10-Source Transformations

Text: 4.4 – 4.6

ECEGR 210

Electric Circuits I

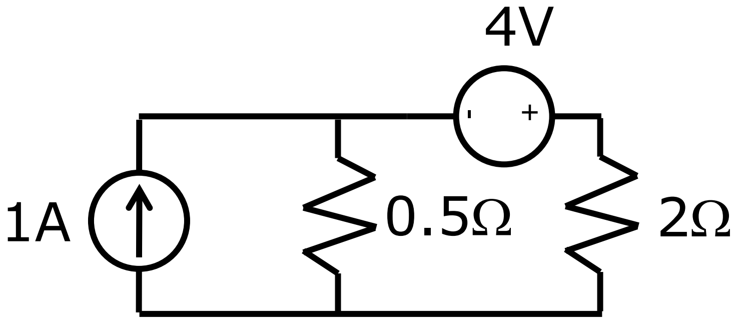
Overview

- Introduction
- Source Transformation
- Thevenin’s Theorem
- Norton’s Theorem

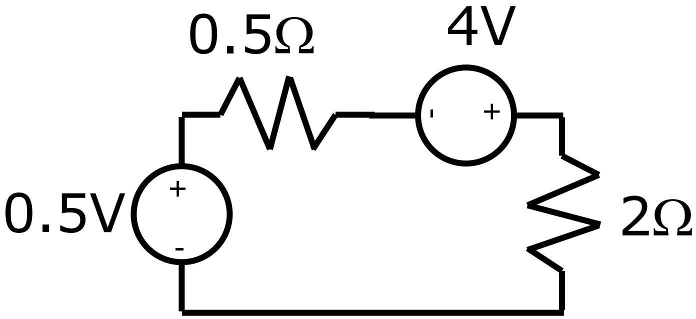
Introduction

- Consider the two circuits (A and B) shown below
- Compute the voltage across the 2Ω resistor in each circuit

Circuit A



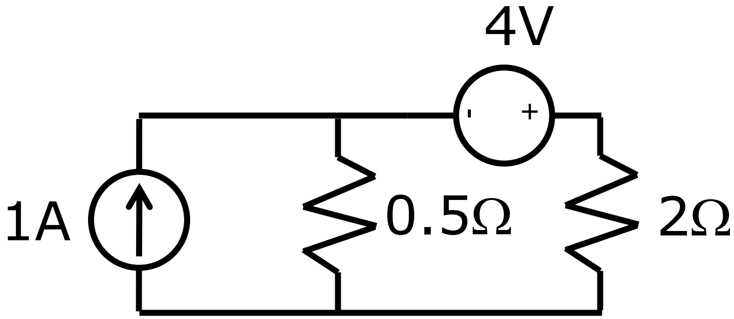
Circuit B



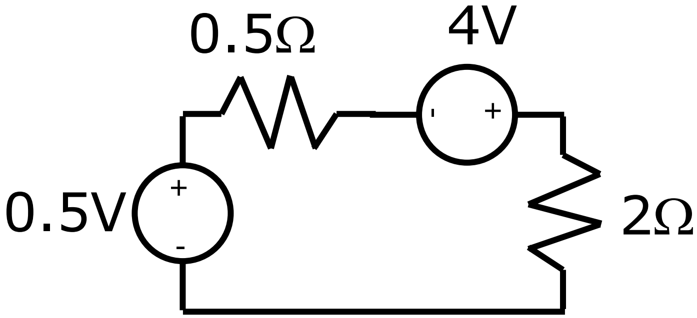
Introduction

- Circuit A (superposition):
  - $V_{R1} = 2I_{R1} = 2 \times 1(0.5/2.5) = 0.4V$ (current source)
  - $V_{R2} = 4(2/2.5) = 3.2V$ (voltage source)
  - $V_R = V_{R1} + V_{R2} = 0.4 + 3.2 = 3.6V$
- Circuit B: $V_R = 4.5(2/2.5) = 3.6V$ (voltage divider)

Circuit A



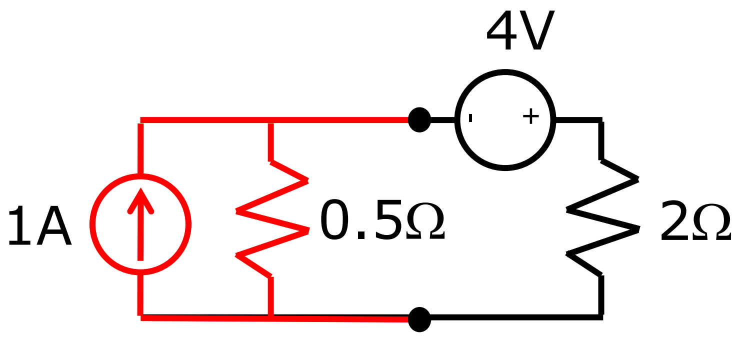
Circuit B



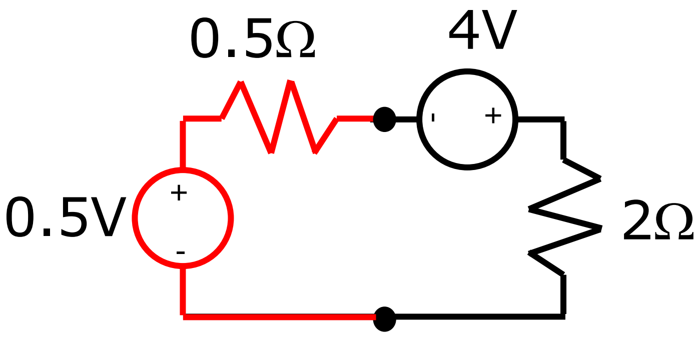
Introduction

- Solving Circuit B was much easier
- Same voltage across (current through) the resistor
- Circuits are <u>equivalent</u> looking into the terminals

