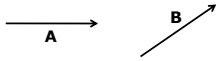
08-Vector Analysis Review

ECEGR 450
Electromechanical Energy Conversion

Vectors and Scalars

- Vectors: have magnitude and direction
  - Force
  - Velocity



- Scalars: no direction
  - Temperature
  - Energy
  - Time

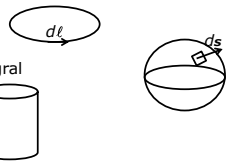
Integrals

Integrals

$\oint_c$: line integral

$\oint_s$: closed surface integral

$\int_v$: volume integral



Dot and Cross Product

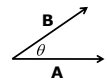
- Dot Product

  $C = \mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta$

  - Scalar
- Cross Product

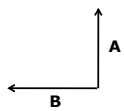
  $\mathbf{C} = \mathbf{A}\times\mathbf{B} = |AB\sin\theta|\,\mathbf{N}$

  - **N** is a unit vector normal to the plane formed by **A** and **B**
  - Use right hand rule to determine direction
  - Vector

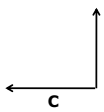


Right Hand Rule

C = **A** x **B**



C is coming out of the slide ●

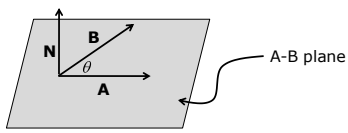


B is going into the slide x

Right Hand Rule

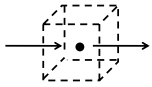
By Right Hand Rule

$\mathbf{C} = \mathbf{A}\times\mathbf{B} \neq \mathbf{B}\times\mathbf{A}$



Divergence

- Divergence
  $c = \nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
  - Measures the outflow of a field from a volume
  - Scalar
  - Positive if net flow is outward
  - Divergence of an incompressible fluid is 0



Curl

Curl

- Vector
- Circulation per unit area of a vector field
- Direction is normal to the surface
- If Curl = 0, then the field is irrotational

$$\mathbf{C} = \nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{N}_x & \mathbf{N}_y & \mathbf{N}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

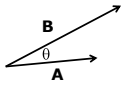
determinant

Example

- Compute the dot product and cross product of **A** and **B**

$\mathbf{A} = 3\angle 5^\circ$

$\boldsymbol{B} = 5\angle 35^\circ$



Example

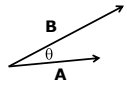
- Compute the dot product and cross product of **A** and **B**

$\mathbf{A} = 3\angle 5^\circ$

$\boldsymbol{B} = 5\angle 35^\circ$

$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta = 15\cos 30^\circ = 12.99$

$\mathbf{A} \times \mathbf{B} = |AB\sin\theta|\,\mathbf{N} = 7.5\mathbf{N}$



- **N** is pointing out of the slide

Example

In Matlab:

```
>> Avector=[3*cosd(5); 3*sind(5); 0];
>> Bvector=[5*cosd(35); 5*sind(35); 0];
>> dot(Avector,Bvector)
ans =
   12.9904
>> cross(Avector,Bvector)
ans =
         0
         0
    7.5000
```