12-Power Transmission

Text: 4.6-4.10

ECEGR 451
Power Systems

Overview

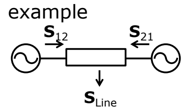
- Conservation of Complex Power
- Short Line Power Transmission
- Power Transmission Visualization
- Radial Line
- Medium and Long Line Power Transmission
- Voltage Collapse
- Surge Impedance Loading

Introduction

- We want to determine the complex power (or P, Q) flowing out of generators and through transmission lines
- Most utility-scale power plants feature synchronous generators
  - |**V**| controlled by exciter, which influences Q
  - δ (power angle) controlled by prime mover, which influences P

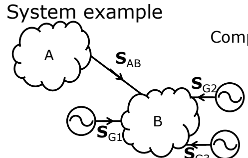
Conservation of Complex Power

- Complex power (and P, Q) are conserved
- Single line example



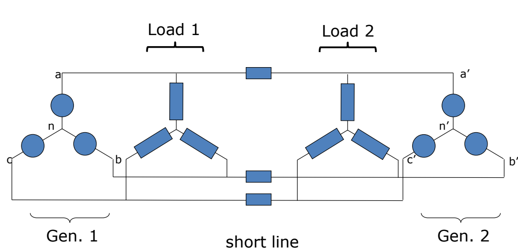
$\mathbf{S}_{12} + \mathbf{S}_{21} = \mathbf{S}_{Line}$

- System example



Complex power consumed by area B:
$\mathbf{S}_{AB} + \mathbf{S}_{G1} + \mathbf{S}_{G2} + \mathbf{S}_{G3}$

Power Transmission



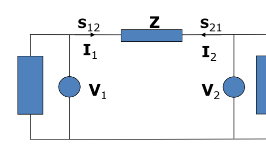
Power Transmission-Short Line

$$\mathbf{S}_{12} = \mathbf{V}_1\mathbf{I}_1^* = \mathbf{V}_1\left(\frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}}\right)^*$$

$$\mathbf{S}_{12} = \frac{|\mathbf{V}_1|^2}{\mathbf{Z}^*} - \frac{\mathbf{V}_1\mathbf{V}_2^*}{\mathbf{Z}^*}$$

$$\mathbf{S}_{12} = \frac{|\mathbf{V}_1|^2}{|\mathbf{Z}|}e^{j\angle\mathbf{Z}} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{|\mathbf{Z}|}e^{j\angle\mathbf{Z}}e^{j\theta_{12}}$$

$$\mathbf{S}_{21} = \frac{|\mathbf{V}_2|^2}{|\mathbf{Z}|}e^{j\angle\mathbf{Z}} - \frac{|\mathbf{V}_2||\mathbf{V}_1|}{|\mathbf{Z}|}e^{j\angle\mathbf{Z}}e^{-j\theta_{12}}$$ by symmetry



per-phase representation

Power Transmission-Short Line

- Magnitudes of $\mathbf{V}_1$ and $\mathbf{V}_2$ are controlled by exciters of the generators and are fairly constant
- θ_{12} can be increased by increasing the mechanical power of Gen. 1 and/or by decreasing the mechanical power of Gen. 2 (recall power angle in ECEGR 450)

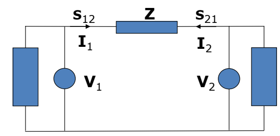
$$\mathbf{S}_{12} = \frac{|\mathbf{V}_1|^2}{|\mathbf{Z}|}e^{j\angle Z} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{|\mathbf{Z}|}e^{j\angle Z}e^{j\theta_{12}}$$

$$\mathbf{S}_{21} = \frac{|\mathbf{V}_2|^2}{|\mathbf{Z}|}e^{j\angle Z} - \frac{|\mathbf{V}_2||\mathbf{V}_1|}{|\mathbf{Z}|}e^{j\angle Z}e^{-j\theta_{12}}$$

Example

Increasing the receiving end voltage ($\mathbf{V}_2$) increases the complex power flowing from generator 1 ($\mathbf{S}_{12}$)

