08-Transposition and Bundling

Text: 3.4 – 3.5

ECEGR 451
Power Systems

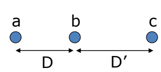
Overview

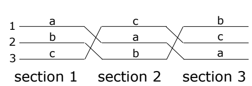
- Transposition
- Bundled Conductors
- Stranded Conductors
- Use of Tables

Transposition

- Symmetrical spacing is not always possible or ideal
- Equal section lengths
- Want the to find the average

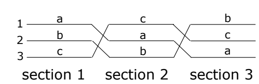
$$\bar{\lambda}_a = \frac{1}{3}\left(\lambda_a^1 + \lambda_a^2 + \lambda_a^3\right)$$





Transposition

$$\bar{\lambda}_a = \frac{1}{3}\left(\lambda_a^1 + \lambda_a^2 + \lambda_a^3\right)$$



using

$$\lambda_k = 2\left\{i_1 \ln\frac{1}{d_{k1}} + i_2 \ln\frac{1}{d_{k2}} + \cdots + i_k \ln\frac{1}{r'_k} + \cdots + i_n \ln\frac{1}{d_{kn}}\right\} \times 10^{-7}$$

$$\bar{\lambda}_a = \frac{1}{3}\left(\frac{\mu_0}{2\pi}\right)\Bigg(\underbrace{i_a \ln\frac{1}{r'} + i_b \ln\frac{1}{d_{12}} + i_c \ln\frac{1}{d_{13}}}_{\text{section 1}} + \underbrace{i_a \ln\frac{1}{r'} + i_b \ln\frac{1}{d_{23}} + i_c \ln\frac{1}{d_{12}}}_{\text{section 2}} + \underbrace{i_a \ln\frac{1}{r'} + i_b \ln\frac{1}{d_{13}} + i_c \ln\frac{1}{d_{23}}}_{\text{section 3}}\Bigg)$$

Transposition

$$\bar{\lambda}_a = \frac{1}{3}\left(\frac{\mu_0}{2\pi}\right)\left(i_a \ln\frac{1}{r'} + i_b \ln\frac{1}{d_{12}} + i_c \ln\frac{1}{d_{13}} + i_a \ln\frac{1}{r'} + i_b \ln\frac{1}{d_{23}} + i_c \ln\frac{1}{d_{12}} + i_a \ln\frac{1}{r'} + i_b \ln\frac{1}{d_{13}} + i_c \ln\frac{1}{d_{23}}\right)$$

- Defining (Geometric Mean Distance)

$$D_m \triangleq \left(d_{12}d_{23}d_{13}\right)^{1/3}$$

- via substitution

$$\bar{\lambda}_a = 2 \times 10^{-7}\left(i_a \ln\frac{1}{r'} + i_b \ln\frac{1}{D_m} + i_c \ln\frac{1}{D_m}\right)$$

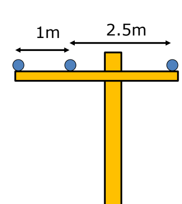
- for each phase

$$\bar{L}_a = 2\left\{\ln\frac{D_m}{r'}\right\} \times 10^{-7}$$

$\bar{L}_a = \bar{L}_b = \bar{L}_c$ balanced, consistent with use of per-phase analysis

Example

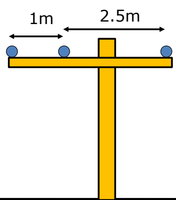
Compute D_m for the shown transmission line configuration.



Example

Compute D_m for the shown transmission line configuration.

$$D_m = \left(d_{12}d_{23}d_{13}\right)^{1/3} = (1\times 2.5\times 3.5)^{1/3} = 2.06\text{m}$$

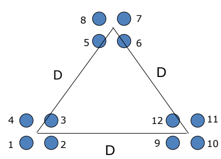


Bundled Lines

- Recall that increasing conductor radius decreases inductance
- Impractical to use very large conductors
  - cost
  - mechanical considerations
- Use bundling instead

Bundled Lines

- Conductors in a bundle form parallel paths (equal current distribution)
- A-phase
  - conductors 1, 2, 3, 4
- B-phase
  - conductors 5, 6, 7, 8
- C-phase
  - conductors 9, 10, 11, 12
- First consider conductor 1



four bundles per phase shown

Bundled Lines

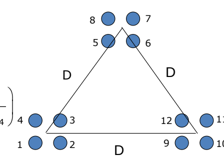
Recall: equilateral, unbundled case

$$\lambda_a = 2\left\{i_a \ln\frac{1}{r'_1} + i_b \ln\frac{1}{D} + i_c \ln\frac{1}{D}\right\} \text{ x } 10^{-7}$$

