13-Self and Mutual Inductance

ECEGR 450
Electromechanical Energy Conversion

Overview

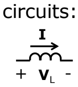
- Introduction
- Self Inductance
- Mutual Inductance

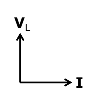
Inductance

- Transformers and other machines have coils of wire wrapped around permeable material
- Transformers are made of one or more inductors on a common core
- We will start with a qualitative description of inductance

Induced vs Applied Voltage

- From circuit theory: $v_L = L\frac{di}{dt}$
- Previously showed that for steady-state AC circuits: $\mathbf{V}_L = \mathbf{I}jX_L$





current lags voltage

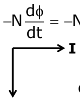
Induced vs Applied Voltage

- Let the current be: $i(t) = 1\cos(\omega t)$
- The flux will be in phase with the current due Ampere's Law $Ni = \mathfrak{R}\phi$

  $\phi(t) = \frac{N}{\mathfrak{R}}\cos(\omega t)$
- Applying Faraday's Law:

  $e = -N\frac{d\phi}{dt} = \frac{N^2}{\mathfrak{R}}\omega\sin(\omega t) = \frac{N^2}{\mathfrak{R}}\omega\cos(\omega t - 90°)$ or

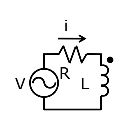
  $e = -N\frac{d\phi}{dt} = -N\frac{d\phi}{di}\frac{di}{dt} = -L\frac{di}{dt}$



current leads voltage

Induced vs Applied Voltage

- Why are induced and applied voltages opposite?
- It all has to do with sign convention
  - V = iR + Ldi/dt (Ldi/dt is on the right hand side)
  - V + e = iR (e is on the left hand side)
- Therefore e = -Ldi/dt

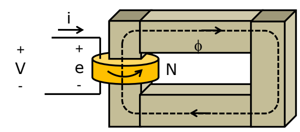


Inductance

- Inductive reactance X_L exists due to Faraday's Law
  - $jX_L = j\omega L$
- The j operator accounts for the 90 degree phase shift between current and induced voltage
- ω accounts for the dependency on frequency
- L is a description of how strong the current links the flux through the coil
- Next we examine inductance

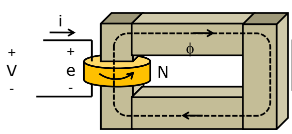
Inductance

- Recall that
  $e = N\frac{d\phi}{dt}$ (note the polarity in the figure)
  - $N\phi$ is also known as the flux linkages (λ)



Inductance

- Self-inductance (inductance) is defined as:
  $L \triangleq N\frac{d\phi}{di}$
- Large inductance: great sensitivity of flux wrt current



Inductance describes how the flux linking a coil changes with the applied current

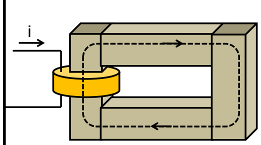
Self Inductance

- Inductance depends on the <u>physical characteristics of the magnetic circuit</u>
- Recall that
  $\phi = \frac{Ni}{\Re}$
  $L \triangleq N\frac{d\phi}{di}$ therefore
  $L = \frac{N^2}{\Re}$
- Inductance is constant if the permeability of the magnetic circuit is itself constant (not the case in ferromagnetic materials)
- We will assume that we are operating in the linear region of B-H curve

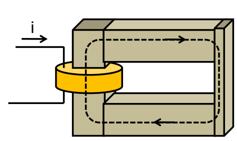
Question

Which circuit has greater inductance?

A.



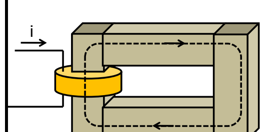
B.



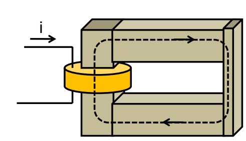
Question

Which circuit has greater inductance?

A.



B.



