14-Ideal Single Phase Transformers

ECEGR 450
Electromechanical Energy Conversion

Overview

- Transformers
- Ideal Single-Phase Transformer
- Transformer Polarity
- Circuit Model

Questions

- Why are transformers used in power systems?
- Is it possible to use a "dc" transformer?
- Are transformers efficient?
- How is the power into a transformer related to the power out of a transformer?

Introduction

Introduction

Introduction

"homemade" Zambian transformer

Introduction

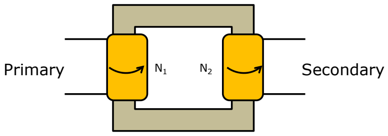
- Review Chapter 2
- Transformers are important electrical-electrical energy conversion components
- One important reason we use AC is because we can easily change the voltage levels, which reduces losses
- Transformers enable this conversion of voltage level
  - High efficiency (up to 99%)
  - No or few moving parts (low maintenance)

Transformers

- Shift between voltage levels
  - generation 11 kV to 30 kV
  - transmission up to 765 kV
  - distribution around 69 kV
  - residential 240/120 V
- Controlling voltages, power flows
  - regulating transformers
- Isolation (dc current)
- Instrument
  - PTs, CTs

Ideal Single-Phase Transformer

- Two magnetically coupled coils
  - Primary: N_1 turns
  - Secondary: N_2 turns
- Primary and secondary can be arbitrarily assigned
- Note direction of windings

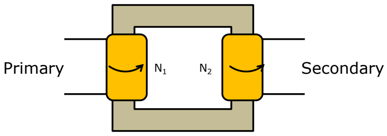


Ideal Single-Phase Transformer

Ideal assumptions

- No flux leakage
- No eddy currents
- No winding resistance
- Near infinite core permeability

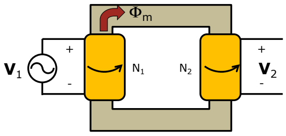
} Recall from magnetic circuits lecture



Single-Phase Transformer

- Primary directly connected to AC voltage source
- Voltage across coil has sinusoidal flux associated with it

$$e = -\frac{d\phi}{dt} \quad \text{(Faraday's Law)}$$



Single-Phase Transformer

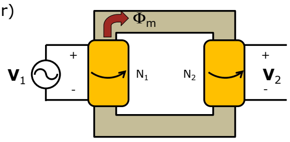
- Same flux passes through each coil
  - Φ_m: mutual flux (phasor)
- Therefore:

$$\mathbf{V}_1 = \frac{d\lambda_1}{dt} = N_1\frac{d\Phi_m}{dt}$$

$$\mathbf{V}_2 = \frac{d\lambda_2}{dt} = N_2\frac{d\Phi_m}{dt}$$

- Rewritten:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \triangleq a \triangleq \frac{1}{n}$$

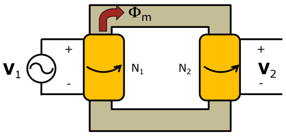


Single-Phase Transformer

- Ratio of voltages is the same as the ratio of turns

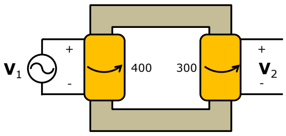
  $\frac{V_1}{V_2} = \frac{N_1}{N_2}$
- Possible to transform voltage level from primary to secondary (and vice versa)
- Note: no current flows
  - (near infinite permeability)

  $\phi = BA = \frac{\mu N i A}{\ell}$



Question

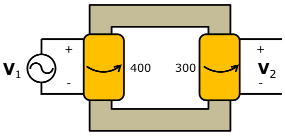
If |$\mathbf{V}_1$| = 120V, is |$\mathbf{V}_2$| greater than 120V?



Question

If |$\mathbf{V}_1$| = 120V, is |$\mathbf{V}_2$| greater than 120V?

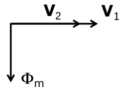
Less than 120V.



