

# 08-Solar Resource Part 3

ECEGR 452

Renewable Energy Systems



# Overview

- Effect of the Atmosphere
- Clearness Index
- Irradiation
- Irradiance Algorithm
- Air Mass Ratio



# What Influences Angle of Incidence?

- Declination ( $\delta$ )
- Latitude ( $\phi$ )
- Tilt ( $\beta$ )
- Time of day (hour angle) ( $\omega$ )



# What Influences Angle of Incidence?

- See Lecture 07-Solar Resource Part 2 for derivation



# What Influences Angle of Incidence?

Extraterrestrial irradiance accounting for the tilt, latitude and declination of a surface at solar noon:

$$\begin{aligned}G_{0T} &= G_{0n} \cos(\theta) = G_{0n} \cos(\phi - \delta - \beta) \\ &= G_{0n} [\cos(\phi) \cos(\delta) \cos(\beta) \\ &\quad - \cos(\phi) \sin(\delta) \sin(\beta) \\ &\quad + \sin(\phi) \sin(\beta) \cos(\delta) \\ &\quad + \sin(\phi) \cos(\beta) \sin(\delta)]\end{aligned}$$

**Important result**



# What Influences Angle of Incidence?

- Now also accounting for time of day
  - $\cos(\theta) = \sin(\delta)\sin(\phi)\cos(\beta)$   
-  $\sin(\delta)\cos(\phi)\sin(\beta)$   
+  $\cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega)$   
+  $\cos(\delta)\sin(\phi)\sin(\beta)\cos(\omega)$

**Important result**



# Simplifications

- If  $\beta = 0$  (no tilt), then  $\theta_z = \theta$  and
  - $\cos(\theta) = \sin(\delta)\sin(\phi) + \cos(\delta)\cos(\phi)\cos(\omega)$
- For surfaces tilted at their latitude
  - $\cos(\theta) = \cos(\delta)\cos(\omega)$
- For surfaces at solar noon
  - $\cos(\theta) = \cos(\phi - \delta - \beta)$



# What Influences Angle of Incidence?

- Try to maximize  $\cos(\theta)$

$$\begin{aligned}G_{0T} &= G_{0n} \cos(\theta) = G_{0n} \cos(\phi - \delta - \beta) \\ &= G_{0n} [\cos(\phi) \cos(\delta) \cos(\beta) \\ &\quad - \cos(\phi) \sin(\delta) \sin(\beta) \\ &\quad + \sin(\phi) \sin(\beta) \cos(\delta) \\ &\quad + \sin(\phi) \cos(\beta) \sin(\delta)]\end{aligned}$$

Adjust tilt to minimize  $\phi - \delta - \beta$





# What Influences Angle of Incidence?

- Try to maximize  $\cos(\theta)$

$$\cos(\theta) = \sin(\delta)\sin(\phi)\cos(\beta)$$

$$-\sin(\delta)\cos(\phi)\sin(\beta)$$

$$+\cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega)$$

$$+\cos(\delta)\sin(\phi)\sin(\beta)\cos(\omega)$$

Track sun east-west to minimize  $\omega$



# Surface Orientation

- General rule of thumb: tilt a PV panel at the latitude
  - Normal to irradiance on equinoxes
  - Too much tilt in summer
  - Too little tilt in winter