Circuit A

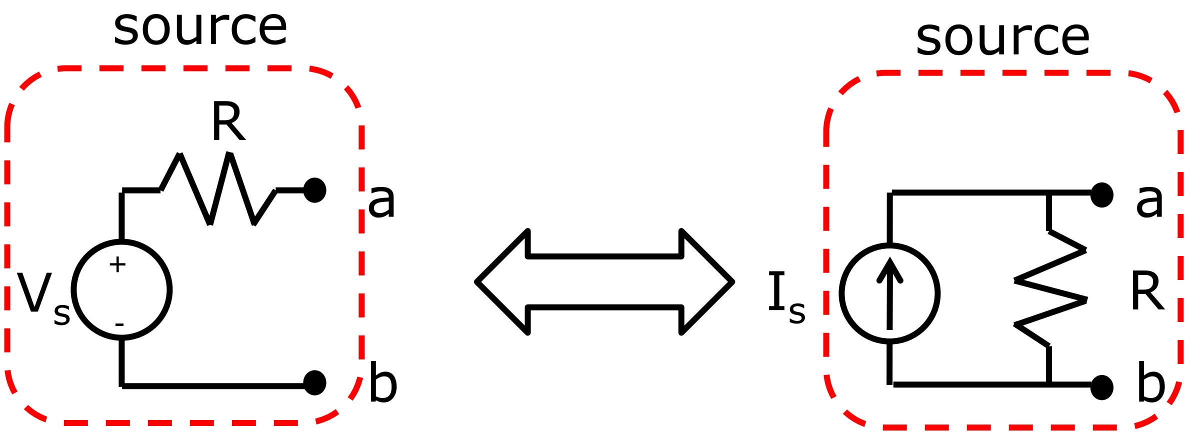


Circuit B



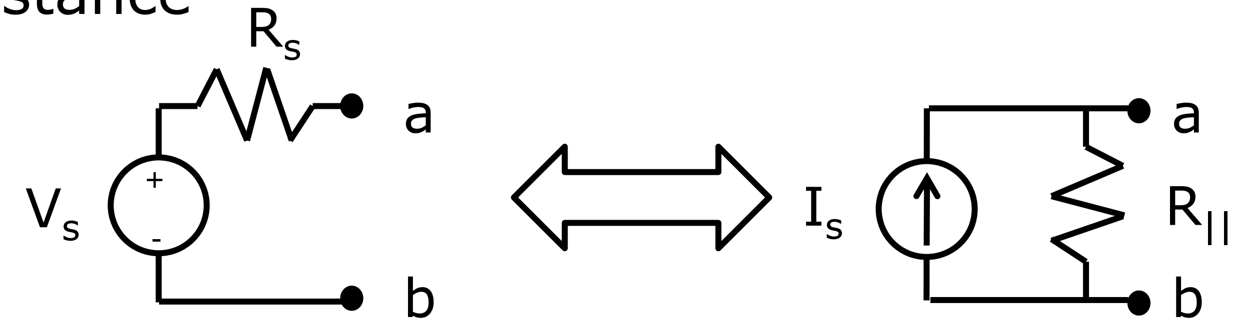
Introduction

- Clearly there can be advantageous of transforming sources
- Source transformations and equivalence are the focus of this lecture



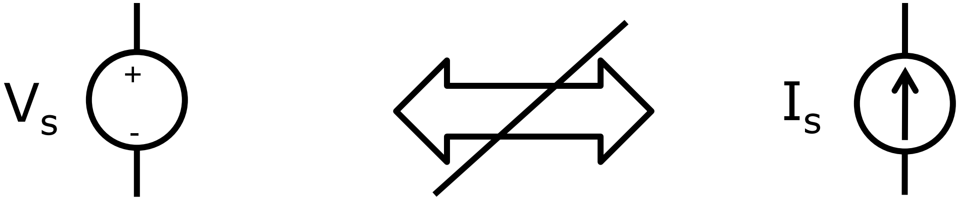
Introduction

- Two ways of modeling real (non-ideal) voltage and current sources are shown
  - R_s: small value, prevents infinite current from flowing if terminals (a,b) are shorted
  - $R_{||}$: large value, prevents infinite voltage at the terminals (a,b) under open circuit conditions
- Generically: can be any voltage source in series with resistance, or any current source in parallel with resistance

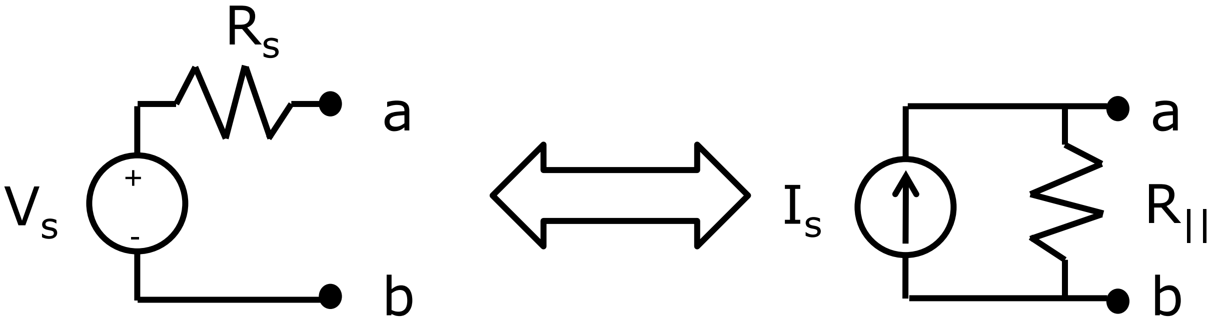


Source Transformation

- Not possible to transform current (voltage) sources to voltage (current) sources directly

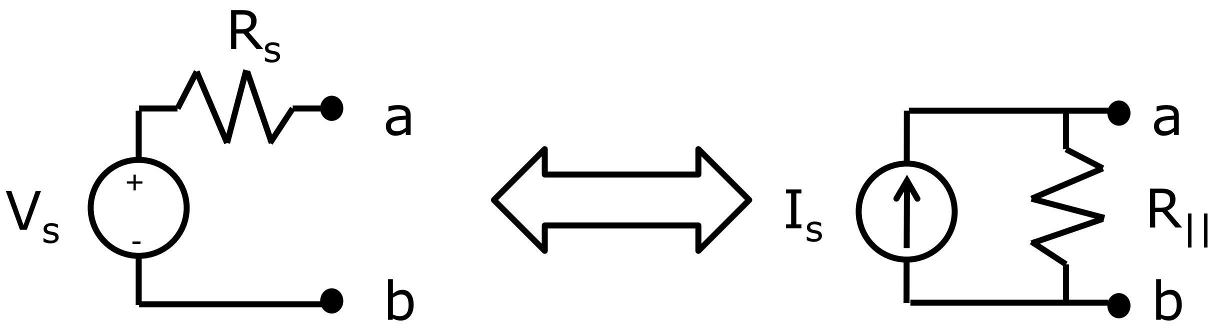


- But we can transform sources with series or parallel resistances as seen by terminals



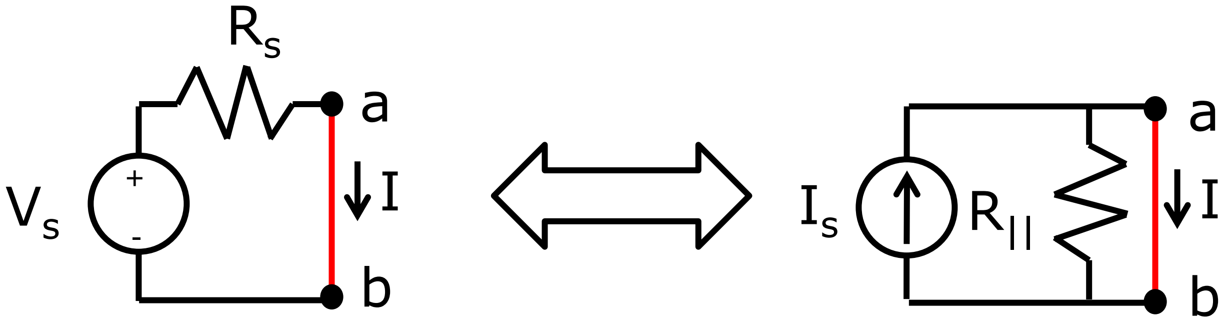
Source Transformations

- For the two circuits to be equivalent, they must have the same i-v characteristics at their terminals under all external circuit connections
  - Due to linearity, only need to verify i-v characteristics under two different external connections (short, open circuit)
- How are V_s, I_s, R_s and $R_{||}$ related?



