12-Magnetic Circuit Analysis

ECEGR 450
Electromechanical Energy Conversion

Overview

- Introduction
- Magnetic Circuit Elements
- Magnetic Circuit Analysis

Questions

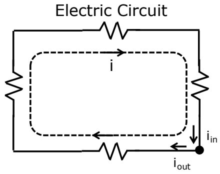
- What is the electric circuit analog of flux?
- What is the magnetic circuit analog of voltage?
- What is the magnetic circuit analog of resistance?

Introduction

- Electrical engineers are trained at solving circuits
- Fortunately, we can derive a loose analogy between magnetic flux and current
- We will be solving "magnetic circuits"
- Assumptions were discussed in previous lecture

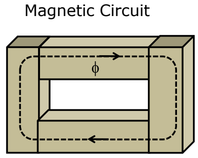
Magnetic Circuit Assumptions

Electric Circuit



current entering node =
current leaving node

Magnetic Circuit

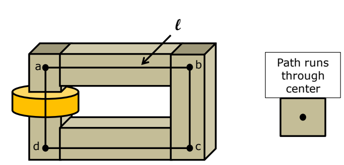


magnetic flux entering a boundary=
magnetic flux leaving a boundary
(from $\oint_s \mathbf{B}\cdot d\mathbf{s} = 0$)

Magnetic Circuit Analysis

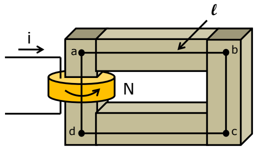
Let ℓ be the mean length of the magnetic path (m)

$\ell = \ell_{ab} + \ell_{bc} + \ell_{cd} + \ell_{da}$



Magnetomotive Force (mmf)

- The current enclosed by the closed path through the core is $\oint_C \mathbf{H} \cdot d\ell = Ni = \mathcal{F}$ (Ampere's Law)
- Where
  - $\mathcal{F}$: magnetomotive force (A-t)
- mmf is analogous to voltage in a circuit

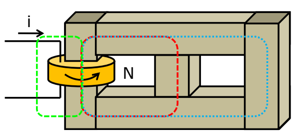


Flux in Magnetic Circuits

- Assuming that **H** is uniform in the material, then
  $H\ell = Ni$
- The magnetic flux density in the material is also uniform and
  $B = \mu H = \frac{\mu Ni}{\ell}$
- The flux is:
  $\phi = BA = \frac{\mu NiA}{\ell}$
  - A: cross sectional area of the material (m^2)

Question

Which path (red, blue or green) results in the greatest mmf?



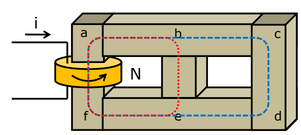
Cross section is uniform

Magnetic Circuit Analysis

mmf is the same, no matter which path is used

$Ni = H_{fa}\ell_{fa} + H_{ab}\ell_{ab} + H_{be}\ell_{be} + H_{ef}\ell_{ef}$

$= H_{fa}\ell_{fa} + H_{ab}\ell_{ab} + H_{bc}\ell_{bc} + H_{cd}\ell_{cd} + H_{de}\ell_{de} + H_{ef}\ell_{ef}$



Cross section is uniform

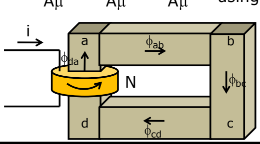
Magnetic Circuit Analysis

$Ni = H_{da}\ell_{da} + H_{ab}\ell_{ab} + H_{bc}\ell_{bc} + H_{cd}\ell_{cd}$

$\mathcal{F} = \frac{B_{da}}{\mu}\ell_{fa} + \frac{B_{ab}}{\mu}\ell_{ab} + \frac{B_{bc}}{\mu}\ell_{bc} + \frac{B_{cd}}{\mu}\ell_{cd}$ using: $H = B/\mu$

$\mathcal{F} = \left(\frac{B_{da}}{\mu}\ell_{fa} + \frac{B_{ab}}{\mu}\ell_{ab} + \frac{B_{bc}}{\mu}\ell_{bc} + \frac{B_{cd}}{\mu}\ell_{cd}\right)\frac{A}{A}$

$\mathcal{F} = \phi_{da}\frac{\ell_{da}}{A\mu} + \phi_{ab}\frac{\ell_{ab}}{A\mu} + \phi_{bc}\frac{\ell_{bc}}{A\mu} + \phi_{cd}\frac{\ell_{cd}}{A\mu}$ using: $\phi = BA$



Cross section is uniform

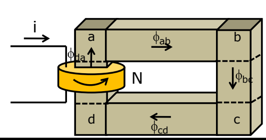
Magnetic Circuit Analysis

$\mathcal{F} = \phi_{da}\frac{\ell_{da}}{A\mu} + \phi_{ab}\frac{\ell_{ab}}{A\mu} + \phi_{bc}\frac{\ell_{bc}}{A\mu} + \phi_{cd}\frac{\ell_{cd}}{A\mu}$