# Where in the world are these PV panels?



Ellensburg



Singapore



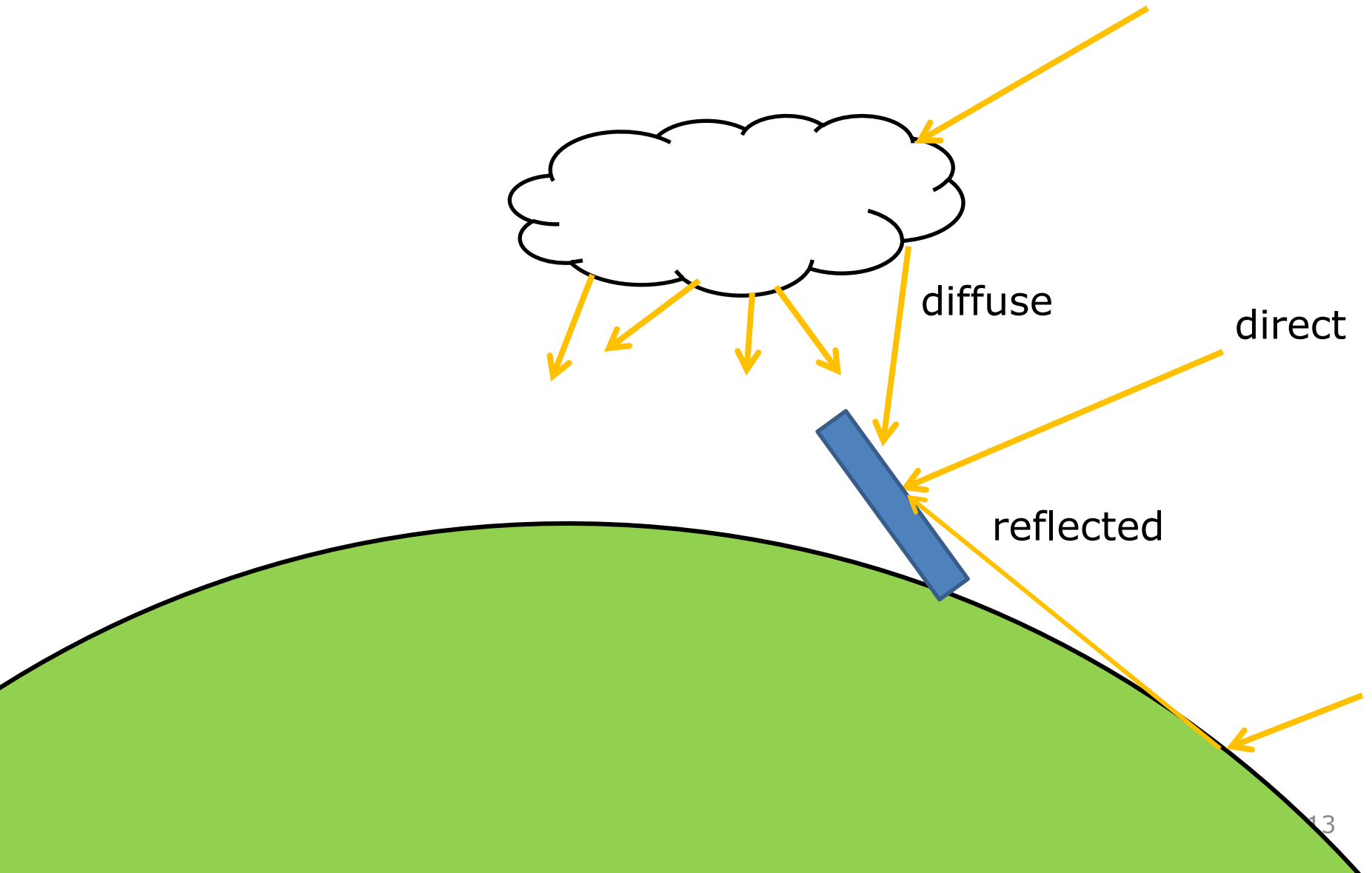
Snohomish



# Introduction

- Previous lectures have focused on extraterrestrial irradiance
- Now we examine irradiance on Earth's surface
  - Resolve GHI into beam and diffuse components
- Irradiance on a surface has 3 components:
  - A: Direct
  - B: Diffuse (from the sky)
  - C: Reflected (reflected from the ground)
  - $G_T = A + B + C$
- Empirical formulae

# Components of Irradiance





# Atmospheric Effects

- Atmosphere: **reflects**, **absorbs** and **scatters** solar radiation
- Net result: reduction in irradiance at the Earth's surface and a non-uniform attenuation of the energy density spectrum
- On **average**, 30% of incident solar irradiance is reflected back into space