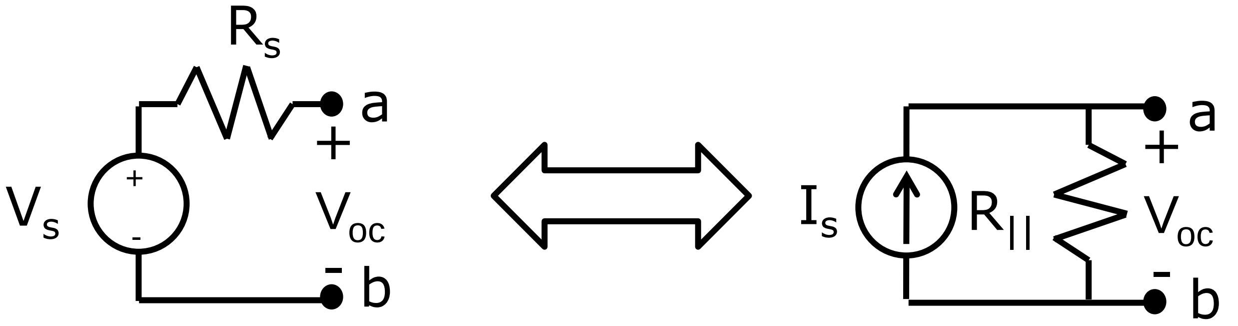
Source Transformation

- Consider when the external circuit is a short
- Both circuits must have same short current out of their terminals
- $I_{sc} = V_s/R_s$
- $I_{sc} = I_s$
- Therefore: $I_s = V_s/R_s$



Source Transformations

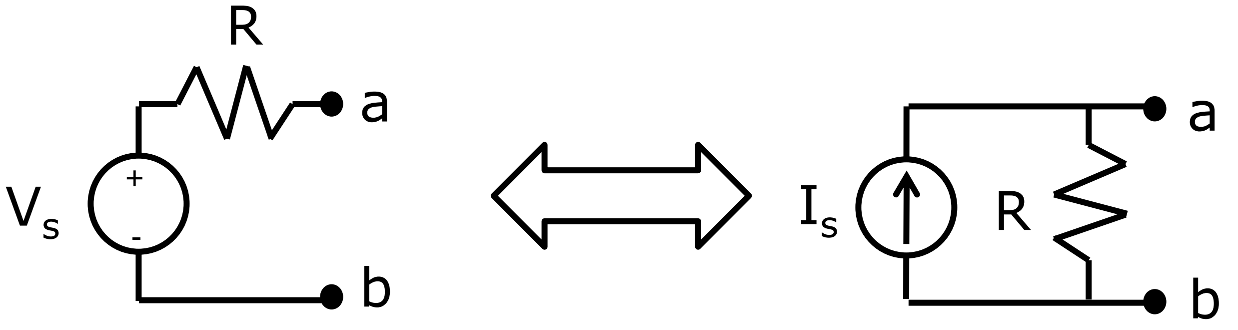
- Consider an open circuit
- Both circuits must same open circuit voltage V_{oc}
- $V_{oc} = V_s$
- $V_{oc} = R_{||} I_s$
- Therefore: $I_s = V_s / R_{||}$



Source Transformations

- Relationships:
  - $I_s = V_s/R_{||}$
  - $I_s = V_s/R_s$
- Therefore
  - $R_s = R_{||} = R$
  - $I_s = V_s/R$
  - $V_s = I_sR$

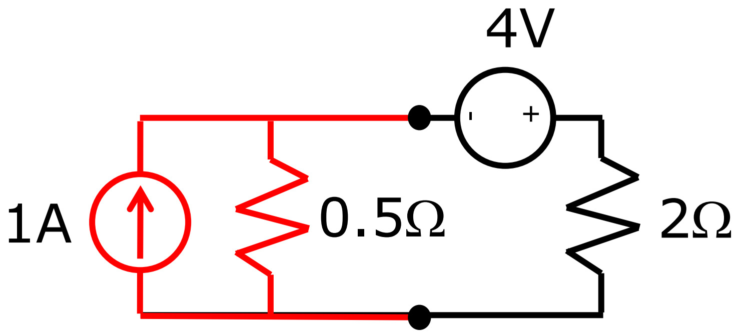
Source transformation equations



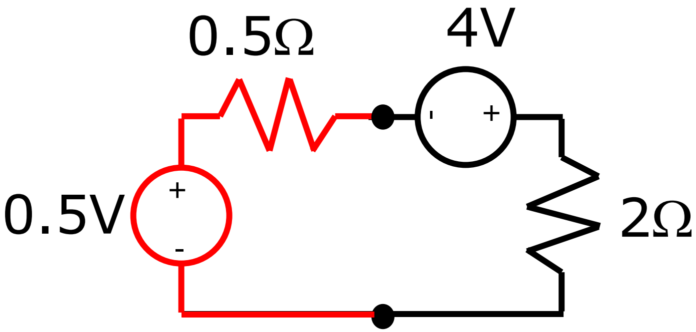
Source Transformations

- Verify the results hold for Circuit A and Circuit B

Circuit A

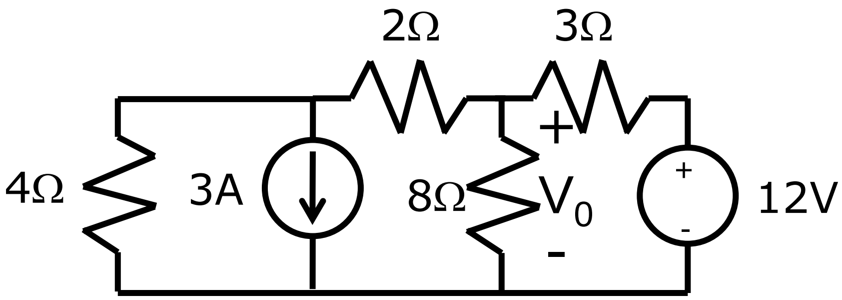


Circuit B



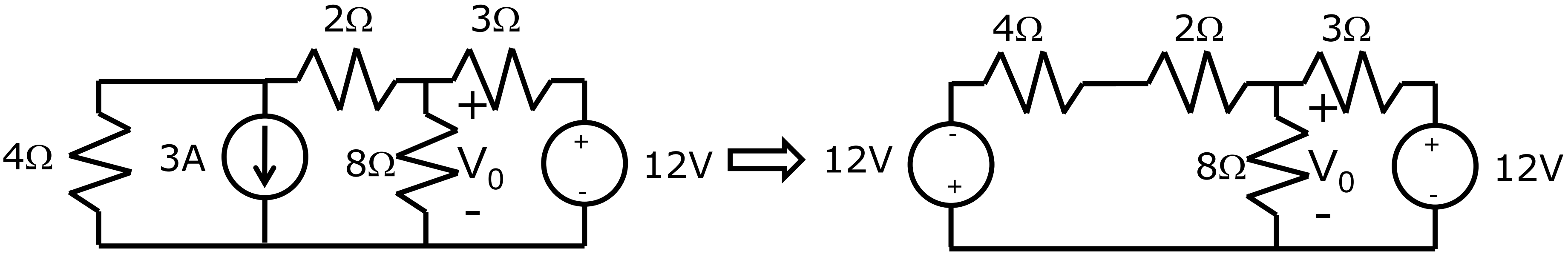
Example

- Use source transformations to find V_0
  - Consider the current source first. Which resistor can we associate it with?
    - 4Ω (they are in parallel)
  - Should we transform them?
    - Yes, the resistor will be in series with the 2Ω resistor, and we can combine the two



