17-Network Matrices

Text: 9

ECEGR 451
Power Systems

Topics

- Component Modeling
  - Short Transmission Line
  - Medium/Long Transmission Line
  - Off-Nominal Transformer
- Ybus
  - Network Solution
  - Generator Modeling
  - Matlab
  - Network Reduction

Network Matrices

- We use matrices because power systems are large
- Convenient to express voltage and current relationships in matrices
  - computationally efficient
  - lends itself to computer implementation
  - will be used later in power flow analysis

Bus Admittance Matrix

- Goal: express network voltage and current relationships as $\boldsymbol{I}=\boldsymbol{Y}_{bus}\boldsymbol{V}$ where:
  - $\boldsymbol{I}$ is a column matrix of node current injections (not identity matrix)
  - $\boldsymbol{Y}_{bus}$ is the admittance matrix
  - $\boldsymbol{V}$ is a column matrix of node voltages
- Each component element of the interconnected network is a branch (even connections to ground)
- Branches are modeled as an admittance **y** (primitive admittance), or impedance **z** (primitive impedance)

Notation Change

- Previously:
  - **z**: impedance per length (e.g. Ω/m)
  - **y**: admittance per length (e.g. S/m)
- Hereafter:
  - **z**: impedance in per unit (length accounted for)
  - **y**: admittance in per unit (length accounted for)

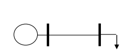
$$\mathbf{y} = \frac{1}{\mathbf{z}}$$

Short Line

- Consider the 2-bus system
- Transmission line a:
  - short model with series impedance = $\mathbf{z}_a$ or series admittance = $\mathbf{y}_a$

$$\mathbf{y}_a = \frac{1}{\mathbf{z}_a}$$

Do not confuse series admittance with shunt admittance

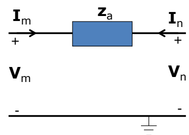


Short Line

Analyzing the circuit:

$$\mathbf{I}_m = \frac{\mathbf{V}_m - \mathbf{V}_n}{\mathbf{z}_a} = \mathbf{y}_a\mathbf{V}_m - \mathbf{y}_a\mathbf{V}_n$$

$$\mathbf{I}_n = -\mathbf{y}_a\mathbf{V}_m + \mathbf{y}_a\mathbf{V}_n$$



In matrix form:

$$\begin{bmatrix}\mathbf{I}_m\\ \mathbf{I}_n\end{bmatrix} = \begin{bmatrix}\mathbf{y}_a & -\mathbf{y}_a\\ -\mathbf{y}_a & \mathbf{y}_a\end{bmatrix}\begin{bmatrix}\mathbf{V}_m\\ \mathbf{V}_n\end{bmatrix}$$

$$\boldsymbol{I} = \boldsymbol{Y}_{bus}\boldsymbol{V}$$

Notation

- $\mathbf{y}_a$ is the branch admittance
- Diagonal elements are self-admittances
- Y_{jk} : refers to the element in the $\boldsymbol{Y}_{bus}$ matrix located in the j^{th} row, k^{th} column

$$\begin{bmatrix}\mathbf{I}_m\\ \mathbf{I}_n\end{bmatrix} = \begin{bmatrix}\mathbf{y}_a & -\mathbf{y}_a\\ -\mathbf{y}_a & \mathbf{y}_a\end{bmatrix}\begin{bmatrix}\mathbf{V}_m\\ \mathbf{V}_n\end{bmatrix}$$

$$\begin{bmatrix}\mathbf{I}_m\\ \mathbf{I}_n\end{bmatrix} = \begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_m\\ \mathbf{V}_n\end{bmatrix}$$

Observations

- In general:
  - $\boldsymbol{Y}_{bus}$ is symmetric
  - Y_{ii}, self-admittances are equal to sum of all the elements connected to that node
  - Y_{ij}, is equal to the negative of the primitive admittance of all components connected between node *i* and *j*