# Atmospheric Effects

- Recall that the atmosphere also causes diffusion of irradiance
  - We have neglected  $G_d$  in all previous derivations
- $G_d$  is difficult to compute, due to the complex geometry of clouds, etc
- Use empirical formula to determine the ratio of  $G_d$  and  $G_b$  for a given GHI and  $G_0$



# Clearness Index

- Basic idea: use ratio of irradiance on surface to irradiance at the top of the atmosphere as a proxy for how cloudy it is.
  - Clouds imply higher  $G_d$  component of  $G_{GHI}$
- clear day:  $G_{GHI}/G_0$  is closer to 1
  - Smaller  $G_d$  component of  $G_{GHI}$
- cloudy day:  $G_{GHI}/G_0$  is closer to 0
  - Larger  $G_d$  component of  $G_{GHI}$





# Clearness Index

- Clearness Index  $k_t$  : ratio of global irradiance received on a horizontal surface to the extraterrestrial irradiance ( $G_{GHI}/G_0$ )
  - Recall:  $G_0$  is the irradiance on a surface (with the same angle of incidence as the horizontal surface) without the atmosphere accounted for (extraterrestrial)
  - Average  $k_t$  values are available for many locations
- Usually, clearness index uses the ratio of radiations over an hour, day or month



# Computing Diffuse Irradiance

- $G_o$  can be computed (see previous lectures)
- Need to know either  $G_{GHI}$  or  $k_t$
- A simple empirical formula is
  - $G_d/G_{GHI} = 1 - 1.13k_t$
  - Reasonably valid for  $0.3 < k_t < 0.8$
- $G_b$  can then be computed from
  - $G_b = G_{GHI} - G_d$



# Computing Beam Irradiance

- Find  $G_b$  and  $G_d$  if  $G_{\text{GHI}} = 800 \text{ W/m}^2$  and  $G_o = 1350 \text{ W/m}^2$



# Computing Beam Irradiance

- Find  $G_b$ ,  $G_d$  if  $G_{\text{GHI}} = 800 \text{ W/m}^2$  and  $G_o = 1350 \text{ W/m}^2$ 
  - $k_t = 800/1350 = 0.593$
  - $G_d/G_{\text{GHI}} = 1 - 1.13k_t$ 
    - $G_d = (1 - 1.13k_t)G_{\text{GHI}} = (0.3304)G_{\text{GHI}} = 264 \text{ W/m}^2$
  - $G_b = 800 - 264 = 536 \text{ W/m}^2$



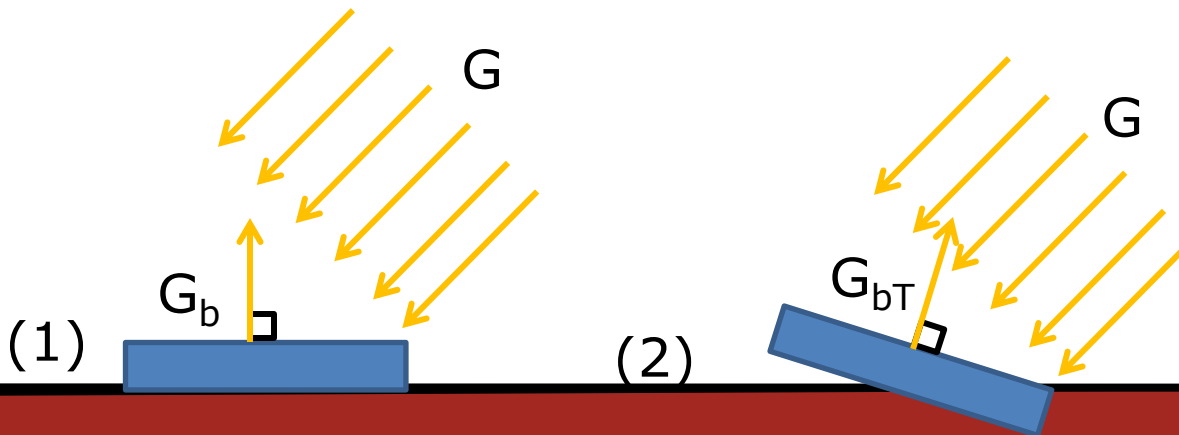
# Computing Beam Irradiance

- Given  $G_b$  how do we find beam irradiance for a tilted surface,  $G_{bT}$ ?
  - remember,  $G_b$  is for a horizontal surface

