mathbf{y} = g_m + jb_m$



$$P_m = \left\{ |V_m|^2 g_m + g_{mn}|V_m|^2 - g_{mn}|V_n||V_m|\cos\theta_{mn} - b_{mn}|V_n||V_m|\sin\theta_{mn} \right\}$$
$$+ \left\{ g_{mp}|V_m|^2 - g_{mp}|V_p||V_m|\cos\theta_{mp} - b_{mp}|V_p||V_m|\sin\theta_{mp} \right\}$$
$$Q_m = -|V_m|^2 b_m - |V_m|^2 \left(\frac{y_{shunt}}{2} \right) + \left\{ -b_{mn}|V_m|^2 - g_{mn}|V_n||V_m|\sin\theta_{mn} + b_{mn}|V_n||V_m|\cos\theta_{mn} \right\}$$
$$+ \left\{ -b_{mp}|V_m|^2 - g_{mp}|V_p||V_m|\sin\theta_{mp} + b_{mp}|V_p||V_m|\cos\theta_{mp} \right\}$$

Power Transmission

- We can write similar expressions for the power out of buses n and p
- You can see that as we add more buses and transmission lines, the equations become quite long
- Let's see if we can simplify
  - use $\boldsymbol{Y}_{bus}$
  - $\boldsymbol{Y}_{bus}$ works with current so we will start out by considering bus current

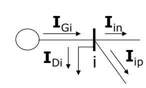
Power Flow Equations

- Total current entering the transmission system at bus i is equal to the current from the generator(s) at bus i minus the current to load at bus i

$$\mathbf{I}_i = \mathbf{I}_{Gi} - \mathbf{I}_{Di} = \sum_{k=1}^{n} \mathbf{I}_{ik} \qquad i = 1, 2, \ldots, n$$

- $\mathbf{I}_i$ is the total current leaving bus i through transmission lines or shunts (not loads)
- Using $\boldsymbol{Y}_{bus}$:

$$\mathbf{I}_i \triangleq \sum_{k=1}^{n} Y_{ik} \mathbf{V}_k \qquad i = 1, 2, \ldots, n$$



yields

$$\boldsymbol{I} = \boldsymbol{Y}_{Bus} \boldsymbol{V}$$

Power Flow Equations

- Now for power

$$\mathbf{S}_i = \mathbf{V}_i \mathbf{I}_i^*$$

- Via substitution

$$\mathbf{S}_i = \mathbf{V}_i \left(\sum_{k=1}^{n} Y_{ik} \mathbf{V}_k \right)^* \qquad \mathbf{I}_i \triangleq \sum_{k=1}^{n} Y_{ik} \mathbf{V}_k$$

- General case

$$\mathbf{S}_i = \mathbf{V}_i \sum_{k=1}^{n} Y_{ik}^* \mathbf{V}_k^* \qquad i = 1, 2 \ldots, n$$

Note: Y_{ik} refers to an element in $\boldsymbol{Y}_{Bus}$

Power Flow Equations

- Recall $\mathbf{S}_i = \mathbf{V}_i \sum_{k=1}^{n} \mathbf{Y}_{ik}^* \mathbf{V}_k^*$

$$\mathbf{V}_i \triangleq |\mathbf{V}_i| e^{j\theta_i} \qquad \theta_{ik} \triangleq \theta_i - \theta_k$$

- via substitution and Euler's Identity

$$\mathbf{S}_i = |V_i| e^{j\theta_i} \sum_{k=1}^{n} Y_{ik}^* |V_k| e^{-j\theta_k}$$
$$= \sum_{k=1}^{n} |V_i||V_k| e^{j\theta_i} e^{-j\theta_k} Y_{ik}^* \qquad Y_{ik} \triangleq G_{ik} + jB_{ik}$$
$$= \sum_{k=1}^{n} |V_i||V_k| e^{j\theta_{ik}} (G_{ik} - jB_{ik})$$
$$= \sum_{k=1}^{n} |V_i||V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

location in $\mathbf{Y}_{bus}$

Power Flow Equations

$$\mathbf{S}_i = \sum_{k=1}^{n} |\mathbf{V}_i||\mathbf{V}_k|(\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

