Transmission Line Capacitance

Text: 3.7—3.10

ECEGR 451
Power Systems

Topics

- Electric Fields Review
- Capacitance
  - capacitance to neutral
  - three phase capacitance
  - asymmetric spacing
  - bundled conductors
- Example

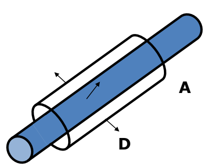
Electric Flux

$\int_A \mathbf{D} \cdot d\mathbf{a} = q_e$ Gauss's Law

D: electric flux density, C/m^2

d**a**: differential area, m^2

q_e: total charge enclosed by surface, C



Electric Field

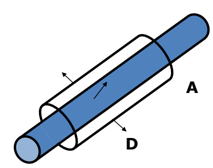
$\mathbf{D} = \varepsilon \mathbf{E}$

E: electric field intensity, v/m

ε: permittivity, F/m

$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

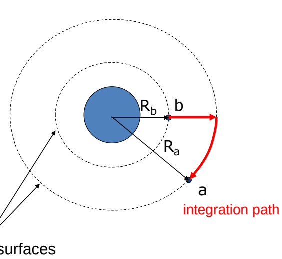


Voltage, Single Conductor

$V_{ab} \triangleq V_a - V_b = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$

$V_{ab} = \int_{R_a}^{R_b} \frac{q}{2\pi\varepsilon R} dR$ from $E = \frac{q}{2\pi R \varepsilon}$

$= \frac{q}{2\pi\varepsilon} \ln \frac{R_a}{R_b}$



Voltage, Two Conductors

$V_{ab} = \frac{q}{2\pi\varepsilon} \ln \frac{R_a}{R_b}$ single conductor

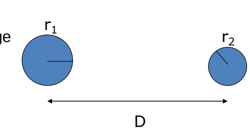
use superposition to determine voltage

$V_{12} = \underbrace{\frac{q_1}{2\pi\varepsilon} \ln \frac{D}{r_1}}_{\text{conductor 1}} + \underbrace{\frac{q_2}{2\pi\varepsilon} \ln \frac{r_2}{D}}_{\text{conductor 2}}$

$= \frac{q_a}{2\pi\varepsilon} \left(\ln \frac{D}{r_1} - \ln \frac{r_2}{D} \right)$

$= \frac{q_a}{2\pi\varepsilon} \left(\ln \frac{D^2}{r_1 r_2} \right)$

assuming equal but opposite charges



Capacitance

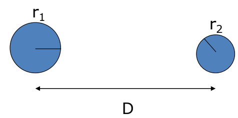
$$c_{12} = \frac{q_1}{V_{12}} \text{ F/m}$$

$$V_{12} = \frac{q_1}{2\pi\varepsilon}\left(\ln\frac{D^2}{r_1 r_2}\right)$$

$$c_{12} = \frac{2\pi\varepsilon}{\ln\left(D^2/r_1 r_2\right)}$$

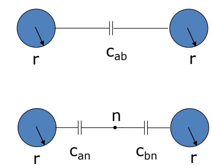
if $r_1 = r_2 = r$

$$c_{12} = \frac{\pi\varepsilon}{\ln D/r}$$



Capacitance to Neutral

- Convenient for per-phase calculations
- Potential difference to neutral is half that of difference between the two conductors



$$c_{ab} = \frac{\pi\varepsilon}{\ln\frac{D}{r}}$$

$$c_n = c_{an} = c_{bn} = \frac{q_a}{\left(V_{ab}/2\right)} = \frac{2\pi\varepsilon}{\ln\frac{D}{r}} \text{ F/m to neutral}$$