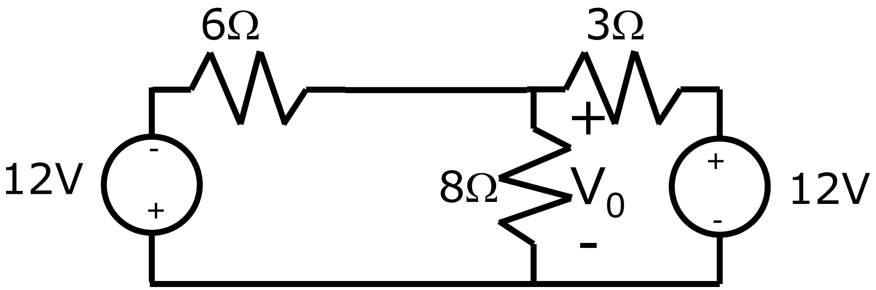
Example

- Transform the source:
  - $R = 4\Omega$
  - $V_s = I_sR = 3 \times 4 = 12V$
- Pay careful attention to the polarity



Example

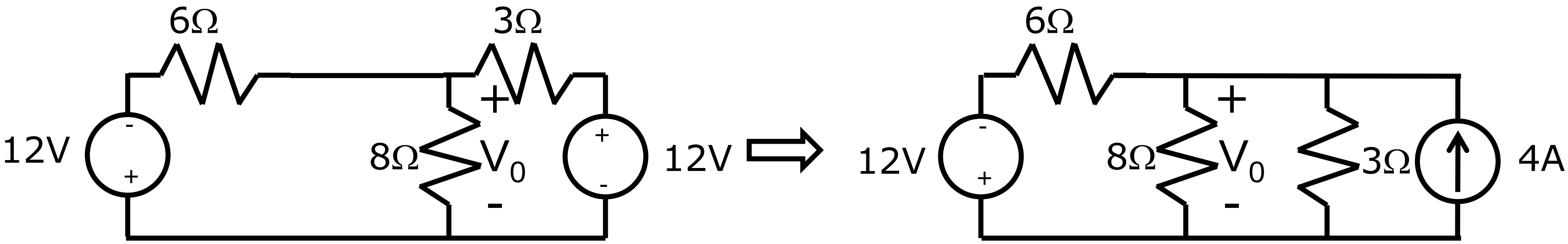
- Transform the 12V source (on the right), if it is beneficial



After combining series resistance

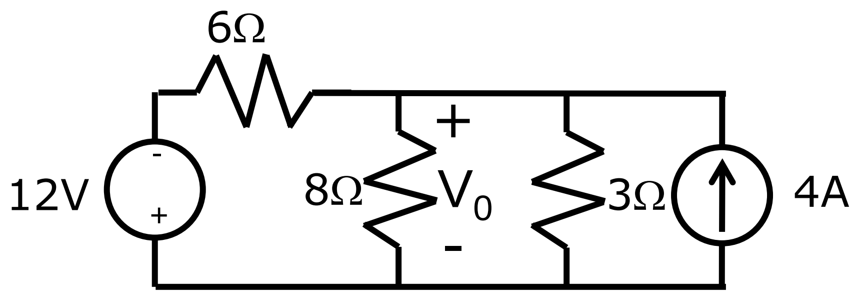
Example

- Yes, beneficial (results in parallel combination with V_0). Combine with 3Ω resistor to get:
  - R = 3Ω
  - $I_s = V_s/R = 12/3 = 4A$



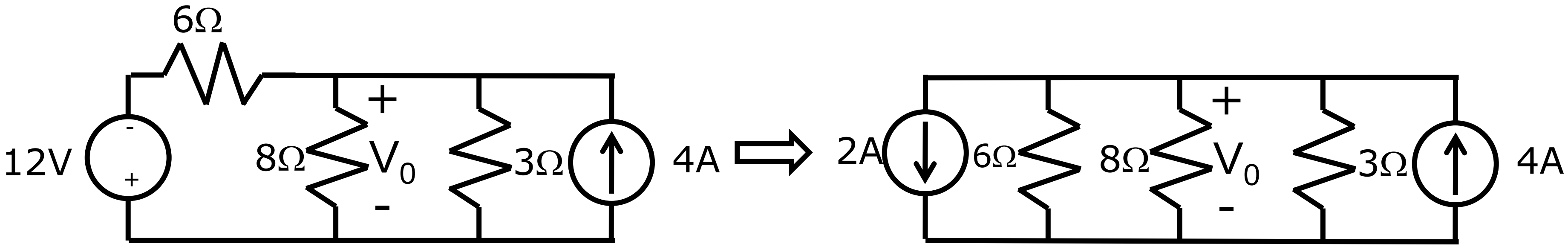
Example

- Now transform voltage source with 6Ω resistor



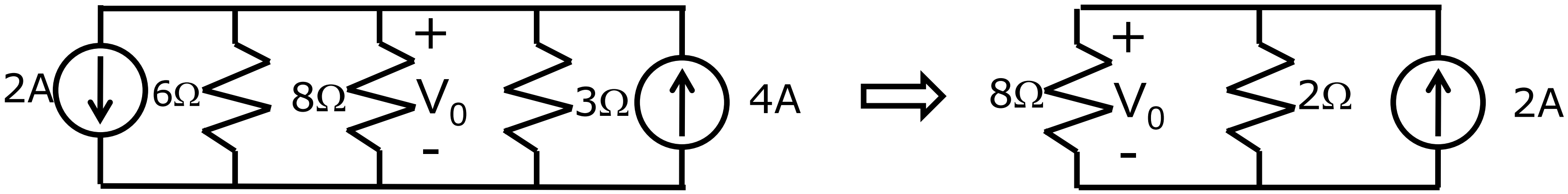
Example

- Result:
  - $R = 6\Omega$
  - $I_s = V_s/R = 12/6 = 2A$



Example

- The rest is easy.
- Current division: $I_0 = (4-2)x(2/10) = 0.4A$
- $V_0 = 3.2V$