- Split into real and reactive parts

$$P_i = \sum_{k=1}^{n} |\mathbf{V}_i||\mathbf{V}_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$$

$$Q_i = \sum_{k=1}^{n} |\mathbf{V}_i||\mathbf{V}_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik})$$

- Spend some time becoming familiar with these equations
- These equations match the equations derived for our simple 2 and 3 bus cases

Power Flow Equations

- To recap: for each bus i, we can find the net power injected (generation – minus load) by

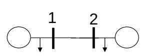
$$P_i = \sum_{k=1}^{n} |\mathbf{V}_i||\mathbf{V}_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$$

$$Q_i = \sum_{k=1}^{n} |\mathbf{V}_i||\mathbf{V}_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik})$$

- G_{ik} and B_{ik} values correspond to the element Y_{ik} in the $\boldsymbol{Y}_{bus}$, and NOT the primitive admittance of line i-k
- If we have voltage magnitude and angle values at each bus, then we can easily determine all the power flows and power injections

Example

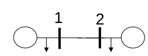
- Voltage at bus 1 is 1.0 p.u. at zero degrees
- Voltage at bus 2 is 0.97 p.u. at -7 degrees
- Series line impedance is 0.01 + j0.1 p.u.
- B_{12} = 0.10 p.u.
- construct $\mathbf{Y}_{bus}$



This is not Y_{bus} element 12, but is the total shunt capacitive reactance of line 1-2. If given in a problem statement, assume B_{mn} is defined as such.

Example

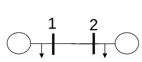
- Voltage at bus 1 is 1.0 p.u. at zero degrees
- Voltage at bus 2 is 0.97 p.u. at -7 degrees
- Series line impedance is 0.01 + j0.1 p.u.
- B_{12} = 0.10 p.u.
- construct $\mathbf{Y}_{bus}$



$$\boldsymbol{Y}_{bus} = \begin{bmatrix} 0.99 - j9.85 & -0.99 + j9.90 \\ -0.99 + j9.90 & 0.99 - j9.85 \end{bmatrix}$$

Example

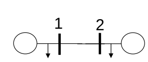
- Voltage at bus 1 is 1.0 p.u. at zero degrees
- Voltage at bus 2 is 0.97 p.u. at -7 degrees
- What is the real power injected into bus 1?



$$\boldsymbol{Y}_{bus} = \begin{bmatrix} 0.99 - j9.85 & -0.99 + j9.90 \\ -0.99 + j9.90 & 0.99 - j9.85 \end{bmatrix}$$

Example

- Voltage at bus 1 is 1.0 p.u. at zero degrees
- Voltage at bus 2 is 0.97 p.u. at -7 degrees
- What is the real power injected into bus 1?



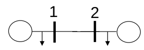
$$\boldsymbol{Y}_{bus} = \begin{bmatrix} 0.99 - j9.85 & -0.99 + j9.90 \\ -0.99 + j9.90 & 0.99 - j9.85 \end{bmatrix}$$

$$P_i = \sum_{k=1}^{n} |\mathbf{V}_i||\mathbf{V}_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$$

$$P_1 = 1.0^2(0.99\cos 0^\circ - 9.85\sin 0^\circ) + 1.0 \times 0.97(-0.99\cos 7^\circ + 9.90\sin 7^\circ) = 1.2$$

Example

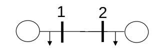
- Voltage at bus 1 is 1 p.u. at zero degrees
- Voltage at bus 2 is 0.97 p.u. at -7 degrees
- What is the reactive power injected into bus 1?