Capacitance and Inductance

- Previously, we found the inductance for a solid, one conductor in a single-phase circuit to be

$$L = 2\left(\ln\frac{D}{r'}\right) \times 10^{-7} \text{ H/m}$$

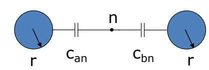
- Capacitance to neutral is

$$c = \frac{2\pi\varepsilon}{\ln\frac{D}{r}} \text{ F/m to neutral}$$

- Note the difference in radius expressions

Capacitance to Neutral

- Once capacitance to neutral is found, capacitive reactance is



$$X_C = \frac{1}{2\pi fc} = \left(\frac{2.862}{f}\right)\left(\ln\frac{D}{r}\right) x10^9 \ \Omega\cdot\text{m to neutral}$$

- Capacitive reactance to neutral for 1 meter of line

Capacitance to Neutral

- As line length increases what happens to
  - resistance?
  - inductive reactance?
  - capacitive reactance?
- For capacitance, you must divide by length, not multiply

$$X_C = \frac{1}{2\pi fC} = \left(\frac{2.862}{f}\right)\left(\ln\frac{D}{r}\right) x10^9 \ \Omega\cdot\text{m to neutral}$$

Symmetrical Three-Phase

- Assume uniform charge distribution
- Use superposition to find voltages

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D}\right)$$

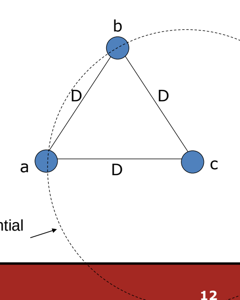
due to q_a and q_b only

$$V_{ab} = \frac{q_c}{2\pi\varepsilon}\ln\frac{D}{D}$$

due to q_c only

equal to zero

equalpotential surface



Symmetrical Three-Phase

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D}\right)$$

similarly

$$V_{ac} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D}{r} + q_c \ln\frac{r}{D}\right)$$

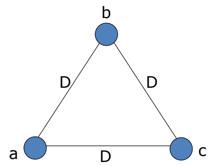
adding

$$V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon}\left(2q_a \ln\frac{D}{r} + \; q_b + q_c \; \ln\frac{r}{D}\right)$$

assuming

$$q_a + q_b + q_c = 0$$

$$V_{ab} + V_{ac} = \frac{3q_a}{2\pi\varepsilon}\left(\ln\frac{D}{r}\right)$$



Symmetrical Three-Phase

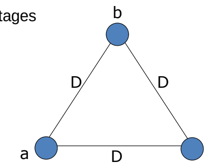
$$V_{ab} + V_{ac} = \frac{3q_a}{2\pi\varepsilon}\left(\ln\frac{D}{r}\right)$$

from our discussion on line and phase voltages

$$V_{ab} = \sqrt{3}V_{an}\angle 30^{\circ}$$

$$V_{ac} = -V_{ac} = \sqrt{3}V_{an}\angle -30^{\circ}$$

$$V_{ab} + V_{ac} = 3V_{an}$$



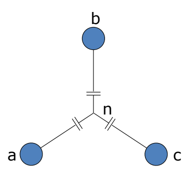
resulting in

$$V_{an} = \frac{q_a}{2\pi\varepsilon}\ln\frac{D}{r} \qquad c_n = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon}{\ln D/r}$$ F/m to neutral

same as single phase!

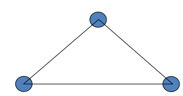
Symmetrical Three-Phase

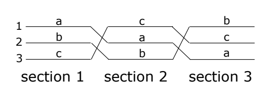
Capacitance to neutral



Asymmetrical Spacing

- Similar approach used for inductance used here
- Assume transposition at equal distances





Asymmetrical Spacing

- section 1

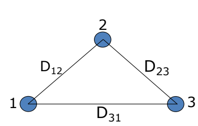
$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{12}}{r} + q_b \ln\frac{r}{D_{12}} + q_c \ln\frac{D_{23}}{D_{31}}\right)$$

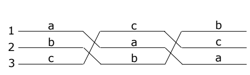
- section 2

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{23}}{r} + q_b \ln\frac{r}{D_{23}} + q_c \ln\frac{D_{31}}{D_{12}}\right)$$

