14-Per Unit

Text: 5.5 – 5.7

ECEGR 451
Power Systems

Topics

- Definitions
- Per Unit Analysis
- Per Unit & Transformers
- Three-Phase Per Unit

Per Unit

- Per unit (p.u.) simplifies system calculations
  - eliminates partitions introduced by transformers
  - erroneous values are easily spotted
- Normalizes values in the system

$$\text{per unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

Per Unit

- Voltage

$$V_{p.u.} = \frac{V}{V_{Base}}$$

- Current

$$I_{p.u.} = \frac{I}{I_{Base}}$$

- Power

$$S_{p.u.} = \frac{S}{S_{Base}}$$

- Impedance

$$Z_{p.u.} = \frac{Z}{Z_{Base}}$$

Note: base values are scalars

Per Unit

- Example

  if V_B is selected to be 125,000 volts and V = 137,500 at zero degrees, then

$$\mathbf{V}_{p.u.} = \frac{137{,}500\angle 0^{\circ}\ \text{V}}{125{,}000\ \text{V}} = 1.1\angle 0^{\circ}$$

- Note: units cancel, so $V_{p.u.}$ is technically unitless, but it is common to add a "per unit" at the end

Base Selection

- Proper selection of base quantities is essential to simplify the problem

  base must be a real scalar

  select base quantities to obey Ohm's Law

$$V_B = I_B Z_B$$

therefore:

$$\frac{\mathbf{V}}{V_B} = \frac{\mathbf{IZ}}{I_B Z_B}$$

$$\Rightarrow \mathbf{V}_{p.u.} = \mathbf{I}_{p.u.}\mathbf{Z}_{p.u.}$$

Per Unit

- We follow the same procedure to define the power base

$$\mathbf{S} = \mathbf{V}\mathbf{I}^*$$

$$\frac{\mathbf{S}}{S_B} = \frac{\mathbf{V}\mathbf{I}^*}{V_B I_B}$$

$$\Rightarrow \mathbf{S}_{p.u.} = \mathbf{V}_{p.u.}\mathbf{I}^*_{p.u.}$$

$$\mathbf{S}_{p.u.} = P_{p.u.} + jQ_{p.u.}$$ note error in the book

$$P_{p.u.} = \frac{P}{S_B}$$

$$Q_{p.u.} = \frac{Q}{S_B}$$

Per Unit

- For impedance:

$$\mathbf{Z}_{p.u.} = R_{p.u.} + jX_{p.u.}$$

$$R_{p.u.} = \frac{R}{Z_B}$$

$$X_{p.u.} = \frac{X}{Z_B}$$

$$Y_B \triangleq \frac{I_B}{V_B} = \frac{1}{Z_B}$$

Example

Let V_B = 100kV and I_B = 50A, compute Z_B and S_B.

Example

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$$V_B = I_B Z_B$$

$$Z_B = \frac{V_B}{I_B} = 2000\Omega$$

$$S_B = V_B I_B = 5\text{MVA}$$

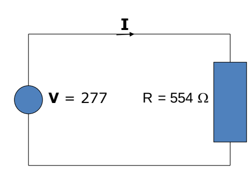
Per Unit Base Selection

- There are two equations that govern the relationship between base values

  $V_B = I_B Z_B$ $\qquad$ $S_B = V_B I_B$
- There are four base values that can be assigned
  - V_B, I_B, Z_B, S_B
- Convention is to select V_B and S_B and compute Z_B and I_B
  - V_B is typically selected such that the voltage levels are between 0.95 to 1.05 p.u.