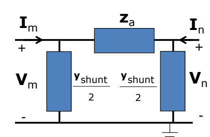
$$\begin{bmatrix}\mathbf{I}_m\\ \mathbf{I}_n\end{bmatrix} = \begin{bmatrix}\mathbf{y}_a & -\mathbf{y}_a\\ -\mathbf{y}_a & \mathbf{y}_a\end{bmatrix}\begin{bmatrix}\mathbf{V}_m\\ \mathbf{V}_n\end{bmatrix} \qquad \begin{bmatrix}\mathbf{I}_m\\ \mathbf{I}_n\end{bmatrix} = \begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_m\\ \mathbf{V}_n\end{bmatrix}$$

Medium/Long Line

Analyzing the circuit:

$$\mathbf{I}_m = \mathbf{V}_m\left(\mathbf{y}_a + \frac{\mathbf{y}_{shunt}}{2}\right) - \mathbf{y}_a\mathbf{V}_n$$

$$\mathbf{I}_n = -\mathbf{y}_a\mathbf{V}_m + \left(\mathbf{y}_a + \frac{\mathbf{y}_{shunt}}{2}\right)\mathbf{V}_n$$



In matrix form:

$$\begin{bmatrix}\mathbf{I}_m\\ \mathbf{I}_n\end{bmatrix} = \begin{bmatrix}\left(\mathbf{y}_a + \frac{\mathbf{y}_{shunt}}{2}\right) & -\mathbf{y}_a\\ -\mathbf{y}_a & \left(\mathbf{y}_a + \frac{\mathbf{y}_{shunt}}{2}\right)\end{bmatrix}\begin{bmatrix}\mathbf{V}_m\\ \mathbf{V}_n\end{bmatrix}$$

Medium/Long Line

- Diagonal element Y_{ii}: add the primitive admittances of all branches connected to node i
- Off-diagonal element Y_{ij}: negative of the primitive admittance connecting i and j

$$\begin{bmatrix}\mathbf{I}_m\\ \mathbf{I}_n\end{bmatrix} = \begin{bmatrix}\left(\mathbf{y}_a + \frac{\mathbf{y}_{shunt}}{2}\right) & -\mathbf{y}_a\\ -\mathbf{y}_a & \left(\mathbf{y}_a + \frac{\mathbf{y}_{shunt}}{2}\right)\end{bmatrix}\begin{bmatrix}\mathbf{V}_m\\ \mathbf{V}_n\end{bmatrix}$$

Example

A transmission line connects Bus 1 and Bus 2 with the following parameters: $z = 0.01 + j0.1$ and $y_{shunt} = j0.15$. Find $\boldsymbol{Y}_{bus}$.

Example

A transmission line connects Bus 1 and Bus 2 with the following parameters: $\mathbf{z}$ = 0.01 + j0.1 and $\mathbf{y}_{shunt}$ = j0.15. Find $\boldsymbol{Y}_{bus}$.

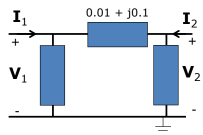
$$\mathbf{y} = \frac{1}{z} = 0.99 - j9.90$$

$$\frac{\mathbf{y}_{shunt}}{2} = j0.075$$

$$Y_{11} = Y_{22} = \mathbf{y} + \frac{\mathbf{y}_{shunt}}{2} = 0.99 - j9.82$$

$$Y_{12} = Y_{21} = -y = -0.99 + j9.9$$

$$Y_{Bus} = \begin{bmatrix} 0.99 - j9.825 & -0.99 + j9.90 \\ -0.99 + j9.90 & 0.99 - j9.825 \end{bmatrix}$$



Observations

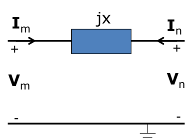
- $\boldsymbol{Y}_{bus}$ is symmetric
- Off-diagonal terms
  - Symmetric
  - Negative real, positive imaginary
  - |real| < |imaginary|
- Diagonal
  - Positive real, negative imaginary
  - |real| < |imaginary|

$$Y_{Bus} = \begin{bmatrix} 0.99 - j9.825 & -0.99 + j9.90 \\ -0.99 + j9.90 & 0.99 - j9.825 \end{bmatrix}$$

Transformer

- Typical transformer model:
  - Ignores resistance
  - Ignores shunt impedance
  - Nominal turns ratio

$$\boldsymbol{Y}_{bus} = \begin{bmatrix} \frac{1}{jx} & -\frac{1}{jx} \\ -\frac{1}{jx} & \frac{1}{jx} \end{bmatrix} = \begin{bmatrix} \mathbf{y} & -\mathbf{y} \\ -\mathbf{y} & \mathbf{y} \end{bmatrix}$$