- $G_b = G \cos \theta_z$

we don't know  $G$

Recall for horizontal surfaces,  $\theta = \theta_z$

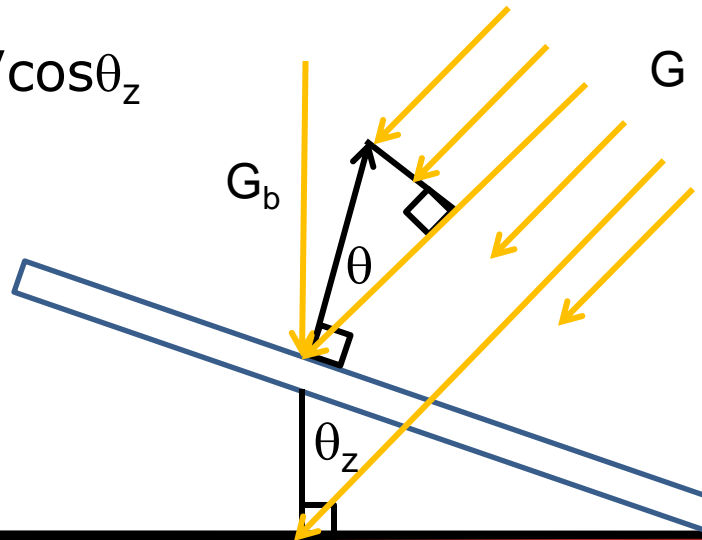




# Computing Beam Irradiance

- $G_{bT} = G \cos \theta$
- $G_b = G \cos \theta_z$ 
  - $G = G_b (1 / \cos \theta_z)$
- Then:
  - $G_{bT} = G_b R_b$
  - Where
    - $R_b = \cos \theta / \cos \theta_z$

Here we know  $G_b$  and can compute  $\theta$  and  $\theta_z$ , but we don't know  $G$ , so we need to eliminate it





# Computing Beam Irradiance

- We now have solved for A
- $G_T = A + B + C$ 
  - $A = G_b R_b$



# Computing Beam Irradiance

- Compute the beam irradiance on a surface tilted at  $15^\circ$  at  $30^\circ$  N on April 15<sup>th</sup>, with a clearness index of 0.75 at solar noon.





## Computing Beam Irradiance

- Compute the beam irradiance on a surface tilted at  $15^\circ$  at  $30^\circ$  N on April 15<sup>th</sup>, with a clearness index of 0.75 at solar noon.
  - $G_d/G_{GHI} = 1 - 1.13k_t$
- We have  $k_t$  and can compute  $G_0$  to find  $G_{GHI}$  using  $k_t = G_{GHI}/G_0$



# Computing Beam Irradiance

- Compute the beam irradiance on a surface tilted at  $15^\circ$  at  $30^\circ$  N on April 15<sup>th</sup>, with a clearness index of 0.75 at solar noon.

$$\phi = 30^\circ$$

$$\beta = 15^\circ$$

$$\omega = 0^\circ$$

$$\delta = \delta_0 \sin\left(\frac{360^\circ (284 + d)}{365}\right) = 23.5^\circ \sin\left(\frac{360^\circ (284 + 105)}{365}\right) = 9.4^\circ$$

$$G_{on}(d) = G_{sc} \left[ 1 + 0.033 \cos\left(2\pi \left(\frac{105}{365}\right)\right) \right] = 1356 \text{ W/m}^2$$



