15-System Model

Text: 5.8 – 5.11

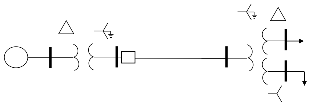
ECEGR 451
Power Systems

Topics

- One-line Diagram
- System Modeling
- Example
- Regulating Transformers

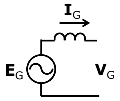
One-Line Diagram

- Generator
- Bus
- Transformer
- Transmission line
- Circuit breaker
- Load



One-Line Diagram

- Simple generator model:
  - $\mathbf{V}_G$: terminal voltage
  - $\mathbf{E}_G$: open circuit voltage
  - $\mathbf{I}_G$: generator current

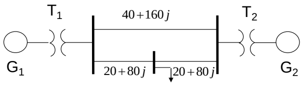


System Model

- We've discussed transmission lines, transformers, per unit, one line diagrams
- Now we put them all together to model the system

System Model

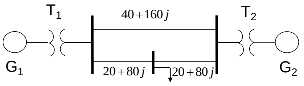
- G_1: 50 MVA, 12.2 kV, X = 0.15 p.u.
- G_2: 20 MVA, 13.8 kV, X = 0.15 p.u.
- T_1: 80 MVA 12.2/161 kV, X = 0.10 p.u.
- T_2: 40 MVA 13.8/161 kV, X = 0.10 p.u.
- Load: 50 MVA, 0.80 PF (lagging), operating at 154 kV



1. Pick a power base for the system

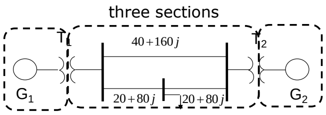
- Common to select power base equal to or near the largest generator in the system
- Let S_B = 100 MVA (three phase)

T_1: 80 MVA 12.2/161 kV, X = 0.10 p.u.
T_2: 40 MVA 13.8/161 kV, X = 0.10 p.u



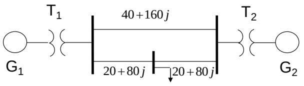
2a. Select voltage base

- Common to select voltage base equal to or near the line-line voltage at any section
  - Must keep track of voltage base and section
  - Sections are separated by transformers
- Let V_B = 132 kV (line-line) at the transmission line section



2b. Compute voltage bases for all sections

- Use transformer ratios (line-line) to relate base voltages between sections
- G1 section: V_{1B}= 132 x (12.2/161) = 10.002 kV
- G2 section: V_{2B} 132 x (13.8/161) = 11.31 kV



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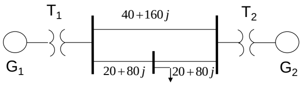
3. Express all impedances in consistent p.u. terms

- All sections have same power bases, but different voltage bases
- Impedances are given with different power and voltage bases
- Convert using: $\mathbf{Z}^{\text{new}}_{\text{p.u.}} = \mathbf{Z}^{\text{old}}_{\text{p.u.}} \left(\frac{V_B^{\text{old}}}{V_B^{\text{new}}}\right)^2 \frac{S_B^{\text{new}}}{S_B^{\text{old}}}$



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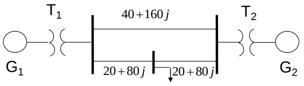
3. Express all impedances in consistent p.u. terms

- G_1: X = 0.15 x (100/50) x (12.2/10.002)² = 0.4463 p.u.
- G_2: X = 0.15 x (100/20) x (13.8/11.31)² = 1.1166 p.u.
- T_1: X = 0.1 x (100/80) x (12.2/10.002)² = 0.1 x (100/80) x (161/132)² = 0.18596 p.u.
- T_2 = 0.1 x (100/40) x (13.8/11.31)² = 0.3719 p.u.

$$\mathbf{Z}^{\text{new}}_{\text{p.u.}} = \mathbf{Z}^{\text{old}}_{\text{p.u.}} \frac{S_B^{\text{new}}}{S_B^{\text{old}}} \left(\frac{V_B^{\text{old}}}{V_B^{\text{new}}}\right)^2$$



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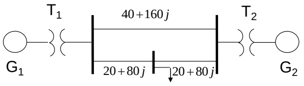
3. Express all impedances in consistent p.u. terms

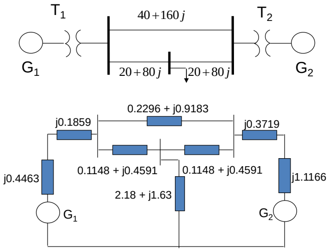
- The transmission line and load:
  - Z_{3B} = (132)²/100 = 174.24 p.u.
  - $\mathbf{Z}_{\text{line,a}}$ = (40 + j160)/174.24 = 0.2296 + j0.9183
  - $\mathbf{Z}_{\text{line,b}}$ = (20 + j80)/174.24 = 0.1148 + j0.4591 p.u.

  and for the load:
  - **S** = 50(.8 + j.6) = 40 + 30j MVA
  - $\mathbf{Z}_{\text{load}}$ = {(154)²/(40 + 30j)}* = 379.456 + j284.529 = 2.18 + j1.63 p.u.