Off-Nominal Transformer

- For off-nominal transformers

$\frac{\mathbf{V}_2}{a}\mathbf{I}_1^* = -\mathbf{V}_2\mathbf{I}_2^*$ (complex power into ideal transformer is preserved)

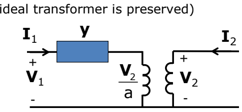
$\mathbf{I}_1 = -a\mathbf{I}_2$

$\mathbf{I}_2 = \frac{-\mathbf{I}_1}{a}$

circuit analysis:

$$\mathbf{I}_1 = \left(\mathbf{V}_1 - \frac{\mathbf{V}_2}{a}\right)\mathbf{y} = \mathbf{V}_1\mathbf{y} - \frac{\mathbf{V}_2\mathbf{y}}{a}$$

$$\mathbf{I}_2 = -\frac{\mathbf{V}_1\mathbf{y}}{a} + \frac{\mathbf{V}_2\mathbf{y}}{a^2} \text{ using } \mathbf{I}_2 = \frac{-\mathbf{I}_1}{a}$$



Off-Nominal Transformer

$$\mathbf{I}_1 = \mathbf{V}_1\mathbf{y} - \frac{\mathbf{V}_2\mathbf{y}}{a}$$

$$\mathbf{I}_2 = -\frac{\mathbf{V}_1\mathbf{y}}{a} + \frac{\mathbf{V}_2\mathbf{y}}{a^2}$$

- In matrix form:

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y} & -\frac{\mathbf{y}}{a} \\ -\frac{\mathbf{y}}{a} & \frac{\mathbf{y}}{a^2} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

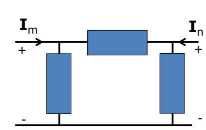
Off-Nominal Transformer

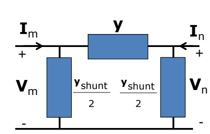
- Off-nominal transformer

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y} & -\frac{\mathbf{y}}{a} \\ -\frac{\mathbf{y}}{a} & \frac{\mathbf{y}}{a^2} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

- Π-circuit:

$$\begin{bmatrix} \mathbf{I}_m \\ \mathbf{I}_n \end{bmatrix} = \begin{bmatrix} \left(\mathbf{y}_a + \frac{\mathbf{y}_{shunt}}{2}\right) & -\mathbf{y}_a \\ -\mathbf{y}_a & \left(\mathbf{y}_a + \frac{\mathbf{y}_{shunt}}{2}\right) \end{bmatrix} \begin{bmatrix} \mathbf{V}_m \\ \mathbf{V}_n \end{bmatrix}$$





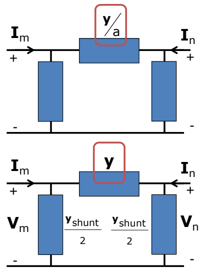
Off-Nominal Transformer

- Off-nominal transformer

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y & -\frac{y}{a} \\ -\frac{y}{a} & \frac{y}{a^2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

- Π-circuit:

$$\begin{bmatrix} I_m \\ I_n \end{bmatrix} = \begin{bmatrix} \left(y_a + \frac{y_{shunt}}{2}\right) & -y_a \\ -y_a & \left(y_a + \frac{y_{shunt}}{2}\right) \end{bmatrix} \begin{bmatrix} V_m \\ V_n \end{bmatrix}$$



Off-Nominal Transformer

- What should the shunt elements be?