- True
- False

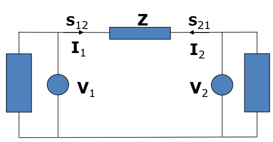


Example

Increasing the receiving end voltage ($\mathbf{V}_2$) increases the complex power flowing from generator 1 ($\mathbf{S}_{12}$)

- True
- False

$$\mathbf{S}_{12} = \frac{|\mathbf{V}_1|^2}{|\mathbf{Z}|}e^{j\angle Z} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{|\mathbf{Z}|}e^{j\angle Z}e^{j\theta_{12}}$$



Power Transmission Visualization

$$\mathbf{S}_{12} = \frac{|\mathbf{V}_1|^2}{|\mathbf{Z}|}e^{j\angle Z} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{|\mathbf{Z}|}e^{j\angle Z}e^{j\theta_{12}} \qquad \mathbf{S}_{21} = \frac{|\mathbf{V}_2|^2}{|\mathbf{Z}|}e^{j\angle Z} - \frac{|\mathbf{V}_2||\mathbf{V}_1|}{|\mathbf{Z}|}e^{j\angle Z}e^{-j\theta_{12}}$$

$$\mathbf{S}_{12} = C_1 - Be^{j\theta_{12}} \qquad -\mathbf{S}_{21} = C_2 + Be^{-j\theta_{12}}$$

where

$$C_1 = \frac{|\mathbf{V}_1|^2}{|\mathbf{Z}|}e^{j\angle Z}, \quad C_2 = -\frac{|\mathbf{V}_2|^2}{|\mathbf{Z}|}e^{j\angle Z}, \quad B = \frac{|\mathbf{V}_1||\mathbf{V}_2|}{|\mathbf{Z}|}e^{j\angle Z}$$

independent of θ_{12}

Power Transmission-Short Line

- If θ_{12} is varied, $\mathbf{S}_{12}$ and $\mathbf{S}_{21}$ sweep out circles in complex plane

sending-end circle

$$S_{12} = C_1 - Be^{j\theta_{12}}$$

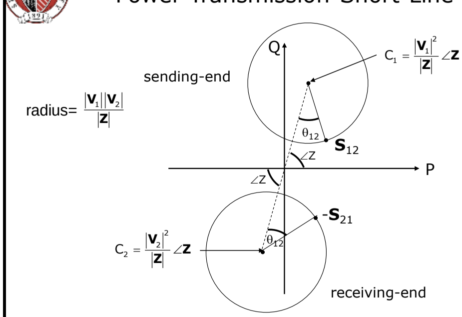
centered at C_1 radius= |B|

receiving-end circle

$$-S_{21} = C_2 + Be^{-j\theta_{12}}$$

centered at C_2 radius= |B|

Power Transmission-Short Line



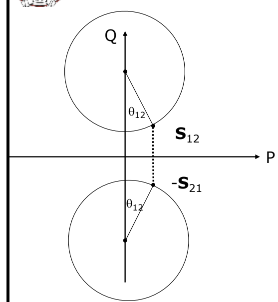
Power Transmission-Short Line

- Observations
  - circles do not intersect if voltage magnitudes are not equal
  - as θ_{12} increases from 0, active power sent and received increases
  - there is a maximum amount of real power sent

    $\theta_{12} = 180^{\circ} - \angle Z$
  - and received

    $\theta_{12} = \angle Z$
- Most transmission lines have small resistance compared to inductance

Power Transmission-Short Line



Lossless line (R = 0)
- $\angle Z = 0^{\circ}$
- $P_{12} = -P_{21}$
- circles lie on Q-axis

Power Transmission-Short Line

- Assume lossless line (R = 0)

$$P_{12} = -P_{21} = \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\sin\theta_{12}$$ reactance of the line

$$P_{12} = -P_{21} = \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\sin\theta_{12}$$ maximum power transfer: $\frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}$

$$Q_{12} = \frac{|\mathbf{V}_1|^2}{X_L} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\cos\theta_{12}$$

$$Q_{21} = \frac{|\mathbf{V}_2|^2}{X_L} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\cos\theta_{12}$$