Example

- Let
  - V_B = 277 V
  - S_B = 69.25 VA
- Compute Z_B and I_B

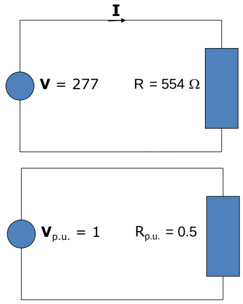


Example

- Let
  - V_B = 277 V
  - S_B = 69.25 VA
- Compute Z_B and I_B

$$Z_B = \frac{V_B^2}{S_B} = 1108\Omega$$

$$I_B = \frac{V_B}{Z_B} = 0.25A$$



equivalent circuit

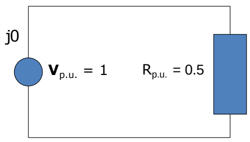
Per Unit Example

- We can easily solve the circuit

$$\mathbf{I}_{p.u.} = \frac{\mathbf{V}_{p.u.}}{Z_{p.u.}} = \frac{1\angle 0^\circ}{0.5} = 2\angle 0^\circ$$

$$\mathbf{S}_{p.u.} = \mathbf{V}_{p.u.}\mathbf{I}^*_{p.u.} = 1\angle 0^\circ \times 2\angle 0^\circ = 2 + j0$$

- We may wish to convert back to current and power:
  - $\mathbf{I}$ = 2/0.25 = 0.5A
  - $\mathbf{S}$ = 2/69.25 = 138.5VA



equivalent circuit

Per Unit & Transformers

- One of the most important advantages of using per unit is realized when transformers are involved

V_{1B}, S_B, I_{1B}, Z_{1B} } side 1 bases

side 2 bases { $V_{2B} = nV_{1B}$, S_B, $I_{2B} = aI_{1B}$, $Z_{2B} = n^2 Z_{1B}$

Per Unit & Transformers

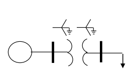
- We then get

$$\mathbf{V}_{2,a'n',p.u.} = \frac{\mathbf{V}_{2,a'n'}}{V_{2B}} = \frac{n\mathbf{V}_{1,an}}{nV_{1B}} = \mathbf{V}_{1,an,p.u.} \qquad \mathbf{I}_{2p.u.} = \frac{\mathbf{I}_2}{I_{2B}} = \frac{a\mathbf{I}_1}{aI_{1B}} = \mathbf{I}_{1p.u.}$$

And can remove the ideal transformer

Example

- Transformer nameplate:
  - 22 kV Y/13.8 kV Y
  - S_{rated} = 60 MVA (3 phase)
  - jX_L = j0.0807 Ohms
- Load:
  - R = 5 Ohms
- Generator:
  - V_{rated}= 22 kV
  - S_{rated}= 60 MVA
- Find the power out of the generator



Example

- Transformer nameplate:
  - 22 kV Y/13.8 kV Y
  - S_{rated} = 60 MVA (3 phase)
  - jX_L = j0.0807 Ohms
- Load:
  - R = 5 Ohms
- Generator:
  - V_{rated}= 22 kV
  - S_{rated}= 60 MVA
- Assign bases
  - generator side
    - V_{1B}= 22kV/(1.73) =12.7 kV
    - S_B=20 MVA
  - load side
    - V_{2B}=13.8kV/(1.73) = 7.9 kV
    - S_B = 20 MVA

Example

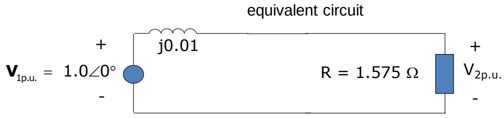
- Assigned bases
  - generator side
    - V_{1B}= 22kV/(1.73) =12.7 kV
    - S_B=20 MVA
  - load side
    - V_{2B}=13.8kV/(1.73) = 7.9 kV
    - S_B = 20 MVA
- Computed bases
  - Z_{1B}= $(V_{1B})^2/S_B$ = 8.07 Ω
  - Z_{2B} = $(V_{2B})^2/S_B$ =3.174 Ω
  - I_{1B} = V_{1B}/Z_{1B} =1574.6 A
  - I_{2B} = 2510.2 A

Example

- Transformer impedance (p.u.)
  - $X_{lp.u.}$ = 0.0807/8.07= 0.01 p.u.
- Load (p.u.):
  - $R_{p.u.}$ = (5/3.174) = 1.576 p.u.
- Generator (p.u.):
  - $V_{p.u.}$= 12.7kV/12.7kV =1 p.u.

Example

- Transformer
  - $X_{lp.u.}$ = 0.01 p.u.
- Load:
  - $R_{p.u.}$ = 1.575 p.u.
- Generator:
  - $V_{p.u.}$= 1 p.u.