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y & -\frac{y}{a} \\ -\frac{y}{a} & \frac{y}{a^2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

- Diagonal elements: sum of admittances connected to them

$$\frac{y}{a} + F = y$$

$$F = y - \frac{y}{a} = y\left(\frac{a-1}{a}\right)$$

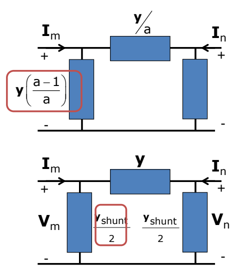
Off-Nominal Transformer

- Off-nominal transformer

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y & -\frac{y}{a} \\ -\frac{y}{a} & \frac{y}{a^2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

- Π-circuit:

$$\begin{bmatrix} I_m \\ I_n \end{bmatrix} = \begin{bmatrix} \left(y_a + \frac{y_{shunt}}{2}\right) & -y_a \\ -y_a & \left(y_a + \frac{y_{shunt}}{2}\right) \end{bmatrix} \begin{bmatrix} V_m \\ V_n \end{bmatrix}$$



Off-Nominal Transformer

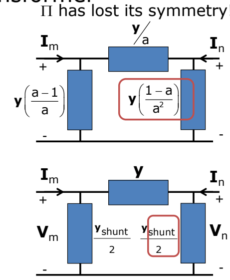
Π has lost its symmetry!

- Off-nominal transformer

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y & -\frac{y}{a} \\ -\frac{y}{a} & \frac{y}{a^2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

- Π-circuit:

$$\begin{bmatrix} I_m \\ I_n \end{bmatrix} = \begin{bmatrix} \left(y_a + \frac{y_{shunt}}{2}\right) & -y_a \\ -y_a & \left(y_a + \frac{y_{shunt}}{2}\right) \end{bmatrix} \begin{bmatrix} V_m \\ V_n \end{bmatrix}$$



Y_{bus} Construction

- Now that we have covered the basic elements we should focus on constructing $\boldsymbol{Y}_{bus}$ for a network
- It is common to have transmission lines described by their primitive impedance (**z**), in p.u. and their shunt admittance, (**B**)
  - note: this means we must divide **B** by two to obtain the admittance on either leg of the Π model

Ybus Construction

General Procedure:

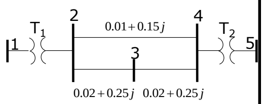
1. Determine the size of $\boldsymbol{Y}_{bus}$
2. Convert all primitive impedances to primitive admittances
3. Pick a bus to start, say bus j:

   Y_{jj} is the sum of the primitive admittances connecting bus j to all other buses and to ground

   Y_{ij} is the negative of the primitive admittance connecting bus j to bus i (zero if they are not connected)
4. Continue until all buses have been computed

Example

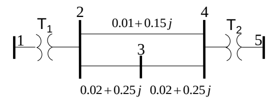
- All impedances are in p.u.
- T_1: $\mathbf{z} = 0.004 + j0.0533$
- T_2: $\mathbf{z} = 0.006 + j0.08$
- Transmission line shunts:
  - line 2-4: $B_{24} = 0.11$
  - line 2-3: $B_{23} = 0.22$
  - line 3-4: $B_{34} = 0.22$



It is assumed that B represents a capacitance

Example

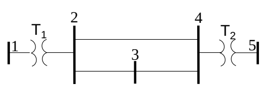
- Calculate primitive admittances:
- $\mathbf{y}_{12} = 1/(.004 + j0.0533) = 1.4 - j18.657$
- $\mathbf{y}_{23} = 1/(0.02 + j0.25) = 0.318 - j3.975$
- $\mathbf{y}_{34} = 0.318 - j3.975$
- $\mathbf{y}_{24} = 0.442 - j6.637$
- $\mathbf{y}_{45} = 0.932 - j12.43$



T_1: z = 0.004 + 0.0533j
T_2: z = 0.006 + j0.08
transmission line shunts:
line 2-4: $B_{24} = 0.11$
line 2-3: $B_{23} = 0.22$
line 3-4: $B_{34} = 0.22$

Example

$$\mathbf{Y}_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} \end{bmatrix}$$



transmission line shunts:
line 2-4: $B_{24} = 0.11$
line 2-3: $B_{23} = 0.22$
line 3-4: $B_{34} = 0.22$

$$Y_{11} = \mathbf{y}_{12} = 1.4 - j18.657$$
$$Y_{12} = Y_{21} = -\mathbf{y}_{12} = -1.4 + j18.657$$
$$Y_{13} = Y_{14} = Y_{15} = Y_{31} = Y_{41} = Y_{51} = 0$$
$$Y_{22} = \mathbf{y}_{12} + \mathbf{y}_{23} + \mathbf{y}_{24} + j\frac{B_{23}}{2} + j\frac{B_{24}}{2}$$
$$= 2.16 - j29.104$$
$$Y_{23} = Y_{32} = -\mathbf{y}_{23} = -0.318 + j3.975$$

