

# 16-Three Phase Transformers

ECEGR 450  
Electromechanical Energy Conversion



## Overview

- Y-Y Transformer Connection
- Y-Y Transformer Analysis
- Delta-Delta Transformer Connection
- Delta-Delta Transformer Analysis

Dr. Louise

2



## Three-Phase Transformer Connections

- We are generally more concerned with three phase transformers
- Three-phase transformers can be composed of three single-phase transformers or be wound on the same core
- Many combinations are possible:
  - Y-Y
  - Delta-Delta
  - Y-Delta
  - Delta-Y } next lecture

Dr. Louise

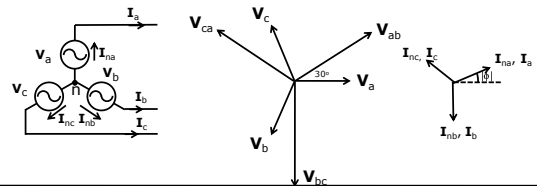
3



## Y-Connected Circuits

- Recall from previous lectures:
  - Line current = Phase current
  - Line voltage = Phase voltage  $\times \sqrt{3} \angle 30^\circ$

Vector Diagrams



Dr. Louise

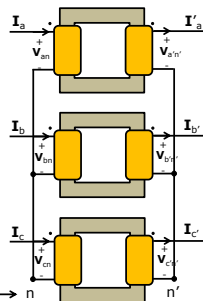
4



## Y-Y Transformer

- Each side has a common point, n or n'
- Neutral points usually grounded
- Tertiary winding (Delta) sometimes connected to avoid distortion if harmonics are present
- Phase voltages appear across the coils on each side
- Insulation is stressed to only 58% of line voltage

Do not confuse this n with 1/a



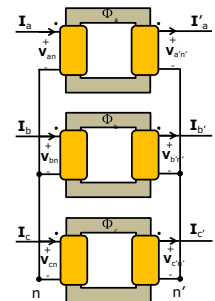
Dr. Louise

5



## Y-Y Transformer Analysis

- Mutual flux in each core are out of phase by  $120^\circ$
- $\Phi_a$  links primary and secondary
- Faraday's law:  $\mathbf{V}_{a'n'}$  and  $\mathbf{V}_{an}$  must be in phase and lead  $\Phi_a$  by  $90^\circ$ 
  - similar result for b, c phases
- Ampere's Law:  $\mathbf{I}_a$  and  $\mathbf{I}_{a'}$  are also in phase
  - similar result for b, c phases



Dr. Louise

6

### Y-Y Transformer Analysis

Phasor Diagram for Ideal Y-Y Connected Transformer

Magnitudes shown assuming  $N_1 < N_2$

Dr. Louie 7

### Three-Phase Transformer Analysis: Y-Y

- Let  $k$  be the transformer voltage gain
- For Y-Y transformers:
 
$$\mathbf{V}_{a'n'} = k\mathbf{V}_{an} = \frac{N_2}{N_1} \mathbf{V}_{an}$$
 therefore  $k = n \leftarrow$  Do not confuse this  $n$  with the neutral point
- $$\mathbf{I}_{a'} = \frac{\mathbf{I}_a}{k} = \frac{\mathbf{I}_a}{n}$$
- $$\mathbf{V}_{a'b'} = \mathbf{V}_{a'n'} \sqrt{3} e^{j\frac{\pi}{6}} = n\mathbf{V}_{an} \sqrt{3} e^{j\frac{\pi}{6}}$$
 (secondary line voltage)
 
$$= \sqrt{3} \left( n \frac{\mathbf{V}_{ab}}{\sqrt{3}} e^{-j\frac{\pi}{6}} \right) e^{j\frac{\pi}{6}} = n\mathbf{V}_{ab}$$
 Recall:  $e^{j\frac{\pi}{6}} = 1 \angle 30^\circ$

Dr. Louie 8

### Three-Phase Transformer Analysis: Y-Y

- Phase voltages
 
$$\mathbf{V}_{a'n'} = n\mathbf{V}_{an}, \mathbf{V}_{b'n'} = n\mathbf{V}_{bn}, \mathbf{V}_{c'n'} = n\mathbf{V}_{cn}$$

$$\mathbf{V}_{a'b'} = n\mathbf{V}_{ab}, \mathbf{V}_{b'c'} = n\mathbf{V}_{bc}, \mathbf{V}_{c'a'} = n\mathbf{V}_{ca}$$
- Line currents
 
$$\mathbf{I}_{a'} = \frac{\mathbf{I}_a}{n}$$

$$\mathbf{I}_{b'} = \frac{\mathbf{I}_b}{n}$$

$$\mathbf{I}_{c'} = \frac{\mathbf{I}_c}{n}$$
 Recall: phase currents = line current for Y connections

primary secondary

Dr. Louie 9

### Per-Phase Y-Y Analysis

- Three phase non-ideal Y-Y transformer model
- Want to model as per-phase equivalent

Magnetizing reactance and core loss (in parallel)

leakage reactance, winding resistance

Dr. Louie 10

### Per Phase Y-Y Analysis

- Per-phase equivalent of non-ideal Y-Y transformer
- Analyze like a single-phase transformer

ideal xfmr

Dr. Louie 11

### Delta-Connected Circuits

- Recall from previous lectures:
  - Line current = Phase current  $\times \sqrt{3} \angle -30^\circ$
  - Line voltage = Phase voltage (but per-phase voltage is line voltage  $\times \frac{1}{\sqrt{3}} \angle -30^\circ$ )

