11-Transmission Line Models Part 2

Text: 4.4-4.5

ECEGR 451
Power Systems

Overview

- Transmission Line Matrix
- Long Line Model
- Medium Line Model
- Short Line Model

Transmission Matrix

$$\mathbf{V}_1 = \mathbf{V}_2 \cosh \gamma X + \mathbf{Z}_c \mathbf{I}_2 \sinh \gamma X$$
$$\mathbf{I}_1 = \frac{\mathbf{V}_2}{\mathbf{Z}_c} \sinh \gamma X + \mathbf{I}_2 \cosh \gamma X$$

General solution for distributed impedance line

Can be expressed in the form

$$\mathbf{V}_1 = A\mathbf{V}_2 + B\mathbf{I}_2$$
$$\mathbf{I}_1 = C\mathbf{V}_2 + D\mathbf{I}_2$$

Where:

$$A = \cosh \gamma X \quad C = \frac{1}{\mathbf{Z}_c} \sinh \gamma X$$
$$B = \mathbf{Z}_c \sinh \gamma X \quad D = \cosh \gamma X$$

transmission parameters

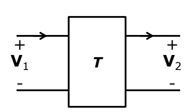
Transmission Matrix

- Transmission matrix:

$$\boldsymbol{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

- The sending and receiving end voltages and currents are concisely related as:

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix}$$



Example

Write the transmission matrix for a transmission line with the following parameters:

$\mathbf{z} = 0.8431\angle 79.04^\circ$ Ω/mi

$\mathbf{y} = 5.105 \text{ x } 10^{-6}\angle 90^\circ$ S/mi

X = 230 miles

Hint: $\gamma \triangleq \mathbf{yz}^{1/2}$

$$\mathbf{Z}_c = \sqrt{\frac{\mathbf{z}}{\mathbf{y}}}$$

Example

Write the transmission matrix for a transmission line with the following parameters:

$\mathbf{z} = 0.8431\angle 79.04^\circ$ Ω/mi

$\mathbf{y} = 5.105 \text{ x } 10^{-6}\angle 90^\circ$ S/mi

$$\gamma = \mathbf{yz}^{1/2} = 0.0002 + j0.0021$$

$$\mathbf{Z}_c = \sqrt{\frac{\mathbf{z}}{\mathbf{y}}} = 404.5 - j38.8$$

A = D = 0.8902 + j0.0208

B = 34.16 + j183.62

C = -0.0000 + j0.0011

Example

- Verify on your own that if

$$\mathbf{V}_2 = \frac{215,000}{\sqrt{3}} = 124,130\angle 0° \text{ V}$$

$$\mathbf{I}_2 = \frac{125,000,000}{3\ 124,130} = 335.7\angle 0° \text{ A}$$

then

$$\mathbf{V}_1 = 137,860\angle 27.77° \text{ V}$$

$$\mathbf{I}_1 = 332.31\angle 26.33° \text{ A}$$

see previous lecture notes

Transmission Matrix

- If sending end voltage and current are known, the receiving end quantities can be computed using:

$$T^{-1} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$ inverse transmission matrix

$$T^{-1}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix}$$

Transmission Matrix

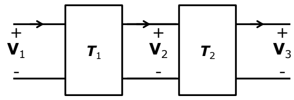
- Multiple transmission line segments can be accounted for using:

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = T_1\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix} = T_2\begin{bmatrix} \mathbf{V}_3 \\ \mathbf{I}_3 \end{bmatrix}$$

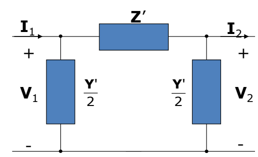
via substitution:

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = T_1T_2\begin{bmatrix} \mathbf{V}_3 \\ \mathbf{I}_3 \end{bmatrix}$$



Lumped-Circuit Equivalent

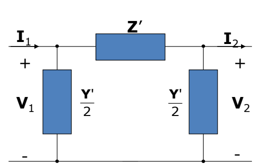
- Another useful form of transmission model is the Lumped-circuit equivalent
  - Yields same result as transmission matrix, and general solution (i.e. not an approximation)



Π-equivalent circuit

Lumped-Circuit Equivalent

- Approach: map **Y**′ and **Z**′ to A, B, C and D
- Next map A, B, C, D to **y** and **z** (already done)
- Express **Y**′ and **Z**′ in terms of **y** and **z** (approximation is introduced here)



Π-equivalent circuit

Lumped-Circuit Equivalent

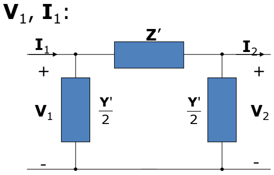
- Equivalent A,B,C,D parameters
- Solving the circuit for $\mathbf{V}_1$, $\mathbf{I}_1$:

$$\mathbf{V}_1 = \mathbf{V}_2 + \mathbf{Z}'\left(\mathbf{I}_2 + \frac{\mathbf{Y}'}{2}\mathbf{V}_2\right)$$

$$= \left(1 + \frac{\mathbf{Z}'\mathbf{Y}'}{2}\right)\mathbf{V}_2 + \mathbf{Z}'\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{\mathbf{Y}'}{2}\mathbf{V}_1 + \frac{\mathbf{Y}'}{2}\mathbf{V}_2 + \mathbf{I}_2$$

$$= \mathbf{Y}'\left(1 + \frac{\mathbf{Z}'\mathbf{Y}'}{4}\right)\mathbf{V}_2 + \left(1 + \frac{\mathbf{Z}'\mathbf{Y}'}{2}\right)\mathbf{I}_2$$



