13-Transformer Review

Text: 5.1 – 5.3

ECEGR 451
Power Systems

Overview

- Transformers
- Ideal Single-Phase
- Non-Ideal Single Phase
- Three-Phase Transformers
- Normal Systems

Introduction

Introduction

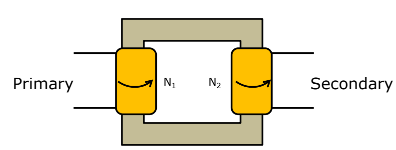
- Transformers are important electrical-electrical energy conversion components
- One important reason we use AC is because we can easily change the voltage levels, which reduces losses
- Transformers enable this conversion of voltage level
  - High efficiency (up to 99%)
  - No or few moving parts (low maintenance)

Transformers

- Shift between voltage levels
  - generation 11 kV to 30 kV
  - transmission up to 765 kV
  - distribution around 69 kV
  - residential 240/120 V

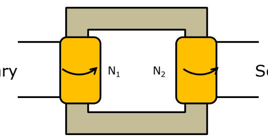
Ideal Single-Phase Transformer

- Two magnetically coupled coils
  - Primary: N_1 turns
  - Secondary: N_2 turns
- Primary and secondary can be arbitrarily assigned
- Note direction of windings



Ideal Single-Phase Transformer

Ideal assumptions
- No flux leakage
- No eddy currents
- No winding resistance
- Near infinite core permeability

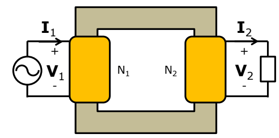


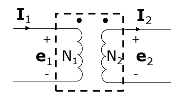
Single-Phase Transformer

- Recall

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2} \triangleq a \triangleq \frac{1}{n}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = a$$





Ideal Transformer

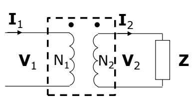
Circuit Model

- Now a load is connected to the secondary
- Solving for $\mathbf{I}_1$

$$\mathbf{I}_1 = \frac{1}{a}\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{1}{a}\frac{\mathbf{V}_2}{\mathbf{Z}} \quad \text{using} \quad \mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{a^2\mathbf{Z}} \quad \text{using} \quad \mathbf{V}_1 = a\mathbf{V}_2$$



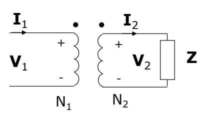
Ideal Transformer

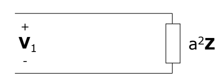
Circuit Model

From this result, <u>it is possible to analyze the circuit only using primary-side voltage and current</u> ($\mathbf{V}_1$, $\mathbf{I}_1$)

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{a^2\mathbf{Z}}$$

Secondary impedance referred to the primary side

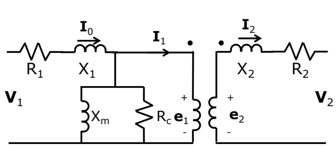




equivalent circuit

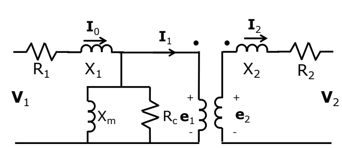
Non-Ideal Model

- Transformer non-idealities:
  - Winding resistance
  - Leakage reactance
  - Magnetizing reactance
  - Core loss



Non-Ideal Model

- For most transformers:
  - $R_c > X_m$
  - $R_1 < X_1$, $R_2 < X_2$

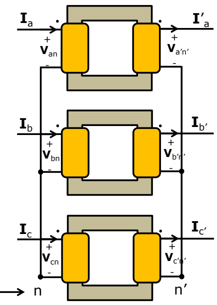


Three-Phase Transformer Connections

- We are generally more concerned with three phase transformers
- Three-phase transformers can be composed of three single-phase transformers or be wound on the same core
- Many combinations are possible:
  - Y-Y
  - Delta-Delta
  - Y-Delta
  - Delta-Y