Vector Diagrams

Using  $\mathbf{V}_{ab}$  as reference

Dr. Louie 12

### Δ-Δ Transformer

- No neutral point
- $\mathbf{V}_{ab}, \mathbf{V}_{a'b'}$  in phase
- $\mathbf{I}_{ab}, \mathbf{I}_{a'b'}$  in phase
  - Similar results for b, c phase
- Line-line voltages appear across the coils on primary and secondary
- Best suited for lower voltage applications

Dr. Louie 13

### Three-Phase Transformer Analysis: Δ-Δ

- Let  $k$  be the transformer voltage gain
- For Δ-Δ transformers:
 
$$\mathbf{V}_{a'b'} = k\mathbf{V}_{ab} = \frac{N_2}{N_1}\mathbf{V}_{ab}$$
 therefore  $k = n$
- $$\mathbf{I}_{a'b'} = \frac{\mathbf{I}_{ab}}{k} = \frac{\mathbf{I}_{ab}}{n}$$
- $$\mathbf{V}_{a'n'} = \frac{\mathbf{V}_{ab}e^{j\frac{\pi}{6}}}{\sqrt{3}} = \frac{n\mathbf{V}_{ab}e^{-j\frac{\pi}{6}}}{\sqrt{3}}$$

$$= \frac{n(\sqrt{3}\mathbf{V}_{an}e^{j\frac{\pi}{6}})e^{-j\frac{\pi}{6}}}{\sqrt{3}} = n\mathbf{V}_{an} \text{ (per phase voltage)}$$

Dr. Louie 14

### Three-Phase Transformer Analysis: Δ-Δ

- Voltage Relationships
  - $\mathbf{V}_{a'b'} = n\mathbf{V}_{ab}$
  - $\mathbf{V}_{b'c'} = n\mathbf{V}_{bc}$
  - $\mathbf{V}_{c'a'} = n\mathbf{V}_{ca}$
- Current Relationships
  - $\mathbf{I}_a = \frac{\mathbf{I}_{ab}}{n}, \mathbf{I}_{b'a'} = \frac{\mathbf{I}_{ab}}{n}$
  - $\mathbf{I}_b = \frac{\mathbf{I}_{bc}}{n}, \mathbf{I}_{c'b'} = \frac{\mathbf{I}_{bc}}{n}$
  - $\mathbf{I}_c = \frac{\mathbf{I}_{ca}}{n}, \mathbf{I}_{a'c'} = \frac{\mathbf{I}_{ca}}{n}$

Dr. Louie 15

### Per-Phase Δ-Δ Transformer

- Need to convert each side of the transformer to Y
- Recall that:  $\mathbf{z}_y = \frac{\mathbf{z}_\Delta}{3}$
- Per-phase equivalent circuit of non-ideal Δ-Δ transformer

Dr. Louie 16

### Example

- Consider three, single-phase transformers. The transformers have the following specifications:
  - 720VA, 360/120V,  $R_H = 18.9\Omega$ ,  $X_H = 21.6\Omega$ ,  $R_L = 2.1\Omega$ ,  $X_L = 2.4\Omega$ ,  $R_{CH} = 8.64k\Omega$ ,  $X_{MH} = 6.84k\Omega$
- Draw the per-phase equivalent circuit if the transformers are connected as Y-Y
- What are the nominal line voltages on each side of the transformer?

Dr. Louie 17

### Example

- Nominal per-phase voltage on primary is 360V and 120V on secondary
  - Nominal primary line voltage: 623.5V
  - Nominal secondary line voltage: 207.8V

Dr. Louie 18



### Example

- Consider three, single-phase transformers. The transformers have the following specifications:
  - 720VA, 360/120V,  $R_H = 18.9\Omega$ ,  $X_H = 21.6\Omega$ ,  $R_L = 2.1\Omega$ ,  $X_L = 2.4\Omega$ ,  $R_{CH} = 8.64k\Omega$ ,  $X_{MH} = 6.84k\Omega$
- Draw the per-phase equivalent circuit if the transformers are connected as  $\Delta$ - $\Delta$
- What are the nominal line voltages on each side of the transformer?

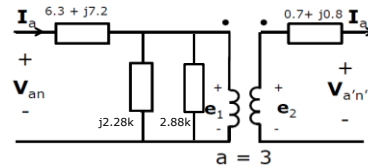
Dr. Louie

19



### Example

- Nominal primary line voltage: 360V
- Nominal secondary line voltage: 120V
- Nominal primary per-phase voltage: 208V
- Nominal secondary per-phase voltage: 69V



Dr. Louie

20



### Summary

- Y-Y, Delta-Delta transformers result in magnitude changes of  $k = n$  from primary to secondary
  - No phase shifting occurs
- Y-Y transformers grant access to neutral point, which is usually grounded to prevent distortion
- Delta-Delta transformers have no neutral point, but are less prone to distortion
- Per phase analysis can be used on Y-Y, Delta-Delta transformers
  - Ensure impedances and voltages are properly converted

Dr. Louie

21