$\phi = \phi_{da} = \phi_{ab} = \phi_{bc} = \phi_{cd}$ — Since flux entering a boundary = flux leaving the boundary $\oint_s \mathbf{B} \cdot d\mathbf{s} = 0$

$\mathcal{F} = \phi\left(\frac{\ell_{da}}{A\mu} + \frac{\ell_{ab}}{A\mu} + \frac{\ell_{bc}}{A\mu} + \frac{\ell_{cd}}{A\mu}\right)$

$\mathcal{F} = \phi(\mathfrak{R}_{da} + \mathfrak{R}_{ab} + \mathfrak{R}_{bc} + \mathfrak{R}_{cd})$ $\left\{ \mathfrak{R} \triangleq \frac{\ell}{\mu A} \right.$ $\mathfrak{R}$: reluctance (A-t/Wb)



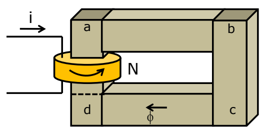
Cross section is uniform

Magnetic Circuit Analysis

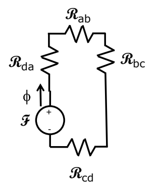
$$\mathcal{F} = \phi(\Re_{da} + \Re_{ab} + \Re_{bc} + \Re_{cd})$$

voltage ($\mathcal{F}$), current (ϕ), resistance ($\Re_{da} + \Re_{ab} + \Re_{bc} + \Re_{cd}$)

Equation can be modeled and solved like a circuit. Important!



Circuit equivalent



Magnetic Circuits

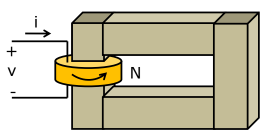
- Note that
  $$\phi = \frac{\mathcal{F}}{\Re} = \frac{NiA\mu}{\ell}$$
- For electric circuits
  $$i = \frac{v}{R} = \frac{v}{\frac{\ell}{\sigma A}}$$
  - σ: conductivity (S/m)

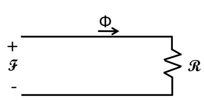
Analogous equations

- Ohm's law for magnetic circuits $\mathcal{F} = \phi\Re$
  - mmf = flux x reluctance
- Checking the units
  - A-t = Wb x (A-t/Wb) = A-t

Magnetic Circuits

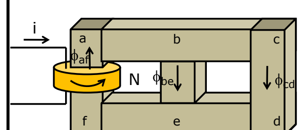
Circuit Quantity	Magnetic Quantity
Voltage, v (volt)	mmf, $\mathcal{F}$ (A-turns)
Current, i (Ampere)	magnetic flux, ϕ, (Wb)
Resistance, R (Ohm)	Reluctance, $\mathcal{R}$, (A-turns/Wb)
Conductivity, σ (S/m)	Permeability, μ (H/m)

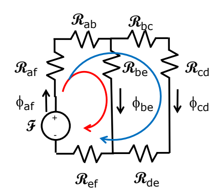




Magnetic Circuit Analysis

KVL and KCL and all other circuit theorems apply to equivalent electric circuit.



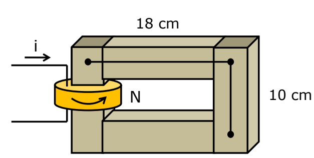


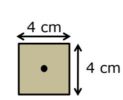
Magnetic Circuit Analysis

- Note: linear circuits assumed in analogy, therefore the magnetic circuit must be linear
  - Linear magnetic circuit = constant permeability
  - Ferromagnetic materials do not have constant permeability (see BH curve)
- Non-linear magnetic circuits can be solved iteratively

Example

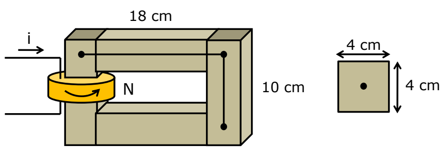
- Compute the flux flowing through the material given:
  - i = 1 A
  - N = 700
  - μ_r = 1000 (assumed to be constant)





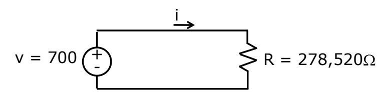
Example

- Want to use: $\mathcal{F} = Ni = \Phi\Re$, $\Re = \frac{\ell}{\mu A}$
- First compute ℓ



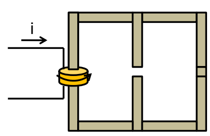
Example

Solution approach is the same as solving for the current in this circuit



Example

- Consider the shown magnetic circuit
  - Note the air gap in the center leg
- Determine:
  - direction of H, B within the circuit
  - which of the three legs has the greatest flux density, and which has the least
  - Which segment has the greatest field intensity
- Draw the equivalent electric circuit



Assume uniform cross section

Magnetic Circuit Analysis

- See Examples 2.8 and 2.9 for more magnetic circuit problems

Summary

- Magnetic circuits can be analyzed in an analogous fashion as electric circuits:
  $v = iR$
  $\mathcal{F} = \phi\Re$
- KVL, KCL, voltage divider, etc. all apply to magnetic circuit
- Reluctance, $\mathcal{R}$, increases with length, and decreases with permeability