# Computing Beam Irradiance

- Now that we have the angles, we can compute

$$G_o = G_{on} \cos \theta_z$$

- Solving for the cosine of the zenith angle ( $\beta = 0$  for zenith angle)

$$\begin{aligned}\cos(\theta_z) &= \sin(\delta)\sin(\phi) + \cos(\delta)\cos(\phi)\cos(\omega) \\ &= \sin(9.4^\circ)\sin(30^\circ) + \cos(9.4^\circ)\cos(30^\circ)\cos(0^\circ) \\ &= 0.936 \text{ [or using } \cos(\theta_z) = \cos(\phi - \delta)\text{]}\end{aligned}$$

- Therefore, for a horizontal surface at the top of the atmosphere:  $G_o = G_{on} \cos \theta_z = 1356 \times 0.936 = 1270 \text{ W/m}^2$



# Computing Beam Irradiance

- Now, solving for GHI:
  - $k_t = G_{GHI}/G_0$
  - $G_{GHI} = 952.5 \text{ W/m}^2$
- Computing the Diffuse Irradiance:
  - $G_d/G_{GHI} = 1 - 1.13k_t$
  - $G_d = 145.25 \text{ W/m}^2$
- Solving for  $G_b$ 
  - $G_b = G_{GHI} - G_d = 807.25 \text{ W/m}^2$



## Computing Beam Irradiance

- The  $G_b$  we computed is for a Horizontal surface, we need to find  $G_{bT}$ , the beam irradiance on a tilted surface
- We can relate  $G_b$  and  $G_{bT}$  by:  $G_{bT} = G_b R_b$
- Where
  - $R_b = \cos\theta / \cos\theta_z$
- We already know  $\cos\theta_z$ , so we need to calculate  $\cos\theta$



# Computing Beam Irradiance

- Solving for the cosine of the angle of incidence:

$$\cos(\theta) = \sin(\delta) \sin(\phi) \cos(\beta)$$

$$- \sin(\delta) \cos(\phi) \sin(\beta)$$

$$+ \cos(\delta) \cos(\phi) \cos(\beta) \cos(\omega)$$

$$+ \cos(\delta) \sin(\phi) \sin(\beta) \cos(\omega)$$

$$\cos(\theta) = \sin(9.4^\circ) \sin(30^\circ) \cos(15^\circ)$$

$$- \sin(9.4^\circ) \cos(30^\circ) \sin(15^\circ)$$

$$+ \cos(9.4^\circ) \cos(30^\circ) \cos(15^\circ) \cos(0^\circ)$$

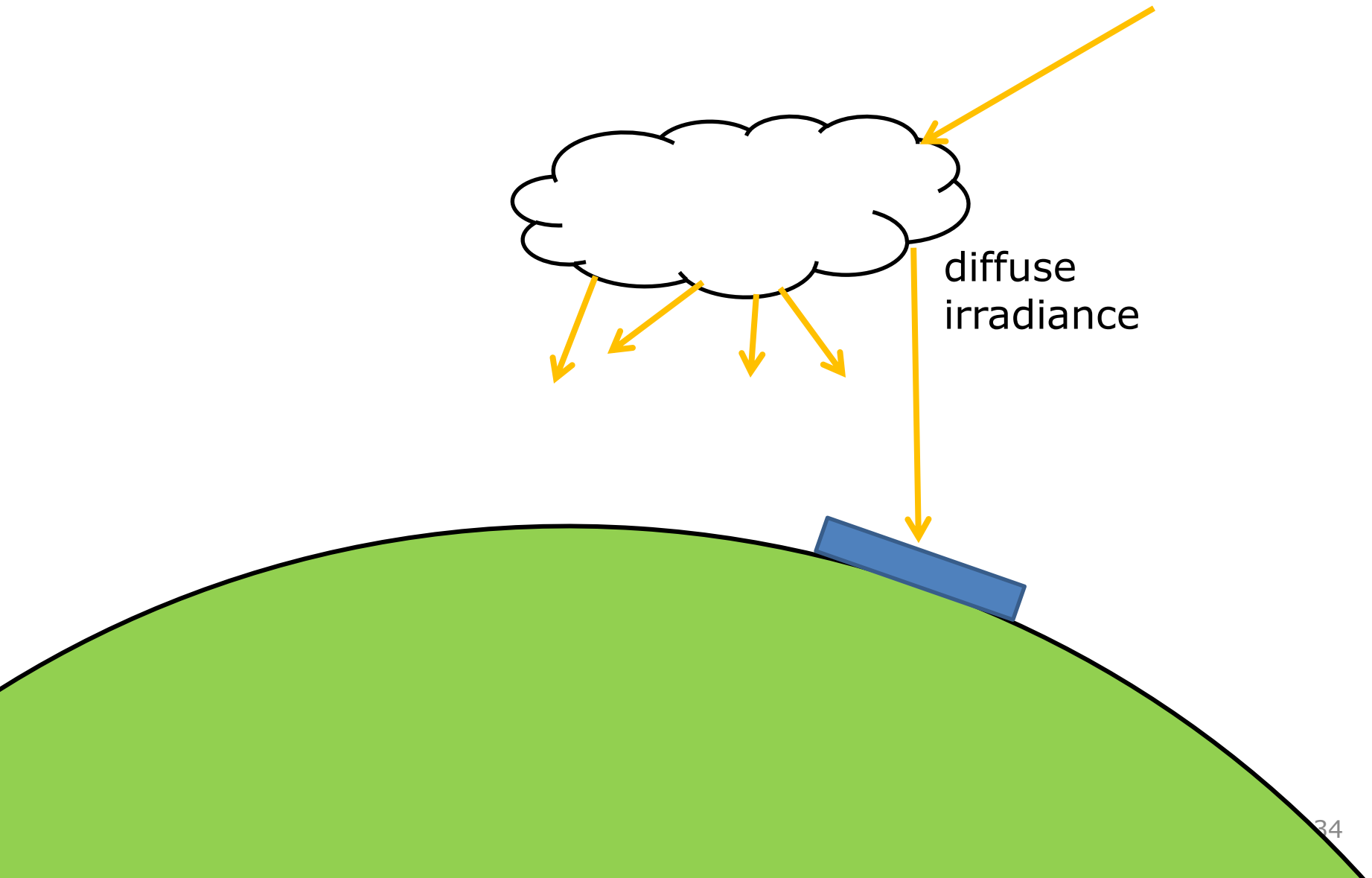
$$+ \cos(9.4^\circ) \sin(30^\circ) \sin(15^\circ) \cos(0^\circ) = 0.995 \quad [\text{or using } \cos(\theta) = \cos(\phi - \delta - \beta)]$$



## Computing Beam Irradiance

- $R_b = \cos\theta / \cos\theta_z = 1.06$
- So that the beam irradiance on the tilted surface is:  $G_{bT} = G_b R_b = 858.1 \text{ W/m}^2$