Y-Y Transformer

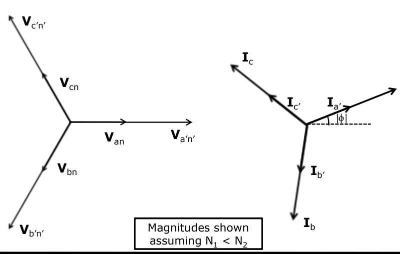
- Each side has a common point, n or n′
- Neutral points usually grounded
- Line-Neutral voltages appear across the coils on each side



Do not confuse this n with 1/a → n

Y-Y Transformer Analysis

Phasor Diagram for Ideal Y-Y Connected Transformer



Three-Phase Transformer Analysis: Y-Y

- Let k be the transformer voltage gain
- For Y-Y transformers:

$$V_{a'n'} = kV_{an} = \frac{N_2}{N_1} V_{an}$$

therefore k = n ← Do not confuse this n with the neutral point

$$I_{a'} = \frac{I_a}{k} = \frac{I_a}{n}$$

$$V_{a'b'} = nV_{ab}$$

Three-Phase Transformer Analysis: Y-Y

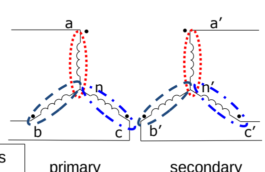
- Phase voltages

$$V_{a'n'} = nV_{an}, \quad V_{a'b'} = nV_{ab}$$
$$V_{b'n'} = nV_{bn}, \quad V_{b'c'} = nV_{bc}$$
$$V_{c'n'} = nV_{cn}, \quad V_{c'a'} = nV_{ca}$$

- Line currents

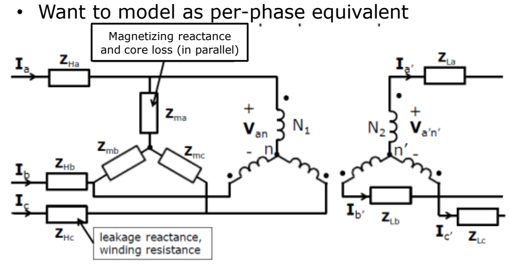
$$I_{a'} = \frac{I_a}{n}$$
$$I_{b'} = \frac{I_b}{n}$$
$$I_{c'} = \frac{I_c}{n}$$

Recall: phase currents = line current for Y connections



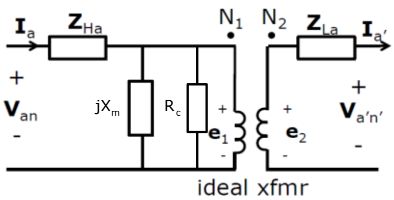
Per-Phase Y-Y Analysis

- Three phase non-ideal Y-Y transformer model
- Want to model as per-phase equivalent



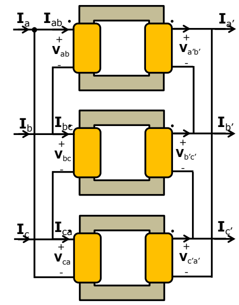
Per Phase Y-Y Analysis

- Per-phase equivalent of non-ideal Y-Y transformer
- Analyze like a single-phase transformer



Δ-Δ Transformer

- No neutral point
- $\mathbf{V}_{ab}$, $\mathbf{V}_{a'b'}$ in phase
- $\mathbf{I}_{ab}$, $\mathbf{I}_{a'b'}$ in phase
  - Similar results for b, c phase
- Line-line voltages appear across the coils on primary and secondary