$$\mathbf{Y}_{bus} = \begin{bmatrix} 0.99 - j9.85 & -0.99 + j9.90 \\ -0.99 + j9.90 & 0.99 - j9.85 \end{bmatrix}$$

Example

- Voltage at bus 1 is 1 p.u. at zero degrees
- Voltage at bus 2 is 0.97 p.u. at -7 degrees
- What is the reactive power injected into bus 1?



$$\mathbf{Y}_{bus} = \begin{bmatrix} 0.99 - j9.85 & -0.99 + j9.90 \\ -0.99 + j9.90 & 0.99 - j9.85 \end{bmatrix}$$

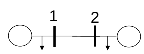
$$Q_i = \sum_{k=1}^{n} |V_i||V_k|\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right)$$

$$Q_1 = -1^2(-9.85) + 1(0.97)\left(-0.99\sin(7) - 9.90\cos(7)\right)$$

$$Q_1 = 0.2 \text{ p.u.}$$

Example

- In this example, the power injected to the bus is also the power transmitted down the power line
- To determine the power out of the generator, we must know the power into the load at bus 1
- If P_{D1} = 1.0 p.u. then the real power from the generator at bus 1 is 2.20 p.u.
- follow a similar procedure for reactive power



Power Flow Equations

- We can use the power flow equations to show the P-θ and Q-|V| relationships
- The relationship can be quantified by find the sensitivity, or partial derivative of P w.r.t. θ and |V|
- From our example:

$$P_1 = P_{12} = |V_1|^2 G_{11} + |V_1||V_2|\left(G_{12}\cos(\theta_{12}) + B_{12}\sin(\theta_{12})\right)$$

$$\frac{\partial P_{12}}{\partial \theta_{12}} = 0 + |V_1||V_2|\left(-G_{12}\sin(\theta_{12}) + B_{12}\cos(\theta_{12})\right)$$

$$\frac{\partial P_{12}}{\partial |V_1|} = 2|V_1|G_{11} + |V_2|\left(G_{12}\cos(\theta_{12}) + B_{12}\sin(\theta_{12})\right)$$

Power Flow Equations

- Evaluating each derivative

$$\frac{\partial P_{12}}{\partial \theta_{12}} = 0 + 0.97\left(0.99\sin(7) + 9.90\cos(7)\right)$$
$$= 9.6$$
$$\frac{\partial P_{12}}{\partial |V_1|} = 2(0.97)0.99 + 0.97\left(0.99\cos(7) + 9.90\sin(7)\right)$$
$$= 4.04$$

- We expect a larger change of real power if the difference in voltage angles change, rather than a voltage magnitude change

Example

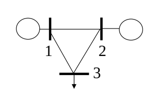
- Now try a three bus system
- Given:

$\mathbf{V}_1 = 1\angle 0^\circ$

$\mathbf{V}_2 = 1.05\angle -3^\circ$

$\mathbf{V}_3 = 0.95\angle -10^\circ$

- Find the power out of each generator and to the load



$$\mathbf{Y}_{bus} = \begin{bmatrix} -j19.98 & j10 & j10 \\ j10 & -j19.98 & j10 \\ j10 & j10 & -j19.98 \end{bmatrix}$$

Example

- start at bus i = 1

$$P_1 = \sum_{k=1}^{n} |V_1||V_k|(G_{1k}\cos\theta_{1k} + B_{1k}\sin\theta_{1k})$$

$$\boldsymbol{Y}_{bus} = \begin{bmatrix} -j19.98 & j10 & j10 \\ j10 & -j19.98 & j10 \\ j10 & j10 & -j19.98 \end{bmatrix}$$

$$\mathbf{V}_1 = 1\angle 0^\circ \quad \mathbf{V}_2 = 1.05\angle -3^\circ \quad \mathbf{V}_3 = 0.95\angle -10^\circ$$

- for k = 1

$$|V_1||V_1|(G_{11}\cos\theta_{11} + B_{11}\sin\theta_{11}) = 0$$

- for k =2

$$|V_1||V_2|(G_{12}\cos\theta_{12} + B_{12}\sin\theta_{12}) = |1||1.05|(0+10\sin 3) = 0.549$$

- for k = 3

$$|V_1||V_3|(G_{13}\cos\theta_{13} + B_{13}\sin\theta_{13}) = |1||0.95|(0+10\sin 10) = 1.65$$

- total real power: 0+0.549+1.65 = 2.19 p.u.