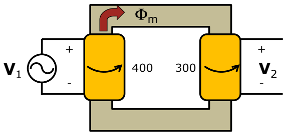
The winding with more turns has greater voltage

Single-Phase Transformer

Phasor Diagram (ideal, no load)



$\mathbf{V}_1$, $\mathbf{V}_2$ in phase.
Φ_m lags voltage by 90°
No current flows



Single-Phase Transformer

- Now a resistive load is connected to the secondary
- $\mathbf{V}_2$ causes $\mathbf{I}_2$ to flow
- Examining mmf

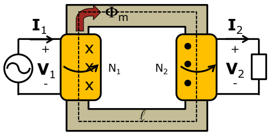
  $\mathfrak{I} = N_1\mathbf{I}_1 - N_2\mathbf{I}_2 = \mathfrak{R}\mathbf{\Phi}_m$
- Infinite permeability

  $\mathfrak{I} = N_1\mathbf{I}_1 - N_2\mathbf{I}_2 = 0 \longleftarrow \mathfrak{R} = \frac{\ell}{\mu A}$

  $N_1\mathbf{I}_1 = N_2\mathbf{I}_2$
- Current gain

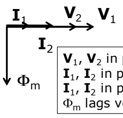
  $\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = a$

Compare to: $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

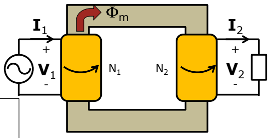


Single-Phase Transformer

Phasor Diagram (ideal, resistive load)



$\mathbf{V}_1$, $\mathbf{V}_2$ in phase.
$\mathbf{I}_1$, $\mathbf{I}_2$ in phase.
$\mathbf{I}_1$, $\mathbf{I}_2$ in phase with Φ_m
Φ_m lags voltage by 90°



Note: |Φ| arbitrarily drawn

Question

In an ideal transformer serving a load, if $|\mathbf{V}_1| > |\mathbf{V}_2|$, is $|\mathbf{I}_1| > |\mathbf{I}_2|$?

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No. The transformer would be creating energy.

Question

How are the transformer input and output power related? Find α in $P_1 = \alpha P_2$

Single-Phase Transformer

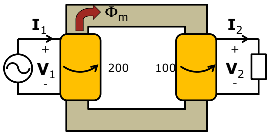
- Power into the transformer

  $P_1 = \mathrm{Re}\{\mathbf{V}_1\mathbf{I}_1{*}\}$
- Power out of the transformer

  $P_2=\mathrm{Re}\{\mathbf{V}_2\mathbf{I}_2^*\}=\mathrm{Re}\{\frac{1}{a}\mathbf{V}_1\mathbf{I}_2^*\}=\mathrm{Re}\{\frac{1}{a}\mathbf{V}_1 a\mathbf{I}_1^*\}=P_1$
- Power is conserved

Question

- Now a load with PF = 0.707 lagging is connected to the secondary
- Draw the phasor diagram of $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, $\mathbf{I}_2$, Φ_m



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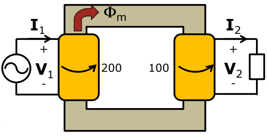
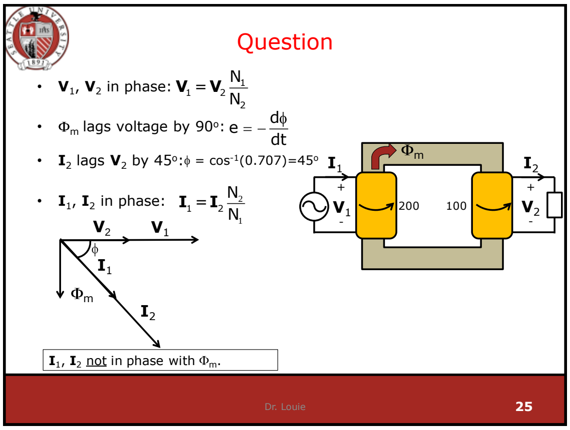