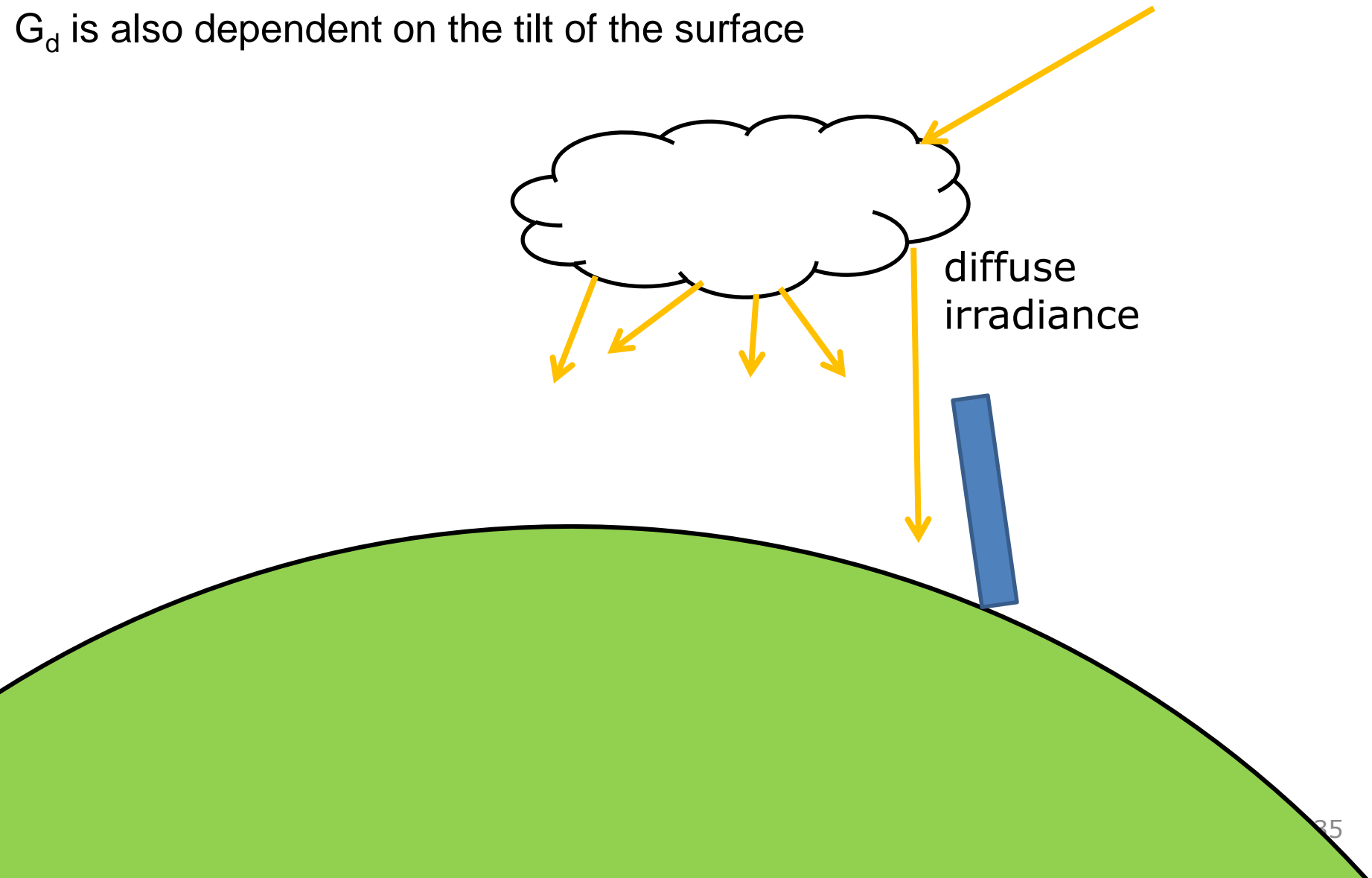
# Atmospheric Effects





# Atmospheric Effects

$G_d$  is also dependent on the tilt of the surface





# Computing Diffuse Irradiance

- We already know  $G_d$
- Now we need to be able to account for the tilting of a surface
  - When  $\beta = 0^\circ$ , all of  $G_d$  is received
  - When  $\beta = 90^\circ$ , half of  $G_d$  is received
  - When  $\beta = 180^\circ$ , no  $G_d$  is received (surface is facing the ground)
- $G_{dT} = \frac{1}{2}G_d(1 + \cos\beta)$ 
  - This assumes isotropic conditions