$y_{12} = 1.4 - j18.657$
$y_{23} = 0.318 - j3.975$
$y_{34} = 0.318 - j3.975$
$y_{24} = 0.442 - j6.637$
$y_{45} = 0.932 - j12.43$

Example

$$\begin{bmatrix} (1.4 - j18.66) & (-1.4 + j18.66) & 0 & 0 & 0 \\ (-1.4 + j18.66) & (2.16 - j29.10) & (-0.318 + j3.98) & X & X \\ 0 & (-0.318 + j3.98) & X & X & X \\ 0 & X & X & X & X \\ 0 & X & X & X & X \end{bmatrix}$$

complete the matrix

Example

$$\begin{bmatrix} (1.4 - j18.66) & (-1.4 + j18.66) & 0 & 0 & 0 \\ (-1.4 + j18.66) & (2.16 - j29.10) & (-0.318 + j3.98) & (-0.442 + j6.64) & 0 \\ 0 & (-0.318 + j3.98) & (0.636 - j7.73) & (-3.18 + j3.98) & 0 \\ 0 & (-0.442 + j6.64) & (-3.18 + j3.98) & (1.692 - j22.88) & (-0.932 + j12.43) \\ 0 & 0 & 0 & (-0.932 + j12.43) & (0.932 - j12.43) \end{bmatrix}$$

admittance matrix

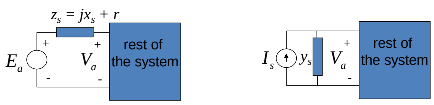
Ybus Properties

- $\mathbf{Y}_{bus}$:
  - symmetric
  - sparse
  - has an inverse if at least one branch connects to the reference

$$\begin{bmatrix} (1.4 - j18.66) & (-1.4 + j18.66) & 0 & 0 & 0 \\ (-1.4 + j18.66) & (2.16 - j29.10) & (-0.318 + j3.98) & (-0.442 + j6.64) & 0 \\ 0 & (-0.318 + j3.98) & (0.636 - j7.73) & (-3.18 + j3.98) & 0 \\ 0 & (-0.442 + j6.64) & (-3.18 + j3.98) & (1.692 - j22.88) & (-0.932 + j12.43) \\ 0 & 0 & 0 & (-0.932 + j12.43) & (0.932 - j12.43) \end{bmatrix}$$

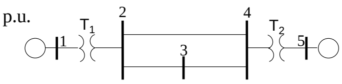
Generator Modeling

- $\boldsymbol{Y}_{bus}$ models the network, what about generators?
- Simple generator model: voltage source (emf) behind an impedance $\mathbf{E}_a = \mathbf{I}_a\mathbf{z}_s + \mathbf{V}_a$
- Sometimes it is more convenient to model as a current source: $\mathbf{I}_s = \frac{\mathbf{E}_a}{\mathbf{z}_s} = \mathbf{I}_a + \mathbf{V}_a\mathbf{y}_s$ where $\mathbf{y}_s = 1/\mathbf{z}_s$



Generator Modeling

- Voltage or current source formation can be used to form $\boldsymbol{V}$ or $\boldsymbol{I}$ in the equation: $\boldsymbol{V} = \boldsymbol{Y}_{bus}\boldsymbol{I}$
- Gen. 1
  - emf: $0.90\angle 0^\circ$ p.u.
  - z_s = j1.25 p.u.
- Gen. 5
  - emf: $0.80\angle -70^\circ$ p.u.
  - z_s = j1.25 p.u.