Lumped-Circuit Equivalent

- Desired form

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\mathbf{V}_2\\ \mathbf{I}_2\end{bmatrix}$$

$$\mathbf{V}_1=\left(1+\frac{\mathbf{Z}'\mathbf{Y}'}{2}\right)\mathbf{V}_2+\mathbf{Z}'\mathbf{I}_2$$

$$\mathbf{I}_1=\mathbf{Y}'\left(1+\frac{\mathbf{Z}'\mathbf{Y}'}{4}\right)\mathbf{V}_2+\left(1+\frac{\mathbf{Z}'\mathbf{Y}'}{2}\right)\mathbf{I}_2$$

- We then have

$$A=1+\frac{\mathbf{Z}'\mathbf{Y}'}{2} \qquad B=\mathbf{Z}'$$

$$C=\mathbf{Y}'\left(1+\frac{\mathbf{Z}'\mathbf{Y}'}{4}\right) \qquad D=1+\frac{\mathbf{Z}'\mathbf{Y}'}{2}$$

Lumped-Circuit Equivalent

- Next determine **Y**′ and **Z**′ in terms of **y**, **z** (or $\mathbf{Z}_c$ and γ)
- Start with B

$B=\mathbf{Z}_c\sinh\gamma X$

from the lumped equivalent circuit

B=**Z**′ therefore:

$$\mathbf{Z}'=\mathbf{Z}_c\sinh\gamma X=\sqrt{\frac{\mathbf{z}}{\mathbf{y}}}\sinh\gamma X=\mathbf{z}X\frac{\sinh\gamma X}{\gamma X}$$

$$=\mathbf{Z}\frac{\sinh\gamma X}{\gamma X} \qquad \text{defining } \mathbf{Z}\triangleq\mathbf{z}X$$

Lumped-Circuit Equivalent

- We note that usually:

$|\gamma|\ll 1$

- therefore:

$$\frac{\sinh\gamma X}{\gamma X}\approx 1$$

and

$$\mathbf{Z}'=\mathbf{Z}\frac{\sinh\gamma X}{\gamma X}$$

$$\mathbf{Z}'\approx\mathbf{Z}$$

remember, this is an approximation!
we will see later when it is appropriate to use

correction factor

Lumped-Circuit Equivalent

- Now find **Y**′
- Let's look at A

$$A=1+\frac{\mathbf{Z}'\mathbf{Y}'}{2}=1+\frac{\mathbf{Z}_c\sinh\gamma X\,\mathbf{Y}'}{2}$$

$A=\cosh\gamma X$ from the distributed model

- Solving

$$\frac{\mathbf{Y}'}{2}=\frac{\cosh\gamma X-1}{\mathbf{Z}_c\sinh\gamma X}=\frac{1}{\mathbf{Z}_c}\tanh\left(\frac{\gamma X}{2}\right)$$

Lumped-Circuit Equivalent

- We can also write

$$\frac{1}{\mathbf{Z}_c}=\frac{1}{\sqrt{\mathbf{z}/\mathbf{y}}}=\frac{\mathbf{y}}{\sqrt{\mathbf{zy}}}=\frac{\mathbf{y}X}{\gamma X}=\frac{\mathbf{Y}}{\gamma X} \qquad \text{where } \mathbf{Y}\triangleq\mathbf{y}X$$

- So that $\frac{\mathbf{Y}'}{2}=\frac{1}{\mathbf{Z}_c}\tanh\left(\frac{\gamma X}{2}\right)$ becomes $\frac{\mathbf{Y}'}{2}=\frac{\mathbf{Y}}{2}\frac{\tanh\left(\frac{\gamma X}{2}\right)}{\gamma X/2}$
- Usually, for transmission lines $|\gamma X|\ll 1$

$$\left(\tanh\left(\frac{\mathbf{y}X}{2}\right)\Big/\left(\frac{\mathbf{y}X}{2}\right)\right)\approx 1 \quad \text{so} \quad \frac{\mathbf{Y}'}{2}\approx\frac{\mathbf{Y}}{2}$$

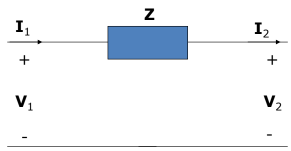
Remember, this is an approximation!
We will see later when it is appropriate to use

Simplified Models

- We don't always need the details of the distributed or equivalent lumped parameter models
- We can use the approximations previously shown to simply the model depending on transmission line length

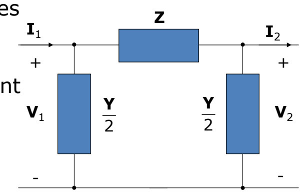
Short Line

- $X < 50$ mi
- Series impedance only $\mathbf{Z} = \mathbf{z}X$



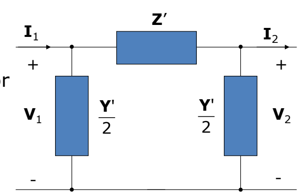
Medium-Length Line

- 150 mi > X >50 mi
- Capacitance becomes too large to ignore
- Also called the nominal Π–equivalent circuit



Long Line

- X > 150 mi
- Use Π-equivalent circuit model (also transmission matrix or general solution)



Summary

- Determined the transmission matrix using distributed parameters

$$\boldsymbol{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix}$$

- Discussed various line models
  - short line (series only; uses Z)
  - medium line (nominal Π-model; uses **Z** and **Y**/2)
  - long line (equivalent Π-model; uses **Z**′ and **Y**′/2)