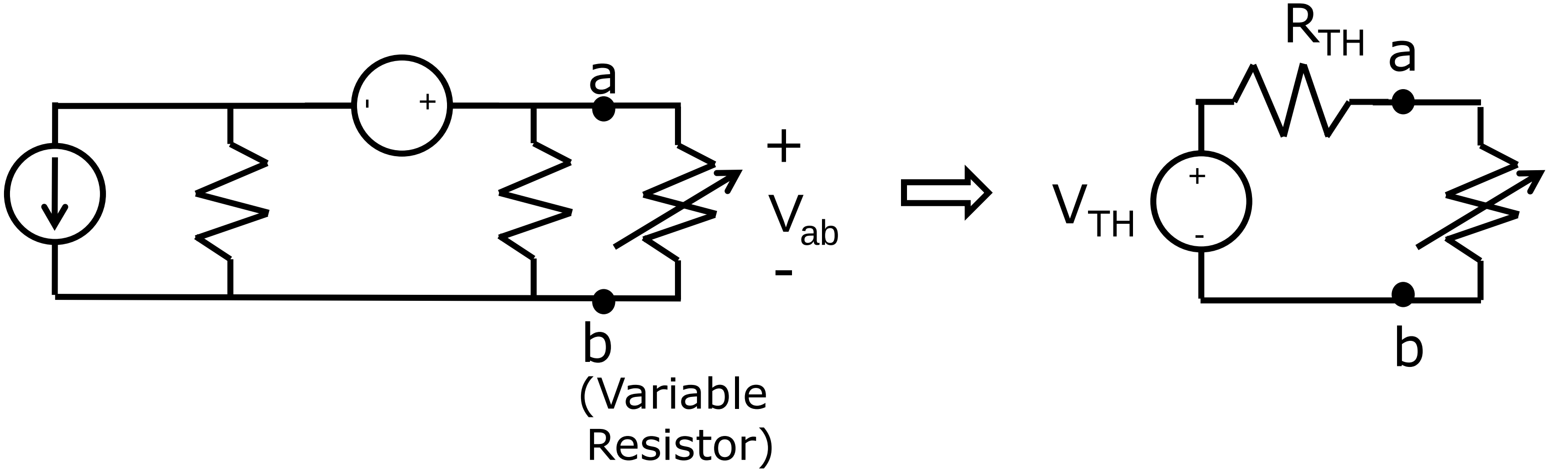


Source Transformations

- Dependent sources are handled using the same procedure
- Be careful

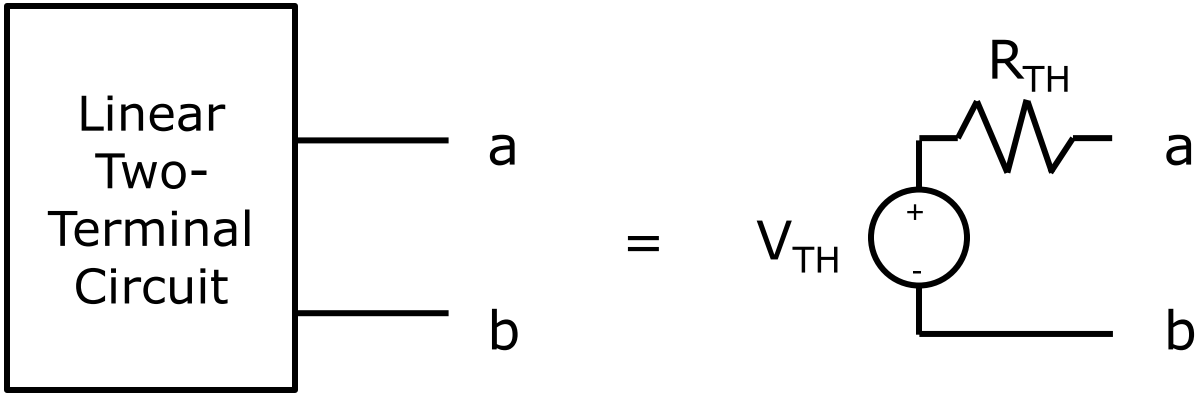
Thevenin's Theorem

- Often, most elements of a circuit are fixed and only one element (the load) changes
- Do not want to re-solve the entire circuit every time the load changes
- Better approach: represent unchanging part of circuit with voltage source with series resistance



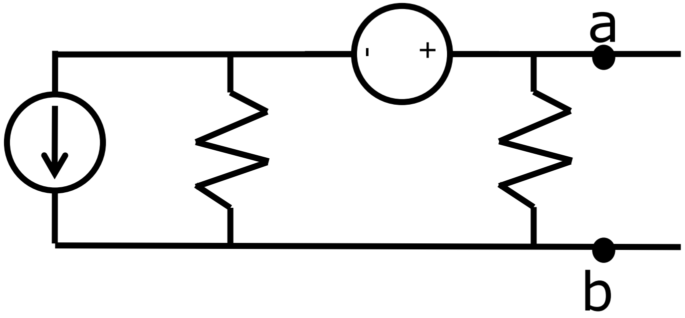
Thevenin's Theorem

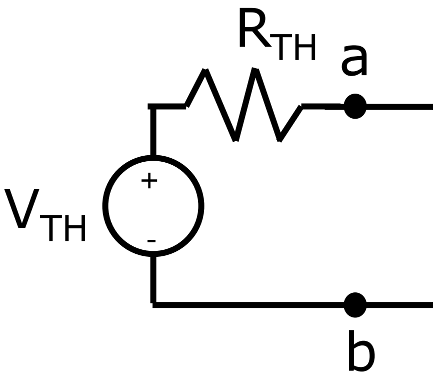
- Thevenin's Theorem: a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source in series with a resistor
- R_{TH}: Thevenin Resistance
- V_{TH}: Thevenin Voltage



Thevenin’s Theorem

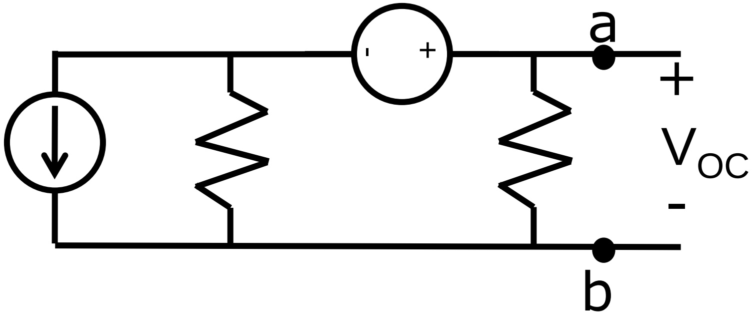
- How do we find V_{TH} and R_{TH}?
- One way: keep applying resistance and source transformations until there is a voltage source in series with a resistance between the terminals

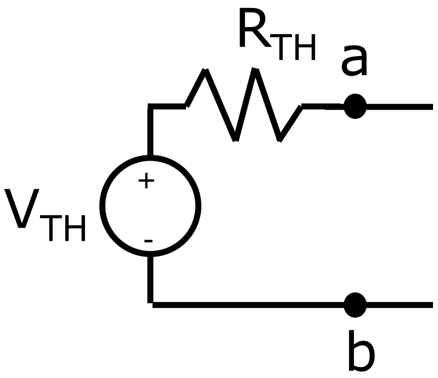




Thevenin’s Theorem

- Better way: recognize that
  - $V_{TH} = V_{OC}$ and
  - R_{TH} = input resistance (looking into terminals a, and b), or $R_{TH} = V_{OC}/I_{SC}$





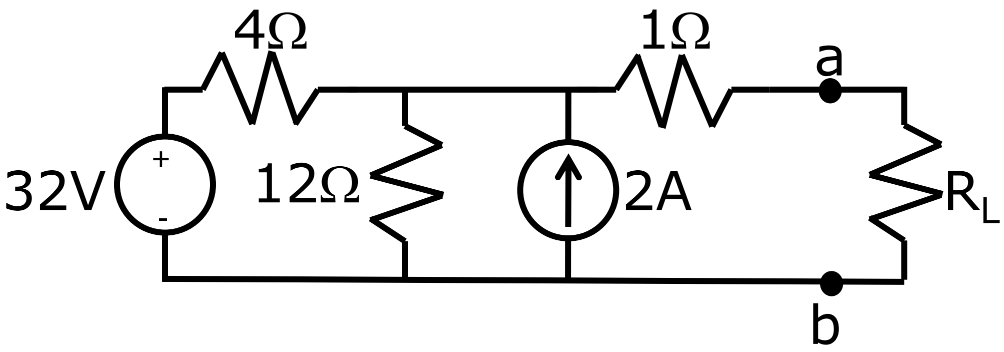
Finding Thevenin Resistance

- No Dependent Sources:
    - short all voltage sources
    - open all current source
    - then find equivalent resistance $R_{TH} = R_{eq}$