Example

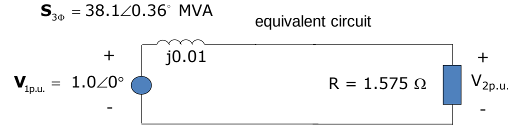
- Solving

$$\mathbf{I}_{p.u.} = \frac{1}{1.575 + 0.01j} = 0.634\angle -0.36^\circ$$

$$\mathbf{S}_{p.u.} = (1\angle 0)(0.634\angle 0.36^\circ) = 0.634\angle 0.36^\circ$$

$$\mathbf{S} = (S_B\mathbf{S}_{p.u.}) = 12.7\angle 0.36^\circ \text{ MVA}$$

$$\mathbf{S}_{3\phi} = 38.1\angle 0.36^\circ \text{ MVA}$$



Example

- To find current:

$$\mathbf{I}_1 = I_{1B}\mathbf{I}_{p.u.} = 1574.6(0.634\angle -0.36^\circ) = 999.73\angle -0.36^\circ \text{A}$$

$$\mathbf{I}_2 = I_{2B}\mathbf{I}_{p.u.} = 2510.2(0.634\angle -0.36^\circ) = 1593.7\angle -0.36^\circ \text{A}$$

Notes on the Example

- Transformer impedances are usually given in p.u. (expressed as %) based on three-phase power and line-line voltage
  - i.e. X_l = 1% = 0.01
- A per-phase diagram using per unit values is called an impedance diagram
- We considered a Y-Y case. For normal systems, phase shifts are ignored. See text page 160 for other xfmr connection notes

Three-Phase Per Unit

- So far we have dealt with per unit as defined by single phase expressions
- Interested in extension to three-phase
  - line-line voltages are used for V_B
  - three-phase power used for S_B

Three-Phase Per Unit

- As with three phase circuits, convention is to always give power base as three-phase, and voltage base as line-line
- Extra notation is given in this lecture and is omitted later

$$S_B^{3\Phi} \triangleq 3S_B$$

$$V_B^{ll} \triangleq \sqrt{3}V_B$$

$$I_B = \frac{S_B^{3\Phi}}{\sqrt{3}V_B^{ll}} = \frac{S_B}{V_B}$$

$$Z_B = \frac{(V_B)^2}{S_B} = \frac{\left(V_B^{ll}/\sqrt{3}\right)^2}{S_B^{3\Phi}/3} = \frac{V_B^{ll}}{S_B^{3\Phi}}$$

$$S_B^{3\Phi} \neq V_B^{ll}I_B$$

Change of Base

- Impedance of machines may not be given in the same base as used in the system
- Base conversion of impedance can save time

$$\mathbf{Z}_{actual} = \mathbf{Z}_{p.u.}^{old}Z_B^{old} = \mathbf{Z}_{p.u.}^{new}Z_B^{new}$$

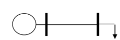
$$\mathbf{Z}_{p.u.}^{new} = \mathbf{Z}_{p.u.}^{old}\frac{Z_B^{old}}{Z_B^{new}}$$

$$\mathbf{Z}_{p.u.}^{new} = \mathbf{Z}_{p.u.}^{old}\left(\frac{V_B^{old}}{V_B^{new}}\right)^2\frac{S_B^{new}}{S_B^{old}}$$

Per Unit (single vs three-phase)

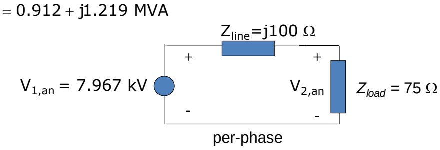
- Given: $\mathbf{Z}_{line}$ = j100 Ω, $\mathbf{Z}_{load}$ = 75 Ω (per phase) and $\mathbf{V}_1$ =13.8 kV (line-line)
- First without using per unit

$$\mathbf{I} = \frac{7.967\text{kV}}{j100+75} = 63.73\angle -53.13^\circ$$



$$\mathbf{S} = \mathbf{V}_{1,an}\mathbf{I}^* = 0.304 + j0.406 \text{ MVA}$$

$$\mathbf{S}_{3\phi} = 3\mathbf{S} = 0.912 + j1.219 \text{ MVA}$$



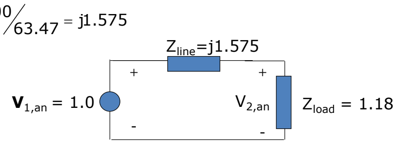
Per Unit (single vs three-phase)