Three-Phase Transformer Analysis: Δ-Δ

- Let k be the transformer voltage gain
- For Δ-Δ transformers:

$$\mathbf{V}_{a'b'} = k\mathbf{V}_{ab} = \frac{N_2}{N_1}\mathbf{V}_{ab}$$

therefore k = n

$$\mathbf{I}_{a'b'} = \frac{\mathbf{I}_{ab}}{k} = \frac{\mathbf{I}_{ab}}{n}$$

$$\mathbf{V}_{a'n'} = n\mathbf{V}_{an}$$

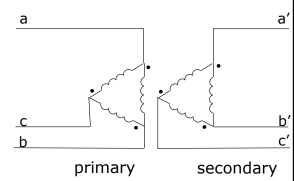
Three-Phase Transformer Analysis: Δ–Δ

- Voltage Relationships

$$\mathbf{V}_{a'b'} = n\mathbf{V}_{ab}$$
$$\mathbf{V}_{b'c'} = n\mathbf{V}_{bc}$$
$$\mathbf{V}_{c'a'} = n\mathbf{V}_{ca}$$

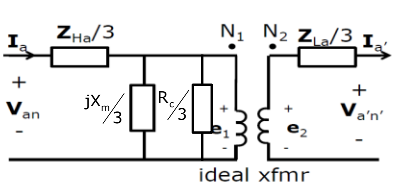
- Current Relationships

$$\mathbf{I}_{a'} = \frac{\mathbf{I}_a}{n}, \quad \mathbf{I}_{b'a'} = \frac{\mathbf{I}_{ab}}{n}$$
$$\mathbf{I}_{b'} = \frac{\mathbf{I}_b}{n}, \quad \mathbf{I}_{c'b'} = \frac{\mathbf{I}_{bc}}{n}$$
$$\mathbf{I}_{c'} = \frac{\mathbf{I}_c}{n}, \quad \mathbf{I}_{a'c'} = \frac{\mathbf{I}_{ca}}{n}$$



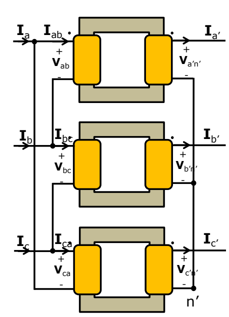
Per-Phase Δ-Δ Transformer

- Need to convert each side of the transformer to Y
- Recall that: $\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$
- Per-phase equivalent circuit of non-ideal Δ-Δ transformer



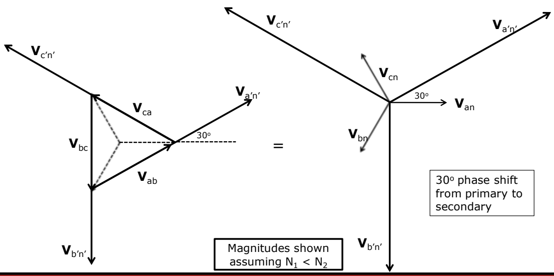
Δ-Y Transformer

- Secondary has a neutral connection
- <u>Line-line voltages appear on the coils on primary</u>
- <u>Line-neutral voltages appear on the coils on the secondary</u>
- Less insulation needed on HV winding



Δ-Y Transformer Analysis

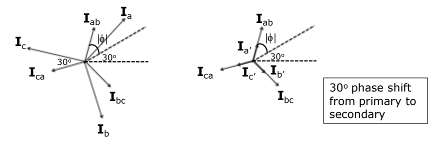
Phasor Diagram for Ideal Delta-Y Connected Transformer



Delta-Y Transformer Analysis

Phasor Diagram for Ideal Delta-Y Connected Transformer

$I_a = I_{ab} - I_{ca}$
$I_b = I_{bc} - I_{ab}$
$I_c = I_{ca} - I_{bc}$



Three-Phase Transformer Analysis: Delta-Y

- Let k be the transformer voltage gain
- For Δ-Y transformers:

$$V_{a'n'} = nV_{ab} = nV_{an}\sqrt{3}e^{\frac{j\pi}{6}} = kV_{an}$$

therefore $k = n\sqrt{3}e^{\frac{j\pi}{6}}$ Important result!