- section 3

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{31}}{r} + q_b \ln\frac{r}{D_{31}} + q_c \ln\frac{D_{12}}{D_{23}}\right)$$





Asymmetrical Spacing

- Averaging the values

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{12}}{r} + q_b \ln\frac{r}{D_{12}} + q_c \ln\frac{D_{23}}{D_{31}}\right)$$ section 1

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{23}}{r} + q_b \ln\frac{r}{D_{23}} + q_c \ln\frac{D_{31}}{D_{12}}\right)$$ section 2

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D_{31}}{r} + q_b \ln\frac{r}{D_{31}} + q_c \ln\frac{D_{12}}{D_{23}}\right)$$ section 3

$$\bar{V}_{ab} = \frac{1}{6\pi\varepsilon}\left(q_a \ln\frac{D_{12}D_{23}D_{31}}{r^3} + q_b \ln\frac{r^3}{D_{12}D_{23}D_{31}} + q_c \ln\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}\right)$$

Asymmetrical Spacing

- simplifying

$$\bar{V}_{ab} = \frac{1}{6\pi\varepsilon}\left(q_a \ln\frac{D_{12}D_{23}D_{31}}{r^3} + q_b \ln\frac{r^3}{D_{12}D_{23}D_{31}} + q_c \ln\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}\right)$$

$$\bar{V}_{ab} = \frac{1}{6\pi\varepsilon}\left(q_a \ln\frac{D_m}{r} + q_b \ln\frac{r}{D_m}\right)$$

where

$$D_m \triangleq D_{12}D_{23}D_{31}{}^{1/3}$$

Asymmetrical Spacing

$$\bar{V}_{ab} = \frac{1}{6\pi\varepsilon}\left(q_a \ln\frac{D_m}{r} + q_b \ln\frac{r}{D_m}\right)$$

- similarly,

$$\bar{V}_{ac} = \frac{1}{6\pi\varepsilon}\left(q_a \ln\frac{D_m}{r} + q_c \ln\frac{r}{D_m}\right)$$

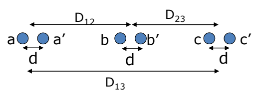
$$\underbrace{3V_{an} = V_{ab} + V_{ac}}_{\text{from before}} = \frac{1}{2\pi\varepsilon}\left(2q_a \ln\frac{D_m}{r} + q_b \ln\frac{r}{D_m} + q_c \ln\frac{r}{D_m}\right)$$

$$3V_{an} = \frac{3}{2\pi\varepsilon} q_a \ln\frac{D_m}{r} \quad \text{using} \quad q_a + q_b + q_c = 0$$

$$c_n = \frac{2\pi\varepsilon}{\ln\frac{D_m}{r}} \text{ F/m}$$

Bundled Solid Conductors, Asymmetrical Spacing

- D_{12} , D_{23}, D_{31} >> d
- Charges are divided equally
- All conductors have radius, r
- Lines are transposed



for one section:

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[\frac{q_a}{2}\left(\ln\frac{D_{12}}{r} + \ln\frac{D_{12}}{d}\right) + \frac{q_b}{2}\left(\ln\frac{r}{D_{12}} + \ln\frac{d}{D_{12}}\right) + \frac{q_c}{2}\left(\ln\frac{D_{23}}{D_{31}} + \ln\frac{D_{23}}{D_{31}}\right)\right]$$

Bundled Conductors

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[\frac{q_a}{2}\left(\ln\frac{D_{12}}{r} + \ln\frac{D_{12}}{d}\right) + \frac{q_b}{2}\left(\ln\frac{r}{D_{12}} + \ln\frac{d}{D_{12}}\right) + \frac{q_c}{2}\left(\ln\frac{D_{23}}{D_{31}} + \ln\frac{D_{23}}{D_{31}}\right)\right]$$