- Dependent Sources:
    - short all voltage sources
    - open all current source
    - Apply test voltage V_0, compute current I_0
    - $R_{TH} = V_0/I_0$

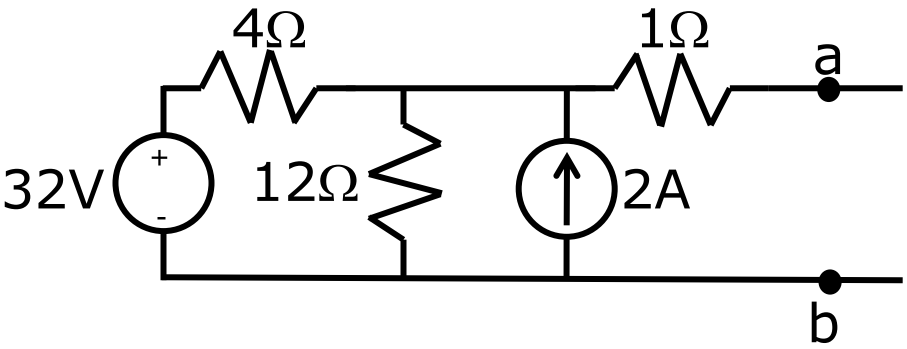
Example

- Find the current through the load resistor if R_L is 6, 16 and 36Ω
- Perfect situation for Thevenin Equivalent
  - Find Thevenin Equivalent, then solve equivalent circuit for various values of R_L



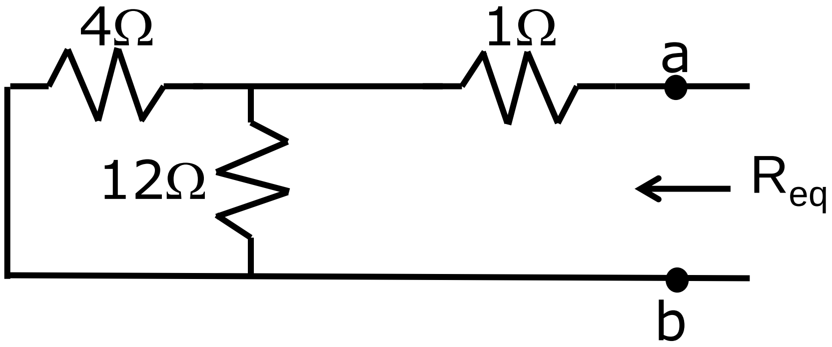
Example

- Start with finding the Thevenin voltage
  - $V_{TH} = V_{OC}$
- By superposition (or mesh analysis)
  - $V_{OC1} = 32(12/16) = 24V$
  - $V_{OC2} = 4x2x(12/16) = 6V$
  - $V_{TH} = V_{OC1} + V_{OC2} =$ 30V



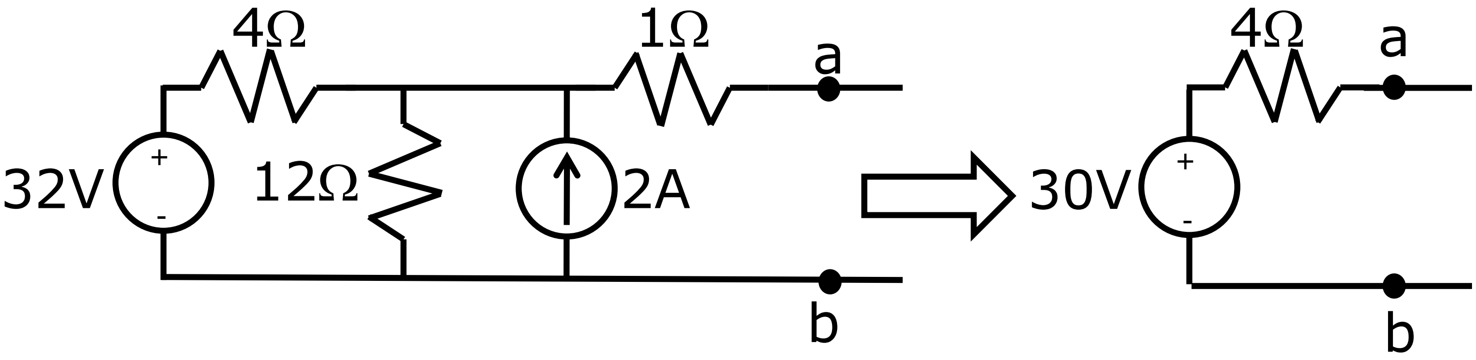
Example

- Now find the Thevenin resistance
  - $R_{TH} = R_{eq}$
  - deactivate all sources
  - $R_{eq} = 1 + (4x12)/16 = 4\Omega$



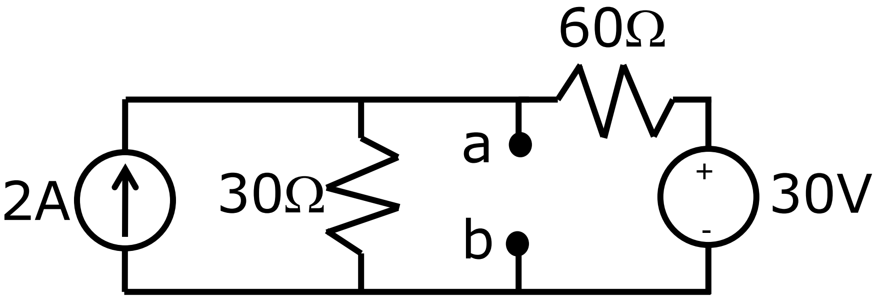
Example

- Thevenin equivalent circuit is shown below
- Current through various load resistances can be easily computed



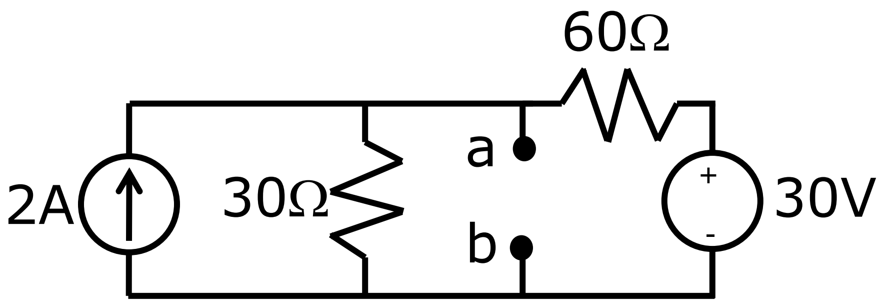
Example

- Find the Thevenin Equivalent of the circuit between the terminals a, b



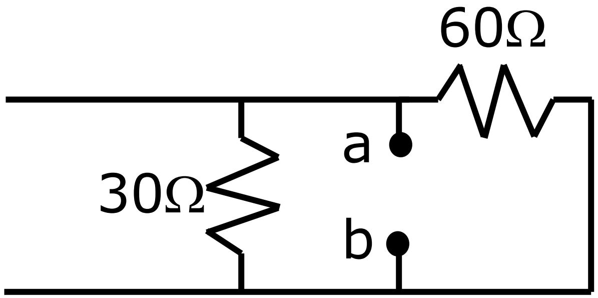
Example

- Via superposition
  - $V_{OC1} = 30x(30/90) = 10V$
  - $V_{OC2} = 30x2(60/90) = 40V$
- $V_{TH} = V_{OC1} + V_{OC2} = 50V$