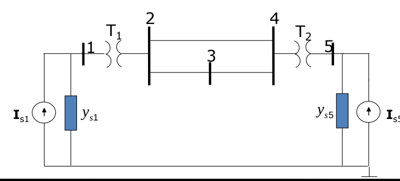
Generator Modeling

- Use a current source model:
- Gen 1: $\mathbf{I}_{s1} = \frac{0.90\angle 0}{j1.25} = 0.72\angle -90^\circ$
- Gen 2: $\mathbf{I}_{s5} = \frac{0.80\angle -70}{j1.25} = 0.64\angle -160^\circ$
- We can now write the bus injection vector as

$$\boldsymbol{I} = \begin{bmatrix} 0.72\angle -90^\circ \\ 0 \\ 0 \\ 0 \\ 0.64\angle -160^\circ \end{bmatrix}$$

Generator Modeling

Here is the same network data as before, but with the generators included as current sources in parallel with admittances



Using Y_{bus}

- Corresponding matrix equation (ignoring resistance to simplify) is:

$$\begin{bmatrix} 0.72\angle -90^\circ \\ 0 \\ 0 \\ 0 \\ 0.72\angle -160^\circ \end{bmatrix} = \begin{bmatrix} -j19.56 & j18.76 & 0 & 0 & 0 \\ j18.76 & -j29.27 & j4.0 & j6.67 & 0 \\ 0 & j4.0 & -j7.78 & j4.0 & 0 \\ 0 & j6.67 & j4.0 & -j23.01 & j12.50 \\ 0 & 0 & 0 & j12.50 & -j13.30 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \\ \mathbf{V}_4 \\ \mathbf{V}_5 \end{bmatrix}$$

- See example 9.7 for details

Network Solution

- We have $\boldsymbol{Y}_{bus}$, now what?
- Most often we know the current from the generators and into the loads, but not the bus voltages
- Find bus voltages through matrix operations

$$\boldsymbol{I} = \boldsymbol{Y}_{bus}\boldsymbol{V} \qquad (Y_{bus})^{-1}\, I = V$$

- What is $(\boldsymbol{Y}_{bus})^{-1}$?

$$(\boldsymbol{Y}_{bus})^{-1} = \text{inv}(\boldsymbol{Y}_{bus}) = \boldsymbol{Z}_{bus}$$

Network Solution

- $\boldsymbol{Z}_{bus}$: bus impedance matrix
  $\boldsymbol{V} = \boldsymbol{Z}_{bus}\boldsymbol{I}$
- note: generally
  $$\begin{bmatrix} \frac{1}{Y_{11}} & \frac{1}{Y_{12}} \\ \frac{1}{Y_{21}} & \frac{1}{Y_{22}} \end{bmatrix} \neq \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$
- $\mathbf{Z}_{bus}$ is:
  - not sparse
  - useful in fault studies and contingency analysis
  - we will discuss in future lectures

Network Solution

- We tend to use $\mathbf{Y}_{bus}$ over $\mathbf{Z}_{bus}$ because $\mathbf{Y}_{bus}$ can be determined by inspection
- Computing $\mathbf{Z}_{bus}$ is computationally burdensome
- We don't need to explicitly find $\mathbf{Z}_{bus}$ to solve our equation
- We can use **LU** factorization (see text section 9.2 for details)
- We will rely on Matlab for matrix computations

Matlab Example

- from Matlab, we found

$$\boldsymbol{Z}_{bus} = \begin{bmatrix} j1.021 & j1.012 & j1.013 & j0.959 & j0.901 \\ j1.012 & j1.055 & j1.056 & j0.999 & j0.939 \\ j1.013 & j1.056 & j1.215 & j1.057 & j0.994 \\ j0.959 & j0.999 & j1.057 & j1.057 & j0.993 \\ j0.901 & j0.939 & j0.994 & j0.993 & j1.009 \end{bmatrix}$$

$$\boldsymbol{V} = \begin{bmatrix} 1.08\angle -30.16^\circ \\ 1.092\angle -31.17^\circ \\ 1.12\angle -32.27^\circ \\ 1.087\angle -33.33^\circ \\ 1.06\angle -34.89^\circ \end{bmatrix}$$

Network Reduction

- Now that we have both the current injected as well as the voltage at each bus, we can compute the power
- We will save that for later
- For large power systems, the size of the $\boldsymbol{Y}_{bus}$ can be a burden on memory (this isn't as true as it used to be)
- Can we reduce its size?