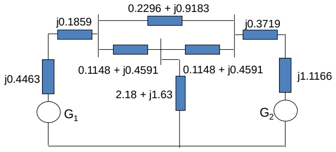
4a. Draw the impedance diagram

- Redrawing the system



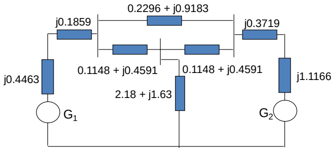
4b. Solve for desired quantities

- Use per-phase analysis



impedance diagram

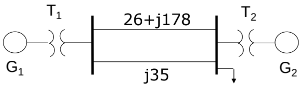
5. Convert back to actual quantities, if needed



impedance diagram

Example

- Specifications
  - G_1: 75 MVA, 10 kV, X = 0.10 p.u.
  - G_2: 75 MVA, 22 kV, X = 0.08 p.u.
  - T_1: 75 MVA 10/365 kV, X = 0.12 p.u.
  - T_2: 80 MVA 24/380 kV, X = 0.14 p.u.
  - Load: $\mathbf{I}_{\text{Load}} = 118.6\angle -10°\text{A}$
  - $\mathbf{E}_{G2} = 25\angle 0°\text{kV}$
- Choose V_B = 365kV at the transmission line and S_B = 75MVA



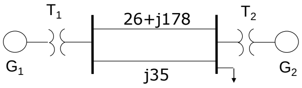
Example

- Section base voltages:

$V_{B3} = 365\text{kV}$

$V_{B1} = 10\text{kV}$

$V_{B2} = 23.05\text{kV}$



Example

- New generator impedances

$$\mathbf{Z}^{\text{new}}_{\text{p.u.}} = \mathbf{Z}^{\text{old}}_{\text{p.u.}}\left(\frac{V_B^{\text{old}}}{V_B^{\text{new}}}\right)^2 \frac{S_B^{\text{new}}}{S_B^{\text{old}}}$$

$$X_{G1} = 0.1\left(\frac{10}{10}\right)^2 \frac{75}{75} = 0.1$$

$$X_{G2} = 0.08\left(\frac{22}{23.05}\right)^2 \frac{75}{75} = 0.073$$

Example

- New transformer impedances

$$\mathbf{Z}_{\text{p.u.}}^{\text{new}} = \mathbf{Z}_{\text{p.u.}}^{\text{old}} \left(\frac{V_B^{\text{old}}}{V_B^{\text{new}}}\right)^2 \frac{S_B^{\text{new}}}{S_B^{\text{old}}}$$

$$X_{T1} = 0.12\left(\frac{10}{10}\right)^2 \frac{75}{75} = 0.12$$

$$X_{T2} = 0.14\left(\frac{24}{23.05}\right)^2 \frac{80}{75} = 0.1423$$

Example

- Transmission line impedance in p.u.

$$Z_B = \frac{V_B^2}{S_B} = \frac{365k^2}{75M} = 1776.3\Omega$$

$$\mathbf{Z}_{L1} = \frac{26 + j178}{1776.3} = 0.015 + j0.1$$

$$\mathbf{Z}_{L2} = \frac{j35}{1776.3} = j0.02$$

Example

- Load current and generator 2 voltage

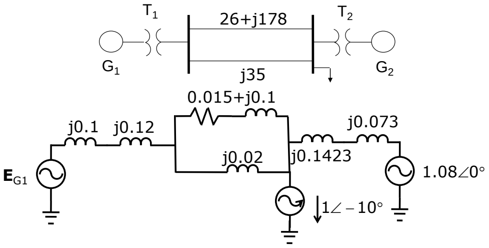
$$I_B = \frac{S_B}{\sqrt{3}V_B} = 118.6A$$

$$\mathbf{I}_{\text{Load}} = \frac{118.6\angle -10^\circ}{I_B} = 1\angle -10^\circ$$