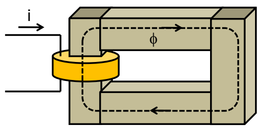
Smaller reluctance. Current gives rise to greater flux

Self Inductance

- Inductance is related to emf by:

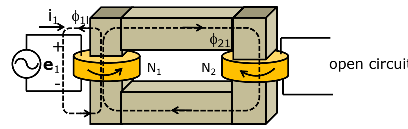
$$\mathbf{e} = N\frac{d\phi}{dt} = N\frac{d\phi}{di}\frac{di}{dt} = L\frac{di}{dt}$$

- A coil with 1 H of inductance will have 1 volt induced in it if the current changes at a rate of 1 A/s
- <u>If we know the inductance, we do not need to compute the flux</u>



Mutual Inductance

- Consider two coils wrapped around a common core
- Let ϕ_1 be the flux that links coil 1
  - Includes leakage flux (ϕ_{1l}) and flux through the core that links coil 2 (ϕ_{21})
  - With coil 2 open: $\phi_1 = \phi_{1l} + \phi_{21}$

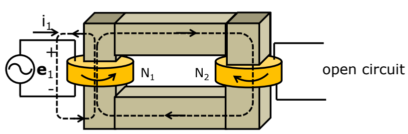


Mutual Inductance

- Induced voltage in coil 1 is:

$$e_1 = N_1\frac{d\phi_1}{dt} = L_1\frac{di_1}{dt}$$

- L_1: self inductance of coil 1



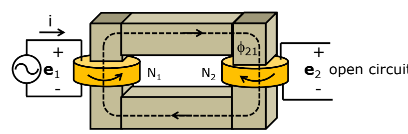
Mutual Inductance

- Induced voltage in coil 2 is:

$$e_2 = N_2\frac{d\phi_{21}}{dt} = N_2\frac{d\phi_{21}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt}$$

$$M_{21} \triangleq N_2\frac{d\phi_{21}}{di_1}$$

  - M_{21}: mutual inductance from coil 1 to coil 2



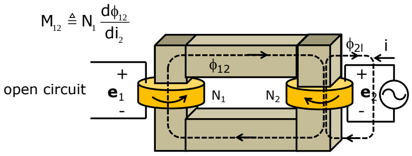
Mutual Inductance

- Similar expressions can be derived in coil 2 if it is connected to the source and coil 1 is open

$$e_2 = N_1\frac{d\phi_2}{dt} = L_2\frac{di_2}{dt}$$

$$e_1 = N_2\frac{d\phi_{12}}{dt} = N_1\frac{d\phi_{12}}{di_2}\frac{di_2}{dt} = M_{12}\frac{di_2}{dt}$$

$$M_{12} \triangleq N_1\frac{d\phi_{12}}{di_2}$$



Mutual Inductance

- We can write:

$$M_{12}M_{21} = N_2\frac{d\phi_{21}}{di_1}N_1\frac{d\phi_{12}}{di_2}$$

- Let k_1 be the fraction of the flux of coil 1 that links coil 2 $\phi_{21} = k_1\phi_1$
- Let k_2 be the fraction of the flux of coil 2 that links coil 1 $\phi_{12} = k_2\phi_2$
- Since $L = N\frac{d\phi}{di}$ we can write: $M_{12}M_{21} = k_1k_2L_1L_2$

Mutual Inductance

- If the system is linear (standard assumption) then:

  $M_{12} = M_{21} = M$

  - M: mutual inductance of coil 1 and coil 2
- And we can reduce

  $M_{12}M_{21} = k_1k_2L_1L_2$ to

  $M = k\sqrt{L_1L_2}$

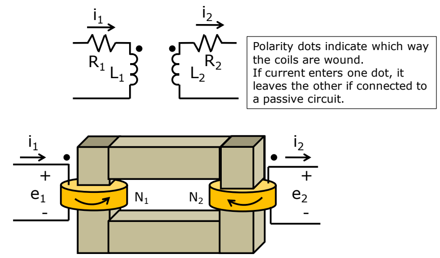
  $k = \sqrt{k_1k_2}$

  - k: coefficient of coupling

k = 1 (tightly coupled coils, no leakage)
k = 0 (magnetically isolated coils)

Mutual Inductance

- The circuit equivalent is:



Example

Two identical coils are wound on the same magnetic core. A current changing at a rate of 2000 A/s in coil 1 induces a voltage of 20 V in coil 2. What is the mutual inductance?