Network Reduction

- Observation: current injection is always zero when there are no external loads or generators connected
- We can eliminate the nodes and reduce the matrix size
- The procedure is also known as Kron Reduction

Network Reduction

- Assume we are given:
  $$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix}$$

1. we can eliminate the third node by expressing $\mathbf{V}_3$ in terms of $\mathbf{V}_1$ and $\mathbf{V}_2$
2. the dependence of $\mathbf{I}_1$ and $\mathbf{I}_2$ on $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$ may be expressed in terms of $\mathbf{V}_1$ and $\mathbf{V}_2$ only
3. collect terms into a new 2 x 2 matrix

Network Reduction

1. We can eliminate the third node by expressing $\mathbf{V}_3$ in terms of $\mathbf{V}_1$ and $\mathbf{V}_2$

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\\ 0\end{bmatrix}=\begin{bmatrix}Y_{11} & Y_{12} & Y_{13}\\ Y_{21} & Y_{22} & Y_{23}\\ Y_{31} & Y_{32} & Y_{33}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\\ \mathbf{V}_3\end{bmatrix}$$

$$0 = Y_{31}\mathbf{V}_1 + Y_{32}\mathbf{V}_2 + Y_{33}\mathbf{V}_3$$

$$\mathbf{V}_3 = -\frac{Y_{31}}{Y_{33}}\mathbf{V}_1 - \frac{Y_{32}}{Y_{33}}\mathbf{V}_2$$

Network Reduction

2. the dependence of $\mathbf{I}_1$ and $\mathbf{I}_2$ on $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$ may be expressed in terms of $\mathbf{V}_1$ and $\mathbf{V}_2$ only

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\\ 0\end{bmatrix}=\begin{bmatrix}Y_{11} & Y_{12} & Y_{13}\\ Y_{21} & Y_{22} & Y_{23}\\ Y_{31} & Y_{32} & Y_{33}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\\ \mathbf{V}_3\end{bmatrix}$$

$$\mathbf{I}_1 = Y_{11}\mathbf{V}_1 + Y_{12}\mathbf{V}_2 + Y_{13}\left(-\frac{Y_{31}}{Y_{33}}\mathbf{V}_1 - \frac{Y_{32}}{Y_{33}}\mathbf{V}_2\right)$$

$$\mathbf{I}_2 = Y_{21}\mathbf{V}_1 + Y_{22}\mathbf{V}_2 + Y_{23}\left(-\frac{Y_{31}}{Y_{33}}\mathbf{V}_1 - \frac{Y_{32}}{Y_{33}}\mathbf{V}_2\right)$$

using

$$\mathbf{V}_3 = -\frac{Y_{31}}{Y_{33}}\mathbf{V}_1 - \frac{Y_{32}}{Y_{33}}\mathbf{V}_2$$

Network Reduction

3. collect terms into a new 2 x 2 matrix

$$\mathbf{I}_1 = \mathbf{Y}_{11}\mathbf{V}_1 + \mathbf{Y}_{12}\mathbf{V}_2 + \mathbf{Y}_{13}\left(-\frac{\mathbf{Y}_{31}}{\mathbf{Y}_{33}}\mathbf{V}_1 - \frac{\mathbf{Y}_{32}}{\mathbf{Y}_{33}}\mathbf{V}_2\right)$$

$$\mathbf{I}_2 = \mathbf{Y}_{21}\mathbf{V}_1 + \mathbf{Y}_{22}\mathbf{V}_2 + \mathbf{Y}_{23}\left(-\frac{\mathbf{Y}_{31}}{\mathbf{Y}_{33}}\mathbf{V}_1 - \frac{\mathbf{Y}_{32}}{\mathbf{Y}_{33}}\mathbf{V}_2\right)$$

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}=\begin{bmatrix}\left(\mathbf{Y}_{11}-\frac{\mathbf{Y}_{13}\mathbf{Y}_{31}}{\mathbf{Y}_{33}}\right) & \left(\mathbf{Y}_{12}-\frac{\mathbf{Y}_{13}\mathbf{Y}_{32}}{Y_{33}}\right)\\ \left(\mathbf{Y}_{21}-\frac{\mathbf{Y}_{23}\mathbf{Y}_{31}}{\mathbf{Y}_{33}}\right) & \left(\mathbf{Y}_{22}-\frac{\mathbf{Y}_{23}\mathbf{Y}_{32}}{\mathbf{Y}_{33}}\right)\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}$$