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{12}}{\sqrt{rd}} + q_b \ln\frac{\sqrt{rd}}{D_{12}} + q_c \ln\frac{D_{23}}{D_{31}}\right]$$

with the lines transposed

$$c = \frac{2\pi\varepsilon}{\ln\left(\frac{D_m}{\sqrt{rd}}\right)} \text{ F/m to neutral}$$

← GMR for capacitance

GMR, GMD for Capacitance Calculations

- In the previous example for the two conductor bundle case:
  - used $rd^{1/2}$ as the GMR
- Recall the GMR for inductance in a two conductor bundle:

$$R_b = r'd^{1/2}$$

- GMDs are the same:

$$D_m = D_{12}D_{23}D_{31}{}^{1/3}$$

Line Capacitance

- Capacitive reactance

$$X_C = \frac{1}{2\pi fC} \ \Omega\cdot\text{m to neutral}$$

$$X_C = \frac{1}{f}\, 1.779 \times 10^6 \left(\ln\frac{D_m}{r}\right) \ \Omega\cdot\text{mi to neutral}$$

- Current associated with the capacitance called *charging current*

Table Use

- Capacitive reactance (unbundled)

$$X_C = \left(\frac{1}{f}\right) 1.779 \times 10^6 \left(\ln \frac{D_m}{r}\right) \ \Omega \bullet \text{mi}$$

- Split into two parts

$$X_C = \underbrace{1.779 \times 10^6 \ \frac{1}{f}\left(\ln \frac{1}{r}\right)}_{X_a'} + \underbrace{1.779 \times 10^6 \ \frac{1}{f} \ln D_m}_{X_d'} \quad \Omega \cdot \text{mi to neutral}$$

  - X_a': 1 ft spacing capacitive reactance, independent of distance
  - X_d': capacitive reactance spacing factor
- Use table, like in inductive reactance calculations

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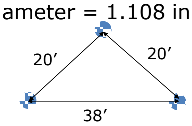
Example

- Single circuit, three-phase line at 60 Hz
- Conductors are ACSR *Drake*
- Find capacitive reactance in Ohm-miles per phase
- Conductor diameter = 1.108 in



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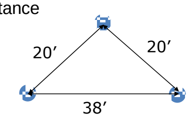
Example

First, the hard way

1) find GMR
2) find GMD
3) calculate capacitance to neutral
4) calculate reactance



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Example

1) find GMR

$$r = \frac{1.108}{24} = 0.0462 \text{ ft}$$

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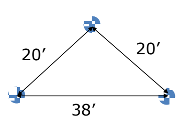
Example

2) find GMD



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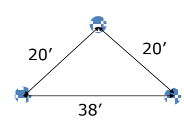
Example

2) find GMD



$D_m = 24.8 \text{ ft} \quad r = 0.0462 \text{ ft}$

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Example

3) calculate capacitance to neutral
4) calculate reactance

Example

3) calculate capacitance to neutral
4) calculate reactance

$$c_n = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln\left(\frac{24.8}{0.0462}\right)} \ F/m$$

$$X_C = \frac{10^{12}}{2\pi \times 60 \times 8.8466 \times 1609} = 0.1864 \times 10^6 \ \Omega \cdot mi \text{ to neutral}$$

Example

Now the easy way

- look up X_a'
- look up X_d'
- add them

Example

Now the easy way

- look up X_a'
- look up X_d'
- add them

$X_a' = 0.0912 \times 10^6$

$X_d' = 0.0953 \times 10^6$

$X_C = 0.0912 + 0.0953 = 0.1865 \times 10^6 \ \Omega \cdot mi$ to neutral

Summary

- Derived equations to calculate shunt capacitive reactance for a transmission line

$$c_n = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon}{\ln GMD/R_b} \text{ F/m to neutral} \quad X_C = \frac{1}{2\pi fc} \ \Omega \cdot m \text{ to neutral}$$

- r is used, not r' in capacitance calculations
- Solution approach is similar to inductance problems
- Once you've mastered determining these parameters we will assume they are given