09-Basic Laws of Electromagnetism

ECEGR 450
Electromechanical Energy Conversion

Overview

- Introduction
- Notation, Variables and Units
- Maxwell's Equations
- Ampere's Force Law
- Torque on a Loop

Questions

- How can electricity be used to create motion?
- How can motion be used to create electricity?
- What is the relationship between electricity and magnetism?

Introduction

- Almost all generators and motors use a magnetic field as a medium for energy conversion
- Magnetic field produced by permanent magnet or a winding with current
- Need to understand Maxwell's equations to appreciate the electromechanical energy conversion process

Notation, Variables and Units

- **Bold**: vector quantity
- **E**: electric field intensity (V/m)
- **B**: magnetic flux density (Wb/m^2) or Tesla (T)
- Φ: magnetic flux, (Wb)
- **H**: magnetic field intensity, (A/m)
- **D**: electric flux intensity (C/m^2)
- **J**: volume current density (C/m^2)
- ρ: volume charge density (C/m^3)
- **F**: force (N)
- q: charge (C)
- **u**: velocity (m/s)

Maxwell's Equations

- Named for James Clerk Maxwell (1831-1879)
- Describe the nature of electromagnetic fields
- Set of four equations
  - Ampere's Law
  - Faraday's Law
  - Gauss's Law (flux)
  - Gauss's Law (charge)

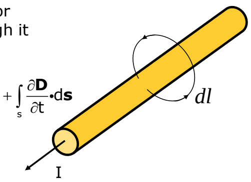
Maxwell's Equations

Faraday's Law and Ampere's Law are especially important for electromechanical energy conversion, so we focus on them

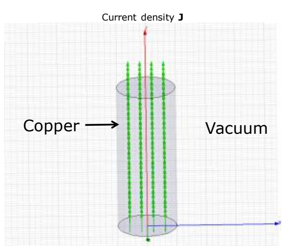
Ampere's Law

- Current through a conductor will produce a circular magnetic field around it
  - Line integral of the magnetic field around the conductor equals the current through it
- Mathematically

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{or} \quad \oint_C \mathbf{H} \cdot d\ell = \oint_s \mathbf{J} \cdot d\mathbf{s} + \int_s \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$$



Ampere's Law

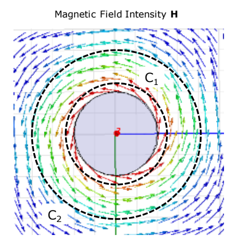


Current through a conductor in Z direction

Ampere's Law

- Direction of **H** field is CCW
  - Right Hand Rule
- Field strength decreases as path length increases
  - Field becomes weaker as distance increases

$$\oint_{C_1} \mathbf{H} \cdot d\ell = \oint_{C_2} \mathbf{H} \cdot d\ell = \text{constant}$$



View "down" Z axis (Current out of slide)

Magnetic Flux Density

- Related to **H** is the Magnetic Flux Density (**B**)
  $\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$
- Where:
  - **B**: magnetic flux density (Wb/m^2)
  - **M**: magnetization field (A/m)
  - μ_0: permeability of free space 4πE-7 (H/m)
  - μ_r: relative permeability

A material is magnetized if $\mathbf{M} \neq 0$
Note: **H** and **M** may have different signs (directions)

Magnetic Flux Density

- Inside <u>linear</u> isotropic homogeneous (LIH)medium

$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$

$\mathbf{M} = \chi\mathbf{H}$

$\mathbf{B} = \mu_0(1+\chi)\mathbf{H}$

$\mathbf{B} = \mu_0\mu_r\mathbf{H} = \mu\mathbf{H}$

Note: permanent magnets are not LIH

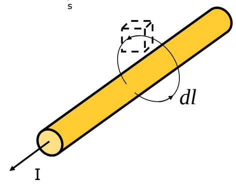
- Where:
  - χ: magnetic susceptibility of the medium
  - μ_r: relative permeability

Note: magnetic susceptibility can be positive or negative

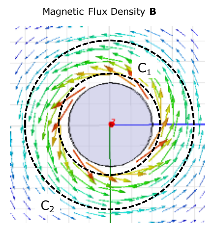
Gauss's Law for Magnetic Fields

- The magnetic flux entering a volume is equal to the magnetic flux leaving it (it is continuous)
- Mathematically

$\nabla \cdot \mathbf{B} = 0$ or $\oint_s \mathbf{B} \cdot d\mathbf{s} = 0$ (divergence is 0)



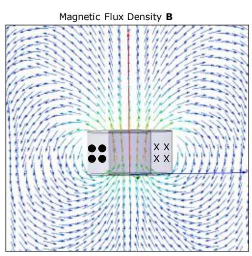
Magnetic Flux Density



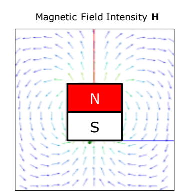
View "down" Z axis
(Current out of slide)

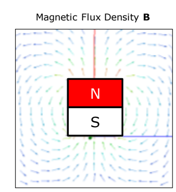
- Direction of **B** field is inline with **H** field (**M** = 0 in a vacuum)
- Field strength decreases as path length increases
  - Field becomes weaker as distance increases