Example

Two identical coils are wound on the same magnetic core. A current changing at a rate of 2000 A/s in coil 1 induces a voltage of 20 V in coil 2. What is the mutual inductance?

$$e_2 = M_{21}\frac{di_1}{dt}$$

$$M_{12} = M_{21} = M$$

$$M = \frac{e_2}{\frac{di_1}{dt}} = \frac{20}{2000} = 0.01\text{H}$$

Example

Two identical coils are wound on the same magnetic core. A current changing at a rate of 2000 A/s in coil 1 induces a voltage of 20 V in coil 2. If L_1 = 25mH, what percentage of the flux set up by coil 1 links coil 2?

Example

Two identical coils are wound on the same magnetic core. A current changing at a rate of 2000 A/s in coil 1 induces a voltage of 20 V in coil 2. If L_1 = 25mH, what percentage of the flux set up by coil 1 links coil 2?

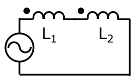
$$L_1 = L_2 = L = 0.25\text{mH}$$

$$M = k\sqrt{L_1L_2} = kL$$

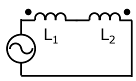
$$k = \frac{0.01}{0.025}\times 100 = 40\%$$

Magnetically Coupled Coils

- It is possible to connect the magnetically coupled coils together
  - series or parallel
- Depending on the polarity the coils can be aiding or opposing
- See 2.6 of text for more details



Series Aiding



Series Opposing

Magnetically Coupled Coils

- For aiding circuits:

$$V_{L1} = L_1\frac{di}{dt} + M\frac{di}{dt} = \frac{di}{dt}(L_1 + M)$$

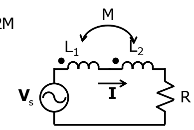
$$V_{L2} = L_2\frac{di}{dt} + M\frac{di}{dt} = \frac{di}{dt}(L_2 + M)$$

M is added because of aiding polarity

$$V_{Leff} = V_{L1} + V_{L2} = \frac{di}{dt}(L_1 + L_2 + 2M) = \frac{di}{dt}L_{eff}$$

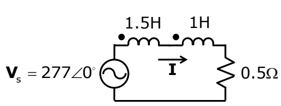
where

$$L_{eff} = L_1 + L_2 + 2M$$



Example

The circuit below operates at 50Hz. The mutual inductance M between the coils is 0.6 H. Compute **I**.



Example

The circuit below operates at 50Hz. The mutual inductance M between the coils is 0.6 H. Compute **I**.

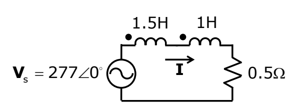
$L_{eff} = 1 + 1.5 + 2\times 0.6 = 3.7$ H

$jX_{eff} = j3.7\times 2\pi\times 50 = j1162\ \Omega$

Mutual inductances add in aiding circuits

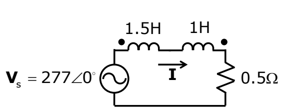
$\mathbf{Z} = 0.5 + j1162\ \Omega$

$\mathbf{I} = 0.238\angle -89.98^\circ$ A



Example

Repeat the problem but with opposing polarity of the inductors.



Example

Repeat the problem but with opposing polarity of the inductors.

$L_{eff} = 1 + 1.5 - 2\times 0.6 = 1.3$ H

$jX_{eff} = j1.3\times 2\pi\times 50 = j408.4\ \Omega$

$\mathbf{Z} = 0.5 + j408.4\ \Omega$

$\mathbf{I} = 0.68\angle -89.93^\circ$ A