For conductor 1

$$\lambda_1 = 2 \text{ x } 10^{-7}\left[\frac{i_a}{4}\left(\ln\frac{1}{r'} + \ln\frac{1}{d_{12}} + \ln\frac{1}{d_{13}} + \ln\frac{1}{d_{14}}\right) + \frac{i_b}{4}\left(\ln\frac{1}{d_{15}} + \ln\frac{1}{d_{16}} + \ln\frac{1}{d_{17}} + \ln\frac{1}{d_{18}}\right) + \frac{i_c}{4}\left(\ln\frac{1}{d_{19}} + \ln\frac{1}{d_{1,10}} + \ln\frac{1}{d_{1,11}} + \ln\frac{1}{d_{1,12}}\right)\right]$$

$$= 2 \text{ x } 10^{-7}\left(i_a \ln\frac{1}{\left(r'd_{12}d_{13}d_{14}\right)^{1/4}} + i_b \ln\frac{1}{\left(d_{15}d_{16}d_{17}d_{18}\right)^{1/4}} + i_c \ln\frac{1}{\left(d_{19}d_{1,10}d_{1,11}d_{1,12}\right)^{1/4}}\right)$$



Bundled Conductors

$$\lambda_1 = 2 \text{ x } 10^{-7}\left(i_a \ln\frac{1}{\left(r'd_{12}d_{13}d_{14}\right)^{1/4}} + i_b \ln\frac{1}{\left(d_{15}d_{16}d_{17}d_{18}\right)^{1/4}} + i_c \ln\frac{1}{\left(d_{19}d_{1,10}d_{1,11}d_{1,12}\right)^{1/4}}\right)$$

- Define
  - Geometric Mean Radius (GMR) of the bundle

$$R_b \triangleq \left(r'd_{12}d_{13}d_{14}\right)^{1/4}$$

  - Geometric Mean Distance (GMD) between bundles

$$D_{1b} \triangleq \left(d_{15}d_{16}d_{17}d_{18}\right)^{1/4} \quad D_{1c} \triangleq \left(d_{19}d_{1,10}d_{1,11}d_{1,12}\right)^{1/4}$$

- via substitution

$$\lambda_1 = 2 \text{ x } 10^{-7}\left(i_a \ln\frac{1}{R_b} + i_b \ln\frac{1}{D_{1b}} + i_c \ln\frac{1}{D_{1c}}\right)$$

Bundled Conductors

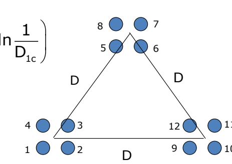
Recap:

$$\lambda_1 = 2 \text{ x } 10^{-7}\left(i_a \ln\frac{1}{R_b} + i_b \ln\frac{1}{D_{1b}} + i_c \ln\frac{1}{D_{1c}}\right)$$

$$R_b \triangleq \left(r'd_{12}d_{13}d_{14}\right)^{1/4}$$

$$D_{1b} \triangleq \left(d_{15}d_{16}d_{17}d_{18}\right)^{1/4}$$

$$D_{1c} \triangleq \left(d_{19}d_{1,10}d_{1,11}d_{1,12}\right)^{1/4}$$



Bundled Conductors

- From previous slide

$$\lambda_1 = 2 \times 10^{-7}\left(i_a \ln\frac{1}{R_b} + i_b \ln\frac{1}{D_{1b}} + i_c \ln\frac{1}{D_{1c}}\right)$$

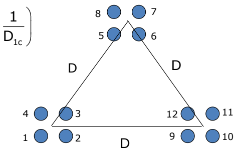
- Assume:

$D_{1b} \approx D_{1c} \approx D$

$i_a + i_b + i_c = 0$

then

$$\lambda_1 = 2 \times 10^{-7}\ i_a \ln\frac{D}{R_b} \quad \text{symmetric}$$



Distance between bundled conductors is << than distance between bundle centers

Bundled Conductors

$$\lambda_1 = 2\left\{i_a \ln\frac{D}{R_b}\right\} \times 10^{-7}$$

$$L_1 = \frac{\lambda_1}{i_a/4} = 8\left\{\ln\frac{D}{R_b}\right\} \times 10^{-7}\ \text{H/m}$$ equal current division

noting

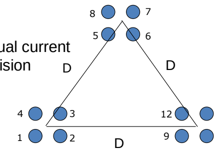
$L_1 \approx L_2 \approx L_3 \approx L_4$

then

$$L_a \approx 2\left\{\ln\frac{D}{R_b}\right\} \times 10^{-7}\ \text{H/m}$$

it follows that

$L_a \approx L_b \approx L_c$ balanced, consistent with use of per-phase analysis



Notes About Bundling

- General GMR for b conductors

$R_b \triangleq \left(r' d_{12}, \ldots, d_{1b}\right)^{1/b}$

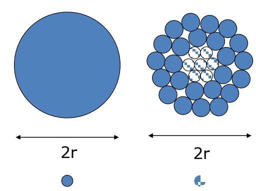
  - If b = 1, then $R_b \triangleq r'$ and the method is generalized
- Bundling increases the effective radius, resulting in
  - lower inductance
  - greater surface area than a single conductor with equal cross section
  - reduced corona

Solid Vs. Stranded Conductors

- Derivations assumed solid conductor
- In practice, most conductors are stranded
- Solid conductors: use r' in the calculations

$r' = re^{\left(-\mu/4\right)}$

- Stranded: look up GMR in the book (A8.1)



alternate representation

Table Use

- Inductive reactance

$$X_L = 2\pi f \quad 2 \times 10^{-7}\left(\ln\frac{D_m}{GMR}\right)\ \Omega/\text{m}$$

- Split into 2 parts

$$X_L = \underbrace{2.022 \times 10^{-3}\ f\left(\ln\frac{1}{GMR}\right)}_{X_a} + \underbrace{2.022 \times 10^{-3}\ f \ln D_m}_{X_d}\ \Omega/\text{mi}$$

  - X_a: 1 ft spacing inductive reactance, independent of distance
  - X_d: inductive reactance spacing factor

Note units of Ω/mi, not Ω/m

Table Use

- Good news, we can use tables to find X_a and X_d
- GMR has been found for must conductors (note the difference between GMR for a conductor, and GMR for a bundle)
  - can use provided GMR instead of r' in the formulas
- Resistance, reactance have been calculated
- Shunts too (more on this later)

Table Use

- Single circuit, three-phase line at 60 Hz
- Conductors are ACSR *Drake*
- Find inductive reactance per mile per phase

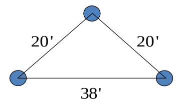
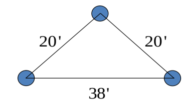


Table Use

First, the hard way

1) find GMR $GMR = 0.0375$ ft (from Table A8.1)
2) find GMD $D_m = (20 \times 20 \times 38)^{1/3} = 24.8$ ft
3) calculate inductance
4) calculate reactance

$$L_a = 2\left\{\ln\frac{D_m}{r'}\right\} \times 10^{-7}$$



$$X_L = (2\pi f)(2 \times 10^{-7})(1609)\left(\ln\frac{24.8}{0.0375}\right) = 0.788\ \Omega/\text{mi per phase}$$

meters to miles

Table Use Steps

1) Look up X_a in Table A8.1
2) Compute GMD
3) Look up X_d in Table A8.2
4) Compute $X_a + X_d$

1. Look up X_a in Table A8.1

Drake: X_a 0.0399 (ft)

2. Compute GMD

$$D_m = (20 \times 20 \times 38)^{1/3} = 24.8 \text{ ft}$$

3. Look up X_d in Table A8.2

X_d at 24 ft X_d at 25 ft

$$X_d = 0.3856 + 0.8(0.3906 - 0.3856) = 0.3896$$

interpolation

4. Compute $X_a + X_d$

- X_L= 0.399 + 0.3896 = 0.788 Ω/mi per phase
- How does this compare to X_L computed without using the table?

Line Inductance Summary

- for transposed lines

$$L_a = 2\left\{\ln\frac{D_m}{r'}\right\} \text{ x } 10^{-7} \text{ H/m}$$

- for bundled conductors

$$L_a = 2\left\{\ln\frac{D}{R_b}\right\} \text{ x } 10^{-7} \text{ H/m}$$

- GMR, X_a, X_d values are found in tables and are helpful in quickly solving problems