# Computing Beam Irradiance

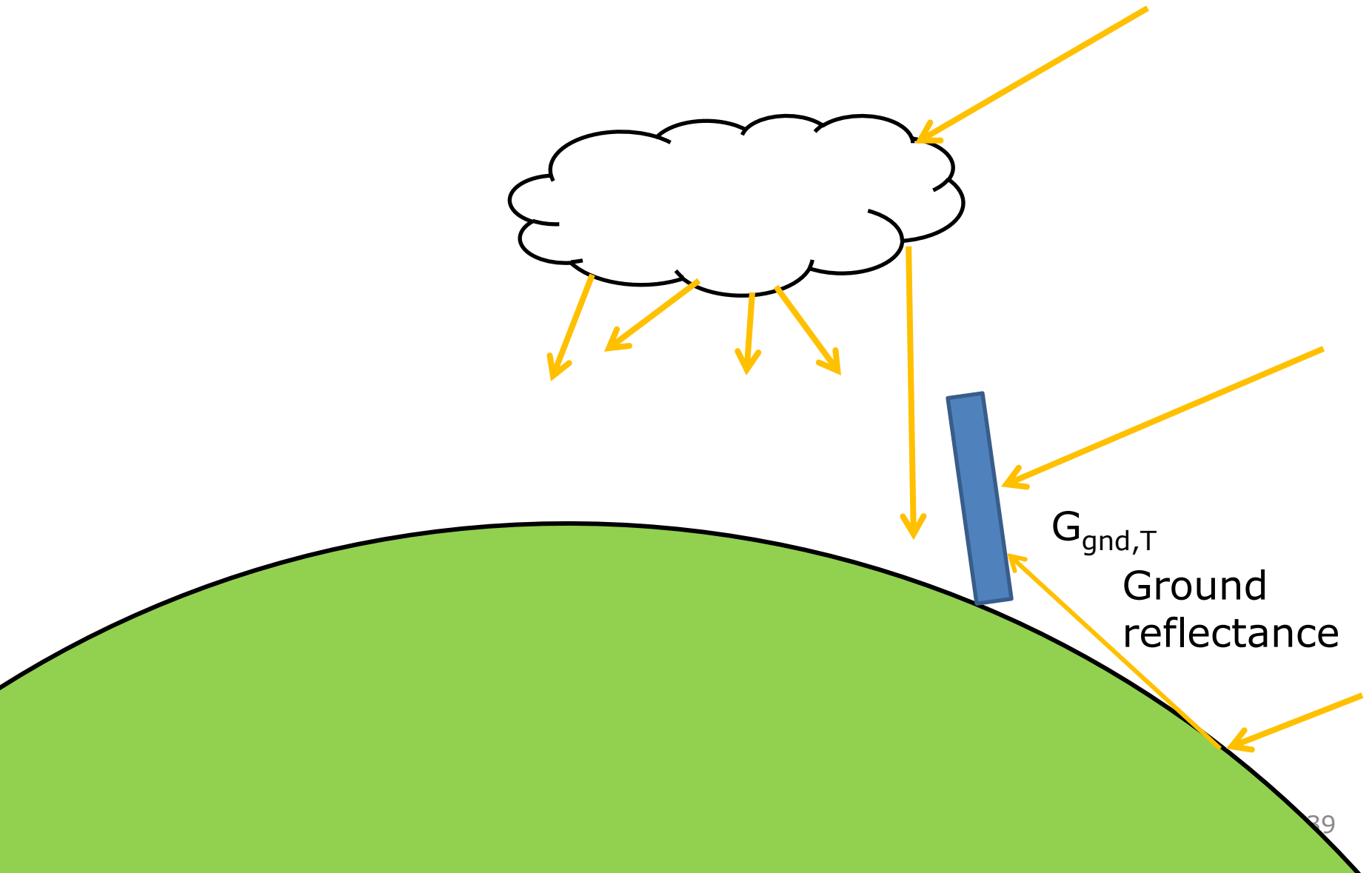
- We now have solved for A and B
- $G_T = A + B + C$ 
  - $A = G_b R_b$
  - $B = \frac{1}{2} G_d (1 + \cos \beta)$



# Ground Reflectance

- Final component is the diffuse irradiance that reflects off the ground
  - Ground albedo ( $\rho$ )