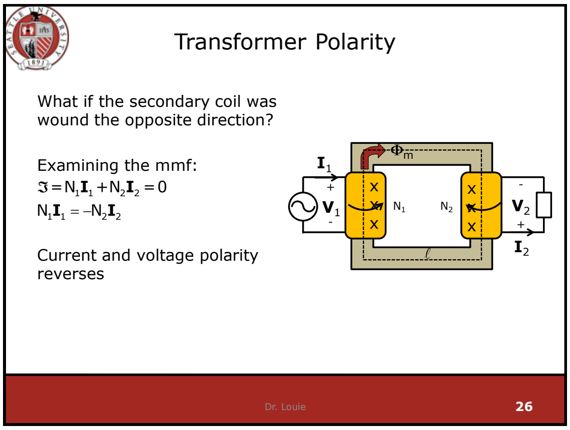
$|\Phi_m|$, $|\mathbf{I}|$ can be arbitrarily drawn with respect to each other



Question

- $\mathbf{V}_1$, $\mathbf{V}_2$ in phase: $\mathbf{V}_1 = \mathbf{V}_2 \frac{N_1}{N_2}$
- Φ_m lags voltage by 90°: $e = -\frac{d\phi}{dt}$
- $\mathbf{I}_2$ lags $\mathbf{V}_2$ by 45°: $\phi = \cos^{-1}(0.707) = 45°$
- $\mathbf{I}_1$, $\mathbf{I}_2$ in phase: $\mathbf{I}_1 = \mathbf{I}_2 \frac{N_2}{N_1}$

$\mathbf{I}_1$, $\mathbf{I}_2$ not in phase with Φ_m.

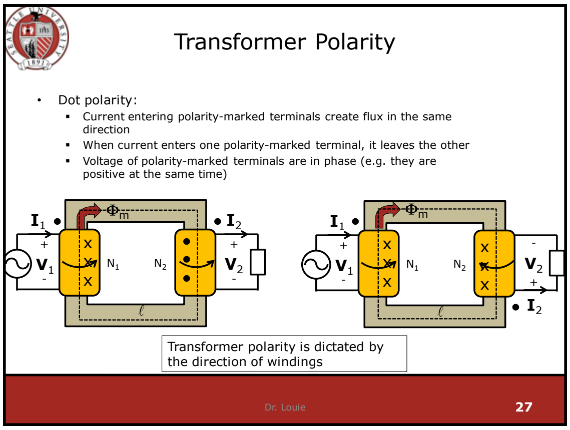


Transformer Polarity

What if the secondary coil was wound the opposite direction?

Examining the mmf:
$\Im = N_1 \mathbf{I}_1 + N_2 \mathbf{I}_2 = 0$
$N_1 \mathbf{I}_1 = -N_2 \mathbf{I}_2$

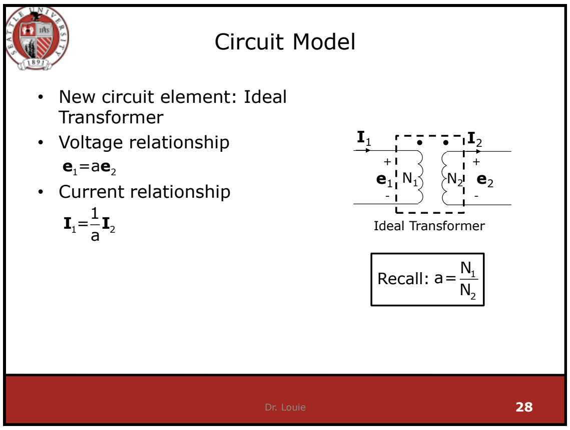
Current and voltage polarity reverses



Transformer Polarity

- Dot polarity:
  - Current entering polarity-marked terminals create flux in the same direction
  - When current enters one polarity-marked terminal, it leaves the other
  - Voltage of polarity-marked terminals are in phase (e.g. they are positive at the same time)