Example

- now bus i = 2

$$P_2 = \sum_{k=1}^{n} |V_2||V_k|(G_{2k}\cos\theta_{2k} + B_{2k}\sin\theta_{2k})$$

- for k = 1

$$|V_2||V_1|(G_{21}\cos\theta_{21} + B_{21}\sin\theta_{21}) = |1.05||1|(10\sin(-3)) = -0.549$$

- for k =2

$$|V_2||V_2|(G_{22}\cos\theta_{22} + B_{22}\sin\theta_{22}) = 0$$

- for k = 3

$$|V_2||V_3|(G_{23}\cos\theta_{23} + B_{23}\sin\theta_{23}) = |1.05||0.95|(10\sin 7) = 1.215$$

- total real power: -0.549+1.21 = 0.667 p.u.

Example

- now bus i = 3

$$P_3 = \sum_{k=1}^{n} |V_3||V_k|(G_{3k}\cos\theta_{3k} + B_{3k}\sin\theta_{3k})$$

- for k = 1

$$|V_3||V_1|(G_{31}\cos\theta_{31} + B_{31}\sin\theta_{31}) = |.95||1|(10\sin(-10)) = -1.649$$

- for k =2

$$|V_3||V_2|(G_{32}\cos\theta_{32} + B_{32}\sin\theta_{32}) = |0.95||1.05|(10\sin(-7)) = -1.215$$

- for k = 3

$$|V_3||V_3|(G_{33}\cos\theta_{33} + B_{33}\sin\theta_{33}) = 0$$

- total real power: -1.649 -1.215 = -2.865 p.u.

Example

- start at bus i = 1

$$Q_1 = \sum_{k=1}^{n} |V_1||V_k|(G_{1k}\sin\theta_{1k} - B_{1k}\cos\theta_{1k})$$

- for k = 1

$$|V_1|^2(G_{11}\sin\theta_{11} - B_{11}\cos\theta_{11}) = -(1^2)(-19.98)$$

- for k =2

$$|V_1||V_2|(G_{12}\sin\theta_{12} - B_{12}\cos\theta_{12}) = 1(1.05)(-10\cos 3) = -10.485$$

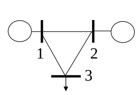
- for k = 3

$$|V_1||V_3|(G_{13}\sin\theta_{13} - B_{13}\cos\theta_{13}) = 1(0.95)(-10\cos 10) = -9.356$$

- total reactive power: 19.98 - 10.48 -9.356 = 0.1365 p.u.

Example

- Following the same procedure we can find reactive power for bus 2 and bus 3
- We can also find the power flow on each line
- If we have bus voltage angle and magnitude, we can completely describe the load and generation, as well as power flows
  - losses (real and reactive) can be computed



Example

- A very common mistake is to assume that the power flowing from bus i to k is found by substituting i=1 and k=2 into:

$$P_i = \sum_{k=1}^{n} |\mathbf{V}_i||\mathbf{V}_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$$

- This is NOT the case, and only would yield the correct answer if the transmission lines are lossless
- To find the power flowing on the individual lines, you can use

$$S_{12} = \mathbf{V}_1(\mathbf{I}_{12})^* + |\mathbf{V}_1|^2(\mathbf{I}_{10})^* = \mathbf{V}_1\left(\sum_{k=1}^{n} Y_{1k}\mathbf{V}_k\right)^*$$

where $\mathbf{I}_{10}$ is the current through the shunts connected to bus 1, if any