$$\mathbf{E}_{G2} = \frac{25\angle 0^\circ}{23.05} = 1.08\angle 0^\circ$$

Example

- Impedance diagram with sources

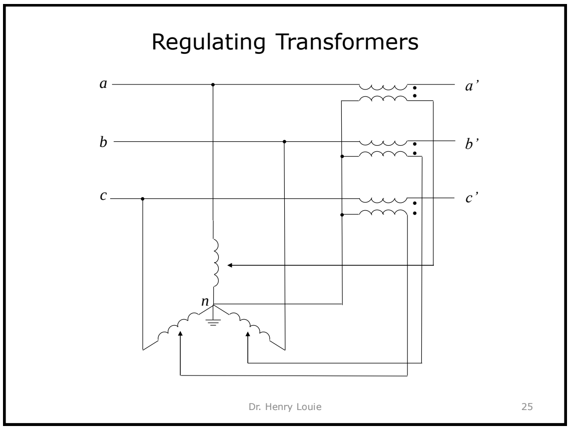


Regulating Transformers

- We can now describe the system model in per unit with the impedance diagram
- We have seen that using per unit on normal systems, we have eliminated the transformers
- However, this is not a general result as there are some transformers that do not “disappear” when per unit normalization is used
  - regulating and off-nominal transformers

Regulating Transformers

- Transformers used to adjust voltage magnitude or phase are called “regulating transformers”
- They do this by adding a small amount (+ or – 10 %) of voltage to the line or phase voltages
- Voltage can also be changed by adjusting the turns ratio of the transformer (called tap changing)
- Tap changing may be automatic and may occur while the transformer is energized (load-tap-changing)



Regulating Transformers

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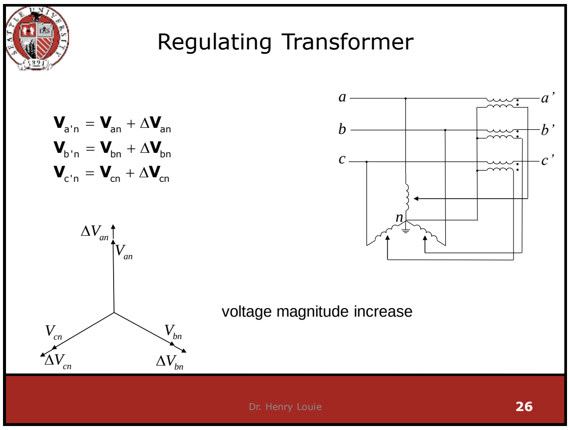

Regulating Transformer

$$\mathbf{V}_{a'n} = \mathbf{V}_{an} + \Delta\mathbf{V}_{an}$$
$$\mathbf{V}_{b'n} = \mathbf{V}_{bn} + \Delta\mathbf{V}_{bn}$$
$$\mathbf{V}_{c'n} = \mathbf{V}_{cn} + \Delta\mathbf{V}_{cn}$$

voltage magnitude increase

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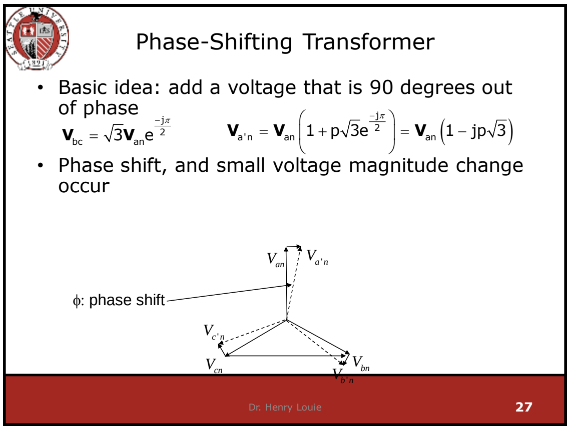

Phase-Shifting Transformer

- Basic idea: add a voltage that is 90 degrees out of phase

$$\mathbf{V}_{bc} = \sqrt{3}\mathbf{V}_{an}e^{\frac{-j\pi}{2}} \qquad \mathbf{V}_{a'n} = \mathbf{V}_{an}\left(1 + p\sqrt{3}e^{\frac{-j\pi}{2}}\right) = \mathbf{V}_{an}\left(1 - jp\sqrt{3}\right)$$

- Phase shift, and small voltage magnitude change occur

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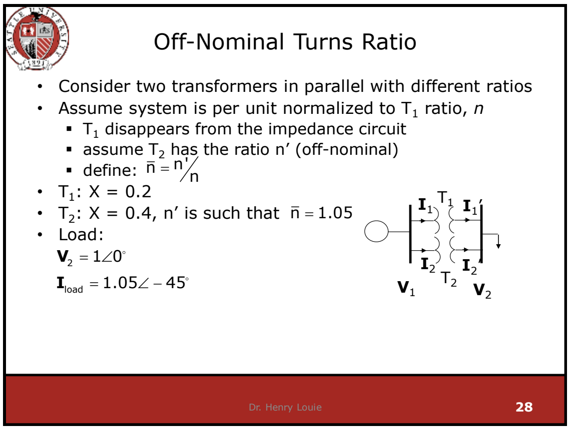