We will derive these expressions later

Power Transmission-Short Line

- Under normal operating conditions: $|\theta_{12}| < 10^{\circ}$
- Approximating

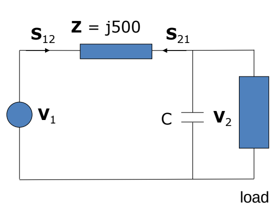
  $\sin\theta_{12} \approx \theta_{12}$
  $\cos\theta_{12} \approx 1$ in radians
- Results in

$$P_{12} = \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\sin\theta_{12} \approx \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\theta_{12}$$ P is sensitive to θ_{12}

$$Q_{12} \approx \frac{|\mathbf{V}_1|^2}{X_L} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}$$ Q is sensitive to voltage magnitude

Example

- Lossless line
- Power consumed by the load is = (10 + j0) MVA
- Assume C is selected to make $|\mathbf{V}_2|$ = 100,000 = $|\mathbf{V}_1|$
- Find the corresponding reactive power injected by the capacitor, Q_C



Example

$P_{12} = -P_{21} = 1$ MW lossless line assumption

$$P_{12} = \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\sin\theta_{12} = 10 \text{ MW}$$

$$(20 \times 10^6)\sin\theta_{12} = 10 \text{ MW} \Rightarrow \theta_{12} = 30^{\circ}$$

$$\theta_1 = 0^{\circ} \Rightarrow \theta_2 = -30^{\circ}$$

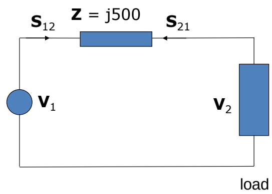
$$Q_{21} = \frac{|\mathbf{V}_2|^2}{X} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X}\cos\theta_{12}$$

$$= (20 - 20\cos 30^{\circ}) \times 10^6 = 2.68 \text{ MVAR}$$

$$Q_C = Q_{21} = 2.68 \text{ MVAR}$$

Example

- Reactive power compensation is removed
- $|\mathbf{V}_1| = 100{,}000$ V
- Can the load of 10 MW be supplied?
- What is $|\mathbf{V}_2|$ for this operating point?

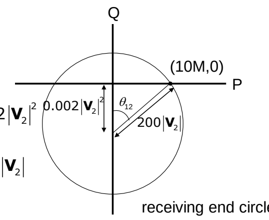


Example

$$-S_{21} = C_2 + Be^{-j\theta_{12}}$$

$$C_2 = \frac{|\mathbf{V}_2|^2}{|\mathbf{Z}|} e^{j\angle Z} = 0.002|\mathbf{V}_2|^2$$

$$B = \frac{|\mathbf{V}_1||\mathbf{V}_2|}{|\mathbf{Z}|} e^{j\angle Z} = 200|\mathbf{V}_2|$$



Example

By Pythagorean theorem:

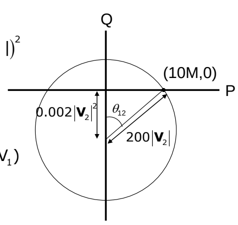
$$\left(0.002\,|\,\mathbf{V}_2\,|^2\right)^2 + \left(10\text{x}10^6\right)^2 = \left(200\,|\,\mathbf{V}_2\,|\right)^2$$

$$x = |\mathbf{V}_2|^2$$

$$0.002^2x^2 - 200^2x + 10e12 = 0$$

$$x = 5e9$$

$$V_2 = 70{,}700 \text{ (very low compared to } V_1)$$



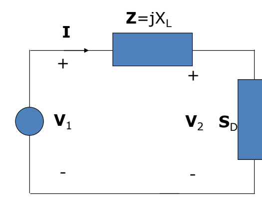
the capacitor bank greatly improved the voltage at bus 2