- Now using per unit with V_B = 13.8/sqrt(3), S_B = 1 MVA

$$Z_B = \frac{(7.967)^2}{1} = 63.47\ \Omega$$

$$\mathbf{Z}_{Load} = 75/63.47 = 1.18$$

$$\mathbf{Z}_{Line} = j100/63.47 = j1.575$$



Per Unit (single vs three-phase)

- Now find the single-phase and three-phase power from the generator:

$$\mathbf{I}_{pu} = \frac{1}{j1.575+1.18} = 0.508\angle -53.13^\circ$$

$$\mathbf{S}_{pu} = \mathbf{V}_{1,pu}\mathbf{I}_{pu}^* = 0.304 + j0.406 \text{ p.u.}$$

$$\mathbf{S} = S_B\mathbf{S}_{pu} = 0.304 + j0.406 \text{ MVA}$$

$$\mathbf{S}_{3\phi,pu} = 3\mathbf{S}_{pu} = 0.912 + j1.219 \text{ p.u.}$$

$$\mathbf{S}_{3\phi} = S_B\left(\mathbf{S}_{pu}\right) = 0.912 + j1.219 \text{ MVA}$$

Per Unit (single vs three-phase)

- We found the single-phase power to be:
  $\mathbf{S}_{pu} = 0.304 + j0.406$ p.u.
  $\mathbf{S} = 0.304 + j0.406$ MVA
  and the three-phase power to be:
  $\mathbf{S}_{3\Phi,pu} = 0.912 + j1.219$ p.u.
  $\mathbf{S}_{3\Phi} = 0.912 + j1.219$ MVA
- We define the three-phase base power as:
  $S_B^{3\Phi} = 3S_B = 3$ MVA
  $$\mathbf{S}_{3\Phi,pu} = \frac{0.912 + j1.219 \text{ MVA}}{3 \text{ MVA}} = 0.304 + j0.406 \text{ p.u.}$$

Per Unit (single vs three-phase)

- If the single phase power is 0.304 + j0.406 p.u., the three phase power is also 0.304 + j0.406 p.u.
- A voltage of 1.0 p.u. in the single phase circuit implies that the line-line voltage is 1.0 p.u.
- When we do a per-phase analysis, the results can be applied the three-phase quantities; just remember to use the correct base when converting back to the actual values
- If we dealing with per unit values only, you need not distinguish between per phase and three phase power, nor line-line voltage and line-neutral voltage

Per Unit (single vs three-phase)

- For example: you are doing a problem with V_B = 22 kV (line-line), and S_B = 100 MVA (three-phase). You do a per-phase analysis and find:
- $\boldsymbol{V}_{1,an,pu}$ = 0.90 p.u.
- $\boldsymbol{S}_{pu}$ = 1.5 p.u. (per phase power)
- What is the line-line voltage at V_1?
  $V_1 = 0.9(22\text{kV}) = 19.8\text{kV}$
- What about the three phase power?
  $S_{3\Phi} = 1.5(100 \text{ MVA}) = 150 \text{ MVA}$

Per Unit (transformer connections)

- If we use the bases corresponding to the nameplate ratings, the per unit reactances have the same numerical values in the Y-Y, Δ-Y, Δ-Δ and Y-Δ connections as in the single phase case
- See example 5.11 for details

Per Unit Analysis Procedure (normal systems)

1. Pick a VA base for the entire system (usually largest generator rating)
2. Arbitrarily select one voltage base. Relate all others by the ratio of the magnitudes of the open-circuit line voltages of each transformer bank
3. Find the impedance bases in the different sections and express all impedances in consistent per unit terms.
4. Draw the impedance diagram for the entire system, and solve for the desired per unit quantities.
5. Convert back to actual quantities, if needed.