Magnetic Fields Around a Solenoid



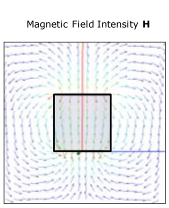
Magnetic Fields External to a Magnet

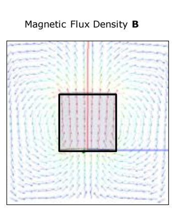




B, **H** leave from North pole and return to the South pole

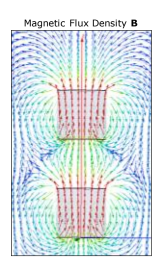
Magnetic Fields Internal to a Magnet



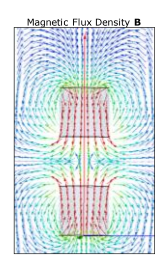


B, **H** have opposite directions inside the magnet. Why?
$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$
B is continuous, and **H** is conservative.

Two Permanent Magnets



Both magnets North in +Z direction



Top magnet North in -Z direction; bottom magnet North +Z direction

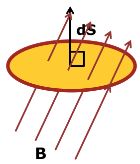
Magnetic Flux

- The total magnetic flux, Φ, passing through a surface is:

$$\Phi = \int_s \mathbf{B} \cdot d\mathbf{s}$$

- Where:

  Φ: magnetic flux (Wb)



Faraday's Law

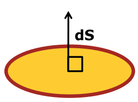
- The change in magnetic flux through a closed path induces a voltage
  - Known as induced electromotive force (emf)
- Mathematically

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{or} \quad \oint_c \mathbf{E} \cdot d\ell = -\int_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

- Understanding Faraday's Law is critical in understanding machines

Farday's Law

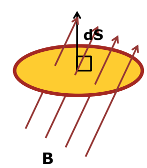
Consider a closed loop of wire in a time-varying magnetic field with differential surface d**S**



Farday's Law

The total magnetic flux, Φ, passing through the loop is:

$$\Phi = \int_s \mathbf{B} \cdot d\mathbf{s}$$



Farday's Law

Voltage is the electric field multiplied by distance so that

$$\oint_c \mathbf{E} \cdot d\ell = -\int_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

$$e = -\int_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} = -\frac{d\Phi}{dt}$$

e: induced electomotive force (emf), (V)

Example

Which of the following are ways to increase the voltage induced in a coil?

A) increase the area enclosed by the coil

B) reduce the resistance of the coil

C) increase the time rate of change of the flux density

Example

Which of the following are ways to increase the voltage induced in a coil?

A) increase the area enclosed by the coil

B) reduce the resistance of the coil

C) increase the time rate of change of the flux density

$$\Phi = \int_s \mathbf{B} \cdot d\mathbf{s}$$

$$e = -\frac{d\Phi}{dt}$$

Example

- If the flux through

Farday's Law of Induction

- Actually, the magnetic flux may be constant in time and we can still induce voltage
  - It's the change in flux that matters
- If the coil is stationary and the magnetic flux changes, then e is known as transformer emf
- If the coil is moving and the magnetic flux is constant, then e is known as motional emf

Important to understand this!

Farday's Law

- A conductor moving in an area of constant flux will have voltage induced in it according to:

  $e = \oint_c (\mathbf{u} \times \mathbf{B}) \cdot d\ell$
  - **u**: velocity of the conductor (m/s)
- Motional emf is due to a force acting on the free electrons in the conductor that moves them to one side or another
  - More on this force later
- Total induced emf is the sum of transformer and motional emf

A side note…

- Units of **B**: Wb/m^2
- Weber: The weber is the magnetic flux which, linking a circuit of one turn, would produce in it an electromotive force of 1 volt if it were reduced to zero at a uniform rate in 1 second
- Examining the units of voltage from: $e = -\int_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} = -\frac{d\Phi}{dt}$
  - Yields: V = ((Wb/m^2)/s) m^2

    $V = \frac{Wb}{sm^2} m^2$

    $Vs = Wb$
- $HB = \frac{Wb}{m^2}\left(\frac{A}{m}\right) = \frac{Vs}{m^2}\left(\frac{A}{m}\right) = \frac{VAs}{m^3} = \frac{Ps}{m^3} = \boxed{\frac{J}{m^3}}$

Energy per unit volume

Gauss's Law for Electric Fields

- The total electric flux leaving a closed volume is equal to the charge enclosed
- Mathematically

  $\nabla \cdot \mathbf{D} = \rho$ or $\oint_s \mathbf{D} \cdot d\mathbf{s} = \int_v \rho dv$

Other Important Equations

- Note:
  - **E** and **H** are fundamental fields
  - **D** and **B** are derived fields
- The fields are related by:

$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_r\varepsilon_0\mathbf{E}$

$\mathbf{B} = \mu\mathbf{H} = \mu_r\mu_0\mathbf{H}$

See a previous slide on applicability of eqns to different media

- Where:
  - ε_0: permittivity of free space 8.85E-12 (F/m)
  - μ_0: permeability of free space 4πE-7 (H/m)
  - ε_r, μ_r: relative permittivity and permeability