Off-Nominal Turns Ratio

- Consider two transformers in parallel with different ratios
- Assume system is per unit normalized to T_1 ratio, *n*
  - T_1 disappears from the impedance circuit
  - assume T_2 has the ratio n' (off-nominal)
  - define: $\bar{n} = n'/n$
- T_1: X = 0.2
- T_2: X = 0.4, n' is such that $\bar{n} = 1.05$
- Load:

$\mathbf{V}_2 = 1\angle 0^\circ$

$\mathbf{I}_{load} = 1.05\angle -45^\circ$

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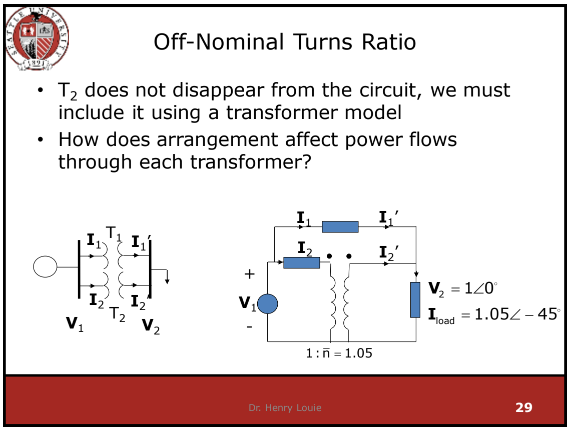

Off-Nominal Turns Ratio

- T_2 does not disappear from the circuit, we must include it using a transformer model
- How does arrangement affect power flows through each transformer?

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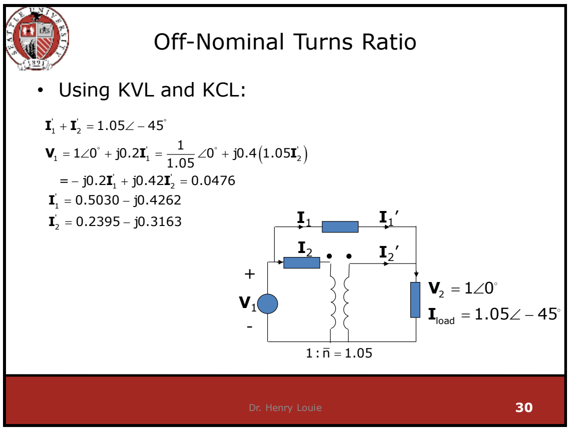

Off-Nominal Turns Ratio

- Using KVL and KCL:

$$\mathbf{I}_1' + \mathbf{I}_2' = 1.05\angle -45^\circ$$
$$\mathbf{V}_1 = 1\angle 0^\circ + j0.2\mathbf{I}_1' = \frac{1}{1.05}\angle 0^\circ + j0.4\left(1.05\mathbf{I}_2'\right)$$
$$= -j0.2\mathbf{I}_1' + j0.42\mathbf{I}_2' = 0.0476$$
$$\mathbf{I}_1' = 0.5030 - j0.4262$$
$$\mathbf{I}_2' = 0.2395 - j0.3163$$

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Off-Nominal Turns Ratio

- Finding the power:

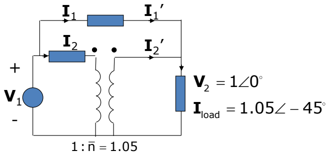
$$\mathbf{S}_{T_1} = \mathbf{V}_2\mathbf{I}_1'^{*} = 0.5030 + j0.4262$$

$$\mathbf{S}_{T_2} = \mathbf{V}_2\mathbf{I}_2'^{*} = 0.2395 + j0.3163$$

- If the turns ratios were the same:

$\mathbf{S}_{T_1} = 0.4950 + j0.4950$ note the large

$\mathbf{S}_{T_2} = 0.2475 + j0.2475$ affect on VARS



Summary

- The system model procedure discussed takes a one line diagram of a power system and produces an impedance diagram
  - per unit is convenient and lends itself to three phase or single phase quantities
  - most transformers disappear from the system
- Regulating transformers can be used to adjust real and reactive power flows through the system
- Next lecture we begin to focus on two other representations of the system using network matrices