θ_{12} can be calculated as 45 degrees

Radial Line

- Lossless radial line
- We want to find the $|\mathbf{V}_2|$ and $\mathbf{S}$ relationship
- Let $\phi = \theta_{V2} - \theta_i$

$$\mathbf{S}_D = \mathbf{V}_2\mathbf{I}^*$$
$$= |\mathbf{V}_2||\mathbf{I}|(\cos\phi + j\sin\phi)$$
$$= P_D + j|\mathbf{V}_2||\mathbf{I}|\sin\phi$$
$$= P_D + j|\mathbf{V}_2||\mathbf{I}|\cos\phi\frac{\sin\phi}{\cos\phi}$$
$$= P_D(1 + j\beta) \qquad \beta = \tan\phi$$



Radial Line

$$P_D = P_{12} = \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\sin\theta_{12}$$

$$Q_D = -Q_{21} = \frac{|\mathbf{V}_2|^2}{X_L} + \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\cos\theta_{12}$$

previous result

$$P_D^2 = \left(\frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\right)^2 \sin^2\theta_{12}$$

$$P_D^2 = \left(\frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\right)^2 \left(-\cos^2\theta_{12} + 1\right) \quad \text{using } \cos^2\theta_{12} + \sin^2\theta_{12} = 1$$

$$\left(Q_D + \frac{|\mathbf{V}_2|^2}{X_L}\right)^2 = \left(\frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\right)^2 \cos^2\theta_{12}$$

Radial Line

$$P_D^2 = \left(\frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\right)^2 \left(-\cos^2\theta_{12} + 1\right)$$

$$\left(Q_D + \frac{|\mathbf{V}_2|^2}{X_L}\right)^2 = \left(\frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\right)^2 \cos^2\theta_{12}$$

$$P_D^2 = -\left(Q_D + \frac{|\mathbf{V}_2|^2}{X_L}\right)^2 + \left(\frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\right)^2 \quad \text{substituting for the cosine term}$$

$$P_D^2 = -\left(\beta P_D + \frac{|\mathbf{V}_2|^2}{X_L}\right)^2 + \left(\frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\right)^2 \quad \text{using } \beta = \tan\phi = \frac{Q_D}{P_D}$$

$$|\mathbf{V}_2|^4 + \left(2\beta P_D X_L - |\mathbf{V}_1|^2\right)|\mathbf{V}_2|^2 + (1+\beta)P_D^2 X_L^{\,2} = 0 \quad \text{arranging for the quadratic equation}$$

Radial Line

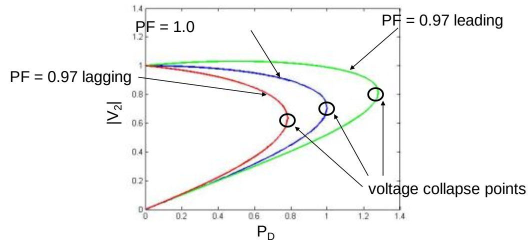
$$|\mathbf{V}_2|^4 + \left(2\beta P_D X_L - |\mathbf{V}_1|^2\right)|\mathbf{V}_2|^2 + (1+\eta)P_D^2 X_L^{\ 2} = 0$$

$$|\mathbf{V}_2|^2 = \frac{|\mathbf{V}_1|^2}{2} - 2\beta P_D X_L \pm \left[\frac{|\mathbf{V}_1|^4}{4} - |\mathbf{V}_2|^2 P_D X_L \left(P_D X_L + \beta|\mathbf{V}_1|^2\right)\right]^{1/2}$$ solutions to quad. eqn

Two solutions! Two receiving-end voltages that supply S_D.
Power system is operated at the higher voltage solution.

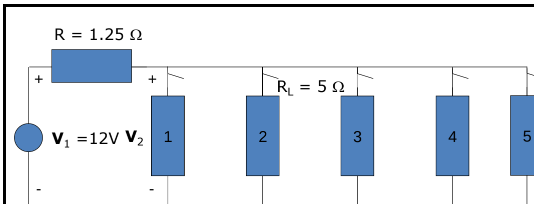
Is there a limit to the power that can be delivered?