$$I_{a'} = \frac{I_{ab}}{n} = \frac{I_a e^{\frac{j\pi}{6}}}{\sqrt{3}n} = \frac{I_a}{k^*}$$

same phase shift as voltage

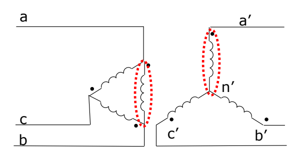
Delta-Y Transformer Analysis

- Voltage relationships

$V_{a'n'} = kV_{an}, \quad V_{a'b'} = kV_{ab}$
$V_{b'n'} = kV_{bn}, \quad V_{b'c'} = kV_{bc}$
$V_{c'n'} = kV_{cn}, \quad V_{c'a'} = kV_{ca}$

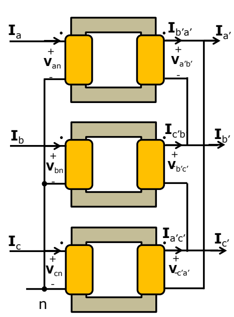
- Line current relationships

$I_{a'} = \frac{I_a}{k^*}$
$I_{b'} = \frac{I_b}{k^*}$
$I_{c'} = \frac{I_c}{k^*}$



Y-Δ Transformer

- No neutral point on secondary
- Line-Neutral voltage across primary coils
- Line-Line voltage across secondary coils



Three-Phase Transformer Analysis: Y-Δ

- Let k be the transformer voltage gain
- For Y-Δ transformers:

$$V_{a'b'} = nV_{an} = \frac{nV_{ab}e^{\frac{-j\pi}{6}}}{\sqrt{3}} = kV_{ab}$$

therefore $k = \frac{ne^{\frac{-j\pi}{6}}}{\sqrt{3}}$ Important result

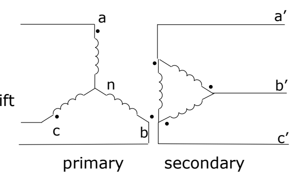
Note: phase shift is -30⁰, whereas for Delta-Y transformer connection in previous slides it is +30°

Three-Phase Transformer Analysis

- It can be shown for line currents:

$$\mathbf{I}_{a'} = \frac{\mathbf{I}_a}{k^*}, \quad \mathbf{I}_{b'} = \frac{\mathbf{I}_b}{k^*}, \quad \mathbf{I}_{c'} = \frac{\mathbf{I}_c}{k^*}$$

same phase-shift as voltages



Question

Is there any concern in connecting a three phase Y-Delta transformer in parallel with a three phase Delta-Y transformer? Assume the transformers are appropriately rated.

Three-Phase Transformer Analysis

- What about power?

$$\mathbf{S} = \mathbf{V}_{a'n'}\mathbf{I}_{a}^{'} = k\mathbf{V}_{an}\left(\frac{\mathbf{I}_a}{k^*}\right)^* = \mathbf{V}_{an}\mathbf{I}_a^*$$ for all configurations

- For ideal three-phase xfmrs, power is conserved, as in the single phase case

Three-Phase Transformer Analysis

- What about impedances?
- For all transformer connections:

secondary impedance referred to the primary $= \frac{1}{|k|^2}\mathbf{Z}_L$

Transformer Phase Shifts

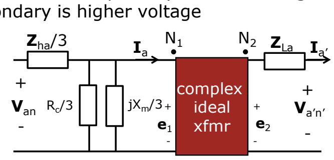
- Various winding connections of Δ-Y, and Y-Δ xfmrs lead to different phase shifts
- Standard is to have phase-neutral voltages advance by 30 degrees when going from low voltage to high voltage
- Previous slides have followed this convention

Per-Phase Analysis (Δ-Y)

- Per-phase equivalent of a non-ideal Δ-Y transformer
- Complex ideal transformer with gain k or k*

$k = \sqrt{3}ne^{j\pi/6}$

  - +30º shift when primary is lower voltage and secondary is higher voltage

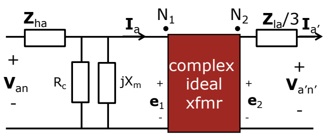


Per-Phase Analysis (Y-Δ)