Example

- A similar statement can be made for the imaginary power flowing from bus 1 to bus 2
- The value corresponding to i=1 and k=2, in general, has no physical meaning, rather it is simply an element in an equation

Power Flow

- In general, when calculating the power flowing from bus m to bus n, the power flowing into the shunts from a π-modeled transmission line are included
- If the complex power flowing from bus m to all other buses is summed and added to the power flowing to any non-transmission line related shunts at bus m, then the resulting power is equal to S_m

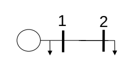
Power Flow Problem

- As in our $\boldsymbol{Y}_{bus}$ discussion, we generally do not know the voltage information at each bus
- Often, we know and can control the voltage magnitude and the real power input at buses with generators
- If a bus only has a load, we only know the real and reactive power demand
- Solving for bus voltage magnitudes and angles is known as the power flow problem

Power Flow Problem

- Solving the power flow equations for |V| and θ is very difficult
- For example, given

$$\mathbf{V}_1 = 1\angle 0$$
$$P_2 = -1.2$$
$$Q_2 = -1$$



- Find $|V_2|$ and θ_2
- Two unknowns, so we need two equations:

$$P_2 = -1.2 = |V_2|^2 G_{22} + 1|V_2|\left(G_{21}\cos\theta_2 + B_{21}\sin\theta_2\right)$$
$$Q_2 = -1.0 = -|V_2|^2 B_{22} + |V_2|1\left(G_{21}\sin\theta_2 - B_{21}\cos\theta_2\right)$$

Power Flow Problem

- Problem is nonlinear, so it is difficult to solve

$$P_2 = -1.2 = |V_2|^2 G_{22} + |1||V_2|\left(G_{21}\cos\theta_2 + B_{21}\sin\theta_2\right)$$
$$Q_2 = -1.0 = -|V_2|^2 B_{22} + |V_2||1|\left(G_{21}\sin\theta_2 - B_{21}\cos\theta_2\right)$$

- Imagine a similar problem on a three bus system (equations are coupled)
- Clearly, finding the exact solution is going to take some work
- Luckily, we don't need an exact solution. Only a solution that is "close" to the answer
- If the exact solution was $|V_2|$ = 0.98554, we might be satisfied with a solution that is $|V_2|$ = 0.98521

Power Flow Problem

- Use some numerical methods to approximate the solution to the power flow problem
- We first need to have a better definition of the problem

Power Flow Problem: Terminology

- Generator Bus: a bus with a connected generator
  - voltage magnitude can be controlled by adjusting the exciter current
  - real power can be controlled by adjusting power to the prime mover
- Load Bus: a bus without a generator
  - real and reactive power are known

Power Flow Problem: Slack Bus

- Since we do not know how much power is flowing on each line, we cannot know in advance the real power out of each generator and load. Why?
  - losses! The amount of losses are not know beforehand
  - Even in a lossless system, the total power generated must equal the power at each load. With a given system load, all but 1 generator real power can be specified
- The generator without its real power specified is known as the "slack bus"

Power Flow Problem: Terminology

- Generator Bus: a bus with a connected generator
  - voltage magnitude can be controlled by adjusting the exciter current
  - real power can be controlled by adjusting power to the prime mover
- Load Bus: a bus without a generator
  - real and reactive power are known
- Slack Bus: a generator bus without its real power specified
  - voltage magnitude is controlled
  - voltage angle is set as the reference
  - usually numbered bus 1
  - Only one slack bus in each system

Power Flow Problem

- For each bus type, there are two variables that can be specified (controlled)
- We can also call
  - generator bus: PV bus
  - load bus: PQ bus
- There are certain cases in which a generator may not be PV, or a load not PQ, for example:
  - certain wind turbines
  - load buses with controlled capacitor banks

Power Flow Problem

- we are interested in solving nonlinear equations using numerical methods
- numerical methods are iterative and provide solutions at are exact to within a tolerance
- We will use Newton-Raphson to solve the problem