Other Important Equations

- Equation of Continuity

$\nabla \cdot \mathbf{J} = \frac{\partial \rho}{\partial t}$

- Lorentz Force Equation

$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

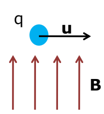
Ampere's Force Law

- Assume a charge q, is moving with velocity **u** through a magnetic field
- By the Lorentz Force equation

$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$

$\mathbf{F} = q\mathbf{u} \times \mathbf{B}$

- A force is exerted on the charge in the direction out of the slide



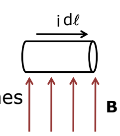
Ampere's Force Law

- A moving charge is current, therefore

$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$

$\mathbf{F} = \int_c i d\ell \times \mathbf{B}$

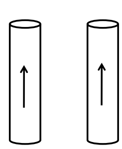
- This is Ampere's force law
- Used to compute torque in machines



Note:dℓ(vector) is the direction of the current

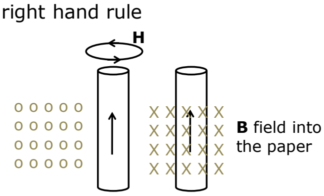
Example

- Consider two conductors, each with current I flowing in the same direction
- Are the conductors attracted to each other or repelled from each other?



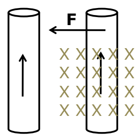
Example

- Consider the magnetic field associated with conductor 1
- From the right hand rule



Example

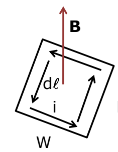
- Using
  $\mathbf{F} = q\mathbf{u} \times \mathbf{B}$
  $\mathbf{F} = \int_c i d\ell \times \mathbf{B}$
- Force on conductor 2 is toward conductor 1
- We can also see that the force on conductor 1 is toward conductor 2



Torque on a Current Loop

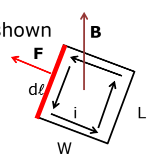
- Consider a loop with width W and length L and current i flowing through it
- Assume a uniform magnetic field **B** is present and perpendicular to the loop

$\mathbf{F} = \int_c i d\ell \times \mathbf{B}$



Torque on a Current Loop

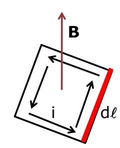
- Consider the side in red
- The direction of the force on this side is computed from
  $\mathbf{F} = \int_c i d\ell \times \mathbf{B}$
  and therefore is in the direction shown



Torque on a Current Loop

Find the direction force on the side colored in red

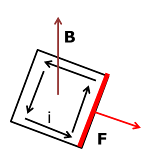
$\mathbf{F} = \int_c i d\ell \times \mathbf{B}$



Torque on a Current Loop

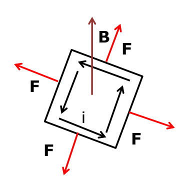
It will be equal in magnitude and opposite in direction as the other side

$\mathbf{F} = \int_c i d\ell \times \mathbf{B}$



Torque on a Current Loop

- We can show that the forces on each side of the conductor net to zero
- No torque developed
- No movement of the conductor

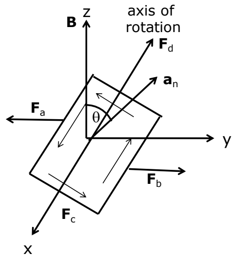


Torque on a Current Loop

- Consider a loop rotated on the x-axis
- From $\mathbf{F} = \int_c i d\ell \times \mathbf{B}$

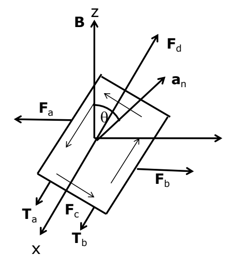
$\mathbf{F}_a = -\mathbf{B} i L \boldsymbol{a}_y$

$\mathbf{F}_b = \mathbf{B} i L \boldsymbol{a}_y$



Torque on a Current Loop

- A torque develops that tends to rotate the loop



Torque on a Current Loop

- The torque **T** is: $\mathbf{T} = \mathbf{r} \times \mathbf{F}$
- Therefore the torque on the a and b sides is:

$\mathbf{T}_a = BiL(W/2)\sin\theta \boldsymbol{a}_x$

$\mathbf{T}_b = BiL(W/2)\sin\theta \boldsymbol{a}_x$

- The total torque on the loop is:

$\mathbf{T} = BiA\sin\theta \boldsymbol{a}_x$ using A= LW

- If there are N coils:

$\mathbf{T} = BiAN\sin\theta \boldsymbol{a}_x$

Summary

- Current gives rise to **H** and **B** fields, which circulate around the conductor in accordance with the Right Hand Rule
- |**H**| decreases as distance from conductor increases
- |**B**| is related to |**H**| by characteristics of the medium
- Change in flux induces a voltage. The change can have mechanical origins
- Current in the presence of a **B** field experiences a force in accordance with Ampere's Force Law