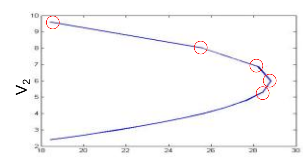
Voltage Collapse



Voltage Collapse

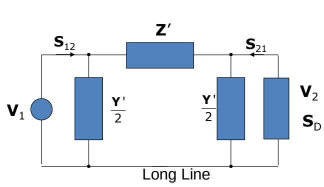
- Voltage collapse is characterized by a precipitous decline in voltage at the receiving end bus as more power is demanded
- Power systems should be designed and operated to avoid this condition
- Voltage collapse is a function of several parameters, including:
  - sending and receiving end voltage magnitudes
  - power factor angle of the load
  - line impedance
- A dc example can be used explain the mechanism of collapse





Power Transmission-Medium and Long Lines

- We now must account for complex power through the shunts



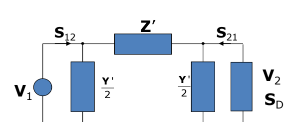
Power Transmission-Medium and Long Lines

- Recall from short lines (sending end)

$$\mathbf{S}_{12} = \frac{|\mathbf{V}_1|^2}{|\mathbf{Z}|} e^{j\angle \mathbf{Z}} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{|\mathbf{Z}|} e^{j\angle \mathbf{Z}} e^{j\theta_{12}}$$

- Now include the sending end shunt

$$\mathbf{S}_{12} = \frac{|\mathbf{V}_1|^2}{|\mathbf{Z}'|} e^{j\angle \mathbf{Z}'} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{|\mathbf{Z}'|} e^{j\angle \mathbf{Z}'} e^{j\theta_{12}} + \frac{\mathbf{Y}'^*}{2}|\mathbf{V}_1|^2$$



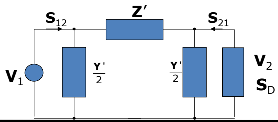
Power Transmission-Medium and Long Lines

- Receiving end short line

$$\mathbf{S}_{21} = \frac{|\mathbf{V}_2|^2}{|\mathbf{Z}|} e^{j\angle Z} - \frac{|\mathbf{V}_2||\mathbf{V}_1|}{|\mathbf{Z}|} e^{j\angle Z} e^{-j\theta_{12}}$$

- Now include the shunts

$$\mathbf{S}_{21} = \frac{|\mathbf{V}_2|^2}{|\mathbf{Z}'|} e^{j\angle Z'} - \frac{|\mathbf{V}_2||\mathbf{V}_1|}{|\mathbf{Z}'|} e^{j\angle Z'} e^{-j\theta_{12}} + \frac{\mathbf{Y}'^*}{2}|\mathbf{V}_2|^2$$



Power Transmission-Medium and Long Lines

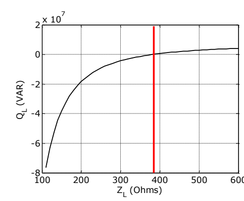
- Note that for long lines, use **Z**′ and **Y**′
- For medium lines, use **Z** and **Y**
- Influence of $\frac{\mathbf{Y}'^*}{2}|\mathbf{V}_1|^2$ and $\frac{\mathbf{Y}'^*}{2}|\mathbf{V}_2|^2$ is usually limited
- How does this affect the sending and receiving end circles?

Surge Impedance Loading (SIL)

- A lossless line terminated in its characteristic impedance ($\mathbf{Z}_c$) is said to be Surge Impedance Loaded (SIL)
- Under these conditions:
  - $|\mathbf{V}_1| = |\mathbf{V}_2|$
  - $|\mathbf{I}_1| = |\mathbf{I}_2|$
- The transmitted power is $P_{SIL} = |\mathbf{V}_1|^2/\mathbf{Z}_c$

Surge Impedance Loading (SIL)

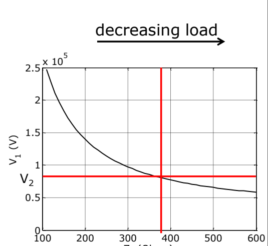
- For lossless lines, Z_c is real
- $Z_L < Z_C$
  - $Q_{12} < 0$
  - Transmission line absorbs reactive power (inductive)
- $Z_L = Z_C$
  - $Q_{12} = 0$
- $Z_L > Z_C$
  - $Q_{12} > 0$
  - Transmission line supplies reactive power (capacitive)



y = j5.38e-3
z = j0.789
Z_c = 383 Ω
Length = 1000 km
V_{2L-L} = 138kv

Surge Impedance Loading (SIL)