Transformer polarity is dictated by the direction of windings

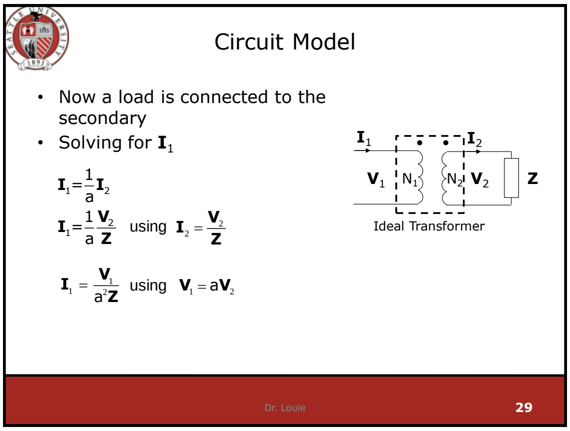


Circuit Model

- New circuit element: Ideal Transformer
- Voltage relationship
  $\mathbf{e}_1 = a\mathbf{e}_2$
- Current relationship
  $\mathbf{I}_1 = \frac{1}{a}\mathbf{I}_2$

Ideal Transformer

Recall: $a = \frac{N_1}{N_2}$



Circuit Model

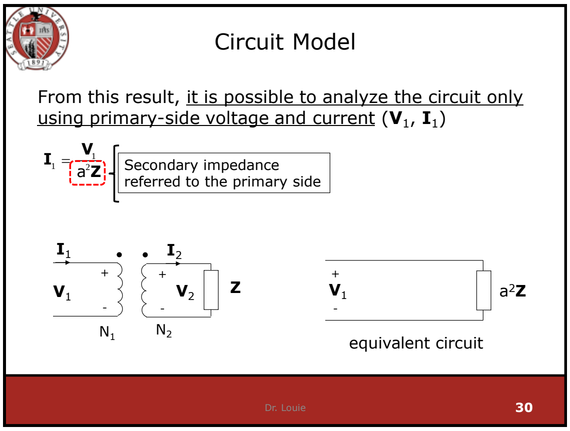
- Now a load is connected to the secondary
- Solving for $\mathbf{I}_1$

$\mathbf{I}_1 = \frac{1}{a}\mathbf{I}_2$

$\mathbf{I}_1 = \frac{1}{a}\frac{\mathbf{V}_2}{\mathbf{Z}}$ using $\mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}}$

$\mathbf{I}_1 = \frac{\mathbf{V}_1}{a^2\mathbf{Z}}$ using $\mathbf{V}_1 = a\mathbf{V}_2$

Ideal Transformer



Circuit Model

From this result, it is possible to analyze the circuit only using primary-side voltage and current ($\mathbf{V}_1$, $\mathbf{I}_1$)

$\mathbf{I}_1 = \frac{\mathbf{V}_1}{a^2\mathbf{Z}}$

Secondary impedance referred to the primary side

equivalent circuit

Example

Consider an ideal transformer with N_1 = 100 and N_2 = 500. The primary is connected to a 100 V source. A load of 100 Ohms is connected to the secondary.

Find the power delivered to the load.

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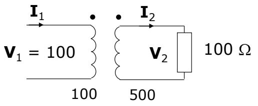
$$a = \frac{N_1}{N_2} = \frac{100}{500} = 0.2$$



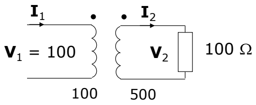
equivalent circuit

Example

Consider an ideal transformer with N_1 = 100 and N_2 = 500. The primary is connected to a 100 V source. A load of 100 Ohms is connected to the secondary.

Find the power delivered to the load.

$$P = \frac{|\mathbf{V}_1|^2}{R} = \frac{10{,}000}{4} = 2{,}500 \text{ W}$$



equivalent circuit

Summary

- Transformers are magnetically coupled coils
- Ratio of turns from primary to secondary is the “turns ratio”. Side with greater number of turns has higher voltage, but lower current
- Ideal transformers: Power in = Power out
- Equivalent circuit is used to analyze transformers. Impedances can be transferred from secondary to primary by scaling by a^2