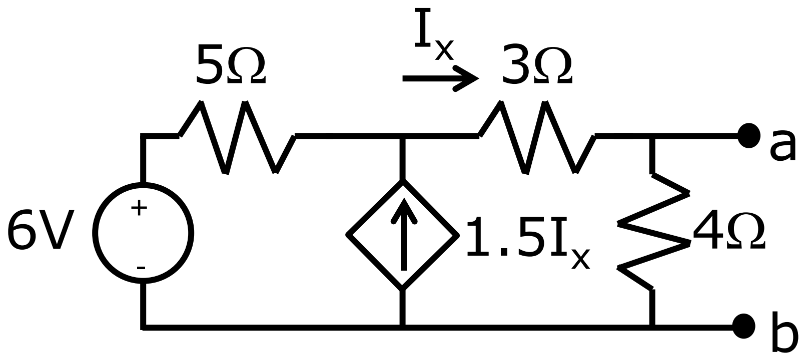
Example

- Now find R_{TH}
  - $R_{TH} = (30x60)/(30+60) = 20\Omega$



Thevenin’s Theorem

- Find the Thevenin equivalent
  - Note the dependent source
- Finding V_{TH} is the same procedure as before



Thevenin's Theorem

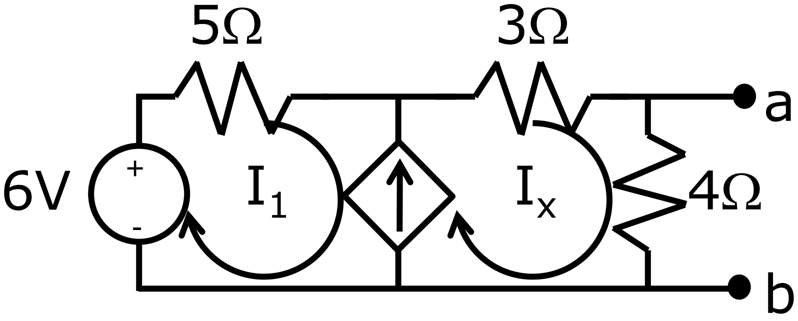
- Mesh Analysis
  - $6 = 5I_1 + 7I_x$ (Supermesh)
  - $1.5I_x + I_1 = I_x$ (current source constraint equation)

Solving…

$I_x = 1.33A$

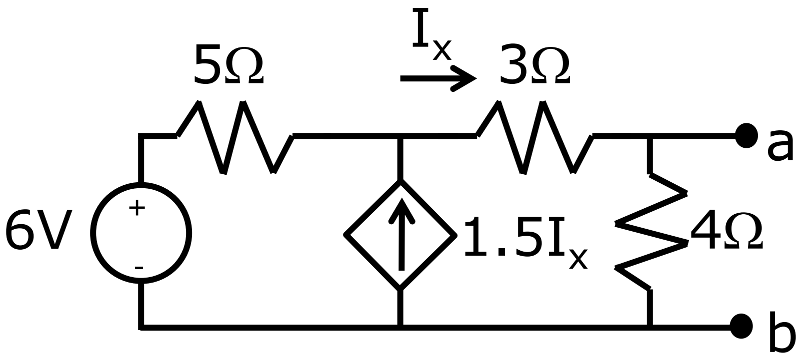
Therefore

$V_{OC} = 1.33x4 = 5.33V = V_{TH}$



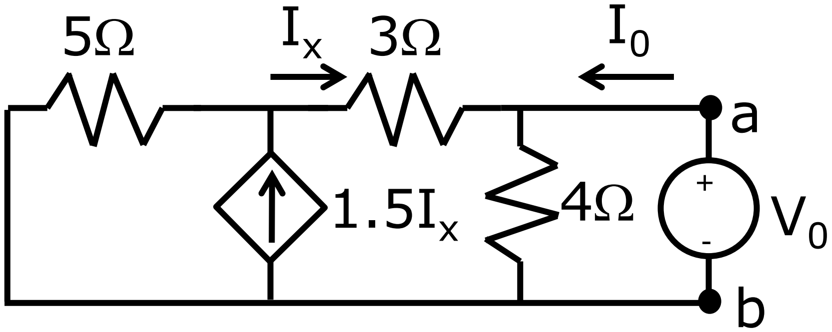
Thevenin’s Theorem

- To find R_{TH}
  - apply either test voltage or current to the terminals
  - deactivate independent sources
  - compute either terminal current or voltage
  - $R_{TH} = V_0/I_0$



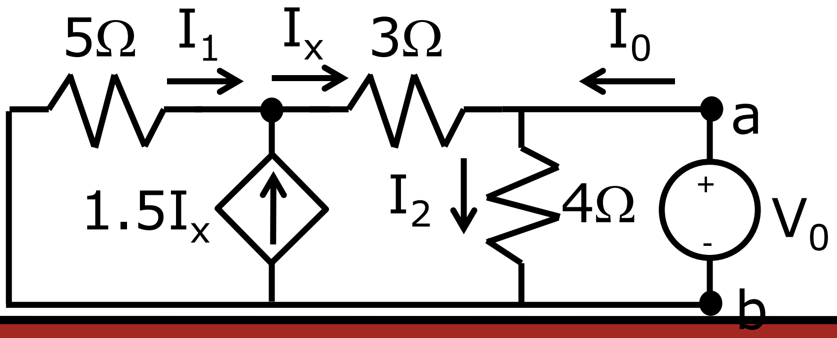
Thevenin’s Theorem

- Apply a test voltage V_0
- Let $V_0 = 1V$
- Now find I_0 (note polarity if I_0)



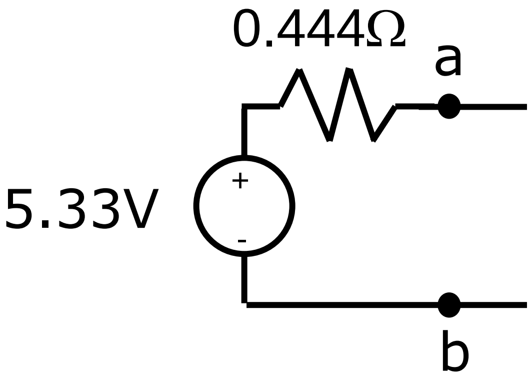
Thevenin’s Theorem

- $I_1 + 1.5I_x = I_x$ (Nodal Analysis)
  - $I_1 + 0.5I_x = 0$
  - -0.2V1 + 0.5(V1 - 1)/3=0
  - -0.0333V1 – 0.16667 =0
  - V_1=-5V
  - I_x = -2A (Ohm’s Law)
  - I_2 = 0.25A (Ohm’s Law)
  - I_0 = 2.25A (KCL)