- Per-phase equivalent of a Y-Δ transformer
- Complex ideal transformer with gain k or k*

$$k = \frac{n e^{j\pi/6}}{\sqrt{3}}$$

  - +30º shift when primary is lower voltage and secondary is higher voltage

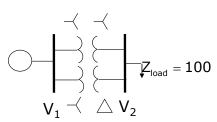


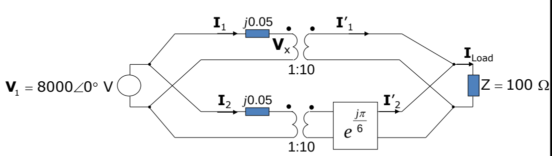
Normal Systems

- Parallel paths in the transmission network can result in circulating currents
- At the heart of the problem are the phase shifts and voltage ratios in transformers
- Systems that are "normal" avoid these problems
- A system is normal if, for each parallel path
  - the product of ideal transformer gain magnitudes is the same
  - the sum of ideal transformer phase shifts is the same

Normal Systems

- $\mathbf{V}_1$: line-neutral = 8000 V
- Y-Y: n =10, X_L = 0.05
- Y-Δ: n =$10\sqrt{3}$, X_L = 0.05
- Find $|\mathbf{I}_1|$, $|\mathbf{I}_2|$, $|\mathbf{I}_{load}|$





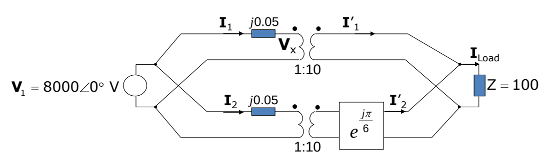
Normal Systems

- KVL for I_1 loop:

$$\mathbf{V}_1 = j0.05\mathbf{I}_1 + \mathbf{V}_x$$

$$= j0.05\mathbf{I}_1 + \frac{1}{10}100\ \mathbf{I}'_1 + \mathbf{I}'_2$$

$$\mathbf{V}_1 =\ 10 + j0.5\ \mathbf{I}'_1 + 10\mathbf{I}'_2 = 8000\angle 0°\ \text{V}$$ replacing $\mathbf{I}_1$ with $10\mathbf{I}_{1'}$



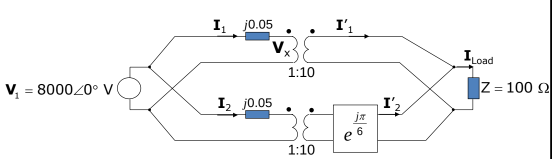
Normal Systems

- KVL around $\mathbf{I}_2$ loop:

$$\mathbf{V}_1 = j0.05\mathbf{I}_2 + \frac{1}{10}e^{\frac{-j\pi}{6}}\ 100\ \ \mathbf{I}'_1 + \mathbf{I}'_2$$

$$\mathbf{V}_1 = 10\mathbf{I}'_1 e^{\frac{-j\pi}{6}} +\ 10 + j0.5\ \mathbf{I}'_2 e^{\frac{-j\pi}{6}}$$ replacing $\mathbf{I}_2$ with $10\mathbf{I}'_2 e^{\frac{-j\pi}{6}}$

$$10\mathbf{I}'_1 +\ 10 + j0.5\ \mathbf{I}'_2 = 8000\angle 30°$$ shifting by 30 deg.



Normal Systems

- Solving:

$$10\mathbf{I}'_1 +\ 10 + j0.5\ \mathbf{I}'_2 = 8000\angle 30°$$

$$10 + j0.5\ \mathbf{I}'_1 + 10\mathbf{I}'_2 = 8000$$

Yields

$$\mathbf{I}'_1 = 3755.0\angle -164.85°\ \text{A}$$

$$\mathbf{I}'_2 = 4527.2\angle 14.88°\ \text{A}$$

$$\mathbf{I}_{load} = 772.5\angle 13.57°\ \text{A}$$

circulating current: only a small portion of the current flows into the load!

Normal Systems