- $Z_L < Z_C$
  - $|V_1| > |V_2|$
  - Sending end voltage higher than receiving
- $Z_L = Z_C$
  - $|V_1| = |V_2|$
- $Z_L > Z_C$
  - $|V_1| < |V_2|$
  - Sending end voltage less than receiving!
  - Ferranti effect



Example

What time of day do you expect the voltage at the end of long transmission lines to be the highest?

Surge Impedance Loading

- See
  - SIL_Movie.mat ($Z_L = Z_C$)
  - SIL_Movie_2.mat ($Z_L = 0.5Z_C$)
  - SIL_Movie_3.mat ($Z_L = 100Z_C$)

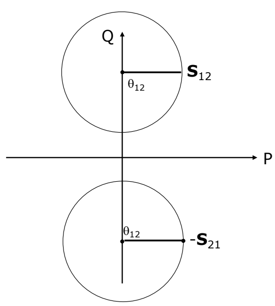
Surge Impedance Loading (SIL)

- See example 4.2, 4.3 for more details

Stability Limits

- Increasing θ_{12} increases the power flow across the line
- For lossless lines, maximum power transmission occurs when $\theta_{12} = 90^o$

$$P_{12} = \frac{|\mathbf{V}_1||\mathbf{V}_2|}{X_L}\sin\theta_{12}$$



Stability Limits

- It is desired to limit $|\theta_{12}|$ for stability reasons
- Consider a lossless long transmission line
- Transmitted power is

$$\mathbf{S}_{12} = \frac{\mathbf{Y}'^*}{2}|\mathbf{V}_1|^2 + \frac{|\mathbf{V}_1|^2}{\mathbf{Z}'^*} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{\mathbf{Z}'^*}e^{j\theta_{12}}$$

- With lossless assumptions

$$\mathbf{Z}' = \mathbf{Z}_c\sinh(\beta X) = j\mathbf{Z}_c\sin(\beta X) \qquad \mathbf{Y}' = \mathbf{Y}\frac{\tanh\left(\gamma X/2\right)}{\gamma X/2} = j\omega C\frac{\tanh\left(\beta X/2\right)}{\beta X/2}$$

- Real power transmitted:

$$\mathrm{Re}\{\mathbf{S}_{12}\} = \mathrm{Re}\left\{\frac{\mathbf{Y}'^*}{2}|\mathbf{V}_1|^2 + \frac{|\mathbf{V}_1|^2}{\mathbf{Z}'^*} - \frac{|\mathbf{V}_1||\mathbf{V}_2|}{\mathbf{Z}'^*}e^{j\theta_{12}}\right\}$$

Stability Limits

- Via Euler's Identity and assuming voltage magnitude at each end of the line are equal:

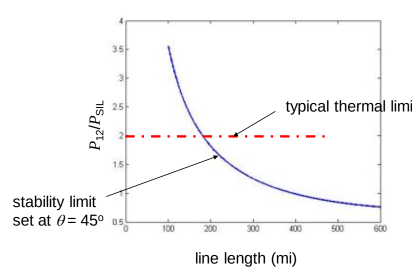
$$\mathrm{Re}\{\mathbf{S}_{12}\} = \mathrm{Re}\left\{\frac{|\mathbf{V}_1||\mathbf{V}_2|}{\mathbf{Z}'^*}e^{j\theta_{12}}\right\} = P_{12} = \frac{|\mathbf{V}_1|^2}{\mathbf{Z}_c}\frac{\sin(\theta_{12})}{\sin(\beta X)}$$

- From the SIL definition:

$$P_{12} = P_{SIL}\frac{\sin(\theta_{12})}{\sin(\beta X)}$$

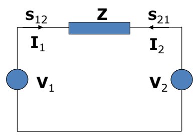
- Limiting θ_{12}, limits maximum power transferred
- For a fixed θ_{12}, as length increases, maximum power transfer decreases
- For very long lines with $\beta x = \pi/2$, P_{SIL} cannot be exceed

Limits



Summary (Short Line)

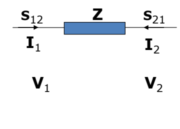
$$S_{12}=\frac{|V_1|^2}{|Z|}e^{j\angle Z}-\frac{|V_1||V_2|}{|Z|}e^{j\angle Z}e^{j\theta_{12}}\qquad S_{21}=\frac{|V_2|^2}{|Z|}e^{j\angle Z}-\frac{|V_2||V_1|}{|Z|}e^{j\angle Z}e^{-j\theta_{12}}$$



per-phase representation

Summary (Short Lossless Line)

$$P_{12}=-P_{21}=\frac{|V_1||V_2|}{X_L}\sin\theta_{12}\qquad Q_{12}=\frac{|V_1|^2}{X_L}-\frac{|V_1||V_2|}{X_L}\cos\theta_{12}$$



per-phase representation

Summary (Short Line)

- Discussed power circles for power transmission analysis

$$S_{12}=\frac{|V_1|^2}{|Z|}e^{j\angle Z}-\frac{|V_1||V_2|}{|Z|}e^{j\angle Z}e^{j\theta_{12}}\qquad S_{21}=\frac{|V_2|^2}{|Z|}e^{j\angle Z}-\frac{|V_2||V_1|}{|Z|}e^{j\angle Z}e^{-j\theta_{12}}$$

- Lossless lines:

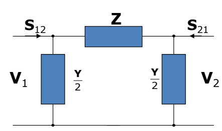
$$P_{12}=-P_{21}=\frac{|V_1||V_2|}{X_L}\sin\theta_{12}\qquad Q_{12}=\frac{|V_1|^2}{X_L}-\frac{|V_1||V_2|}{X_L}\cos\theta_{12}$$

(if a short line model is used)

Summary (Medium Line)

$$\mathbf{S}_{12}=\frac{|\mathbf{V}_1|^2}{|\mathbf{Z}|}e^{j\angle\mathbf{Z}}-\frac{|\mathbf{V}_1||\mathbf{V}_2|}{|\mathbf{Z}|}e^{j\angle\mathbf{Z}}e^{j\theta_{12}}+\frac{\mathbf{Y}^*}{2}|\mathbf{V}_1|^2$$

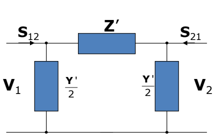
$$\mathbf{S}_{21}=\frac{|\mathbf{V}_2|^2}{|\mathbf{Z}|}e^{j\angle Z}-\frac{|\mathbf{V}_2||\mathbf{V}_1|}{|\mathbf{Z}|}e^{j\angle Z}e^{-j\theta_{12}}+\frac{\mathbf{Y}^*}{2}|\mathbf{V}_2|^2$$



Summary (Long Line)

$$\mathbf{S}_{12}=\frac{|\mathbf{V}_1|^2}{|\mathbf{Z}'|}e^{j\angle\mathbf{Z}'}-\frac{|\mathbf{V}_1||\mathbf{V}_2|}{|\mathbf{Z}'|}e^{j\angle\mathbf{Z}'}e^{j\theta_{12}}+\frac{\mathbf{Y}'^*}{2}|\mathbf{V}_1|^2$$

$$\mathbf{S}_{21}=\frac{|\mathbf{V}_2|^2}{|\mathbf{Z}'|}e^{j\angle Z'}-\frac{|\mathbf{V}_2||\mathbf{V}_1|}{|\mathbf{Z}'|}e^{j\angle Z'}e^{-j\theta_{12}}+\frac{\mathbf{Y}'^*}{2}|\mathbf{V}_2|^2$$



Summary

- Discussed radial lines
  - two operating points: one is stable, one unstable
  - voltage collapse: limits the amount of power that can be supplied to a load
- Stability limits tend to limit power transfer for long lines