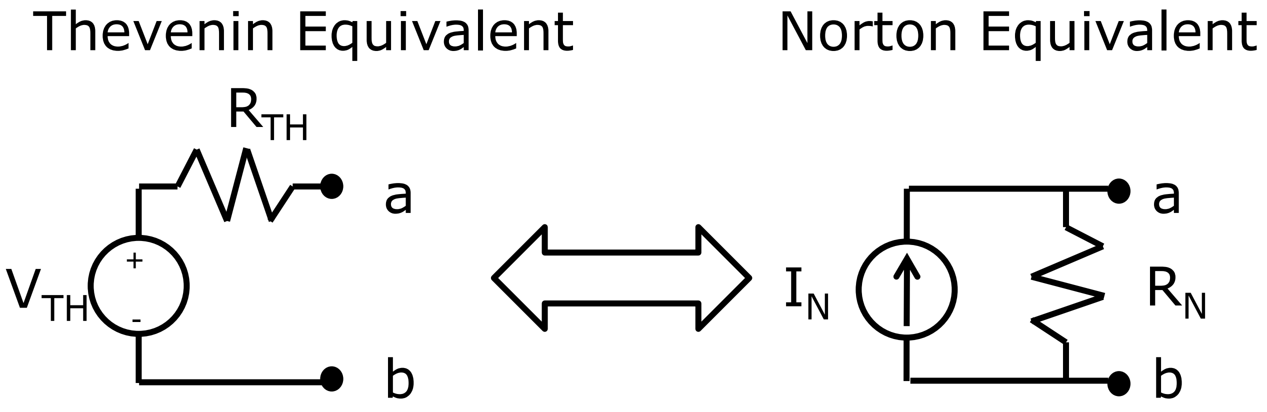
Thevenin’s Theorem

- Therefore $R_{TH} = V_0/I_0 = 1/2.25 = 0.444\Omega$



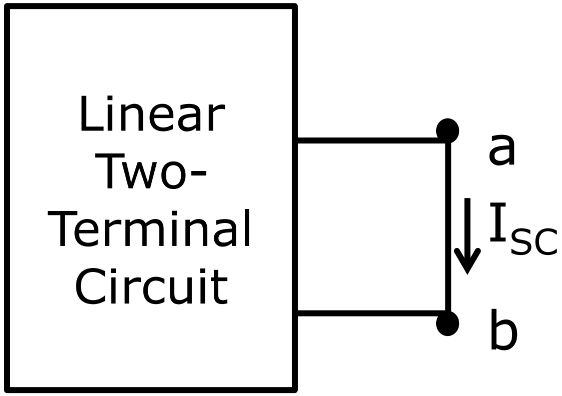
Norton's Theorem

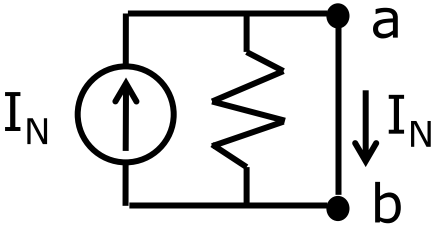
- Thevenin equivalent circuit can be transformed into current source in parallel with a resistor
- From discussion on source transformation:
  - $R_N = R_{TH}$



Norton’s Theorem

- I_N is found by shorting the terminals of the circuit
    - $I_N = I_{SC}$

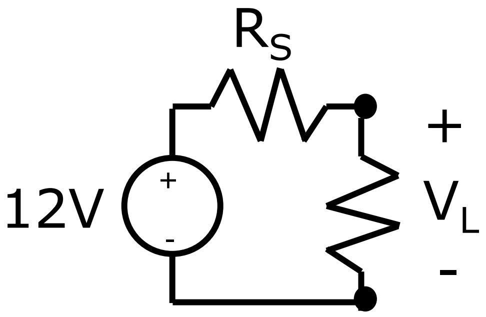


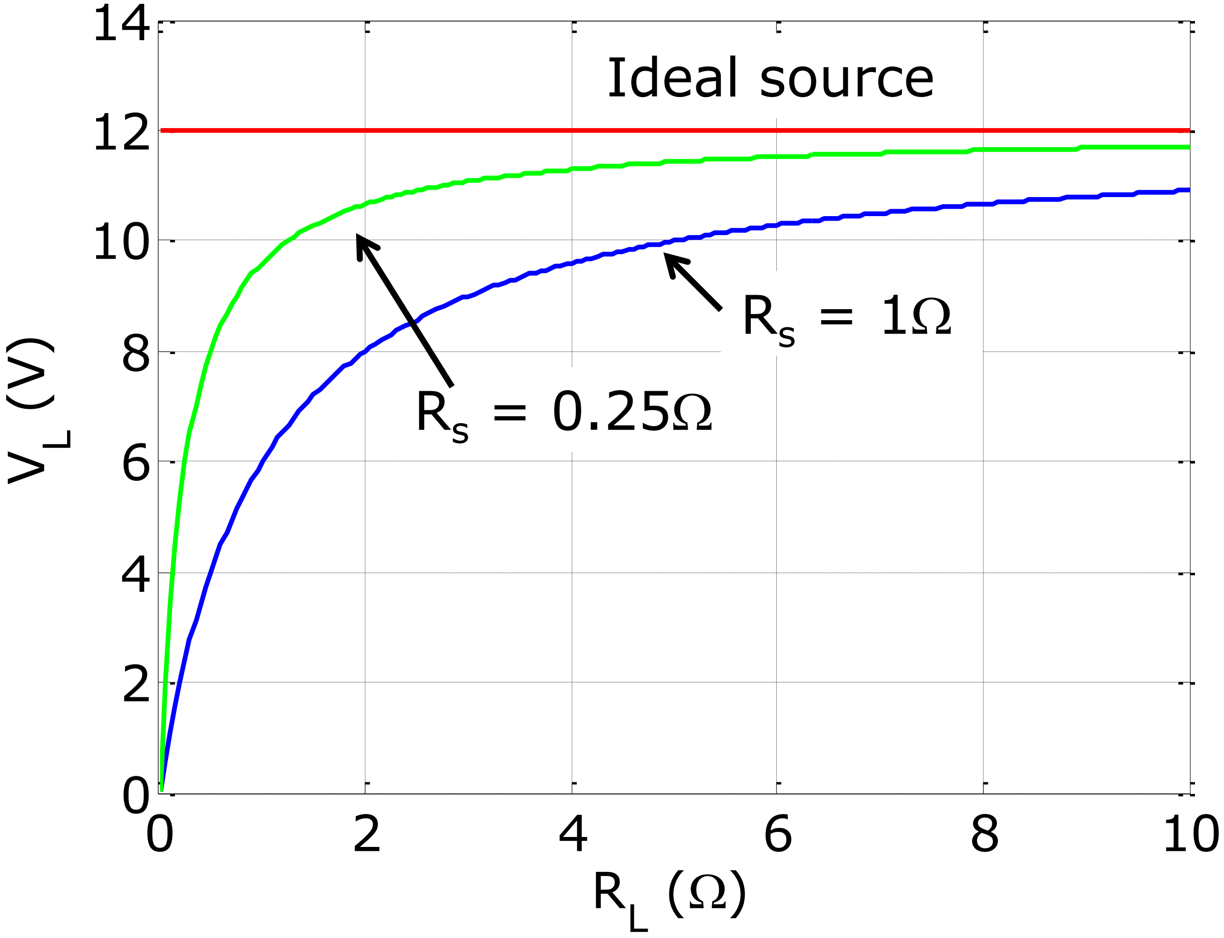


Norton’s Theorem

- Process for finding R_{TH} is identical to that for Thevenin’s theorem
- No Dependent Sources:
  - short all voltage sources
  - open all current source
  - then find equivalent resistance $R_{TH} = R_{eq}$
- Dependent Sources:
  - short all voltage sources
  - open all current source
  - Apply test voltage V_0 (or current), compute current I_0 (or voltage)
  - $R_{TH} = V_0/I_0$

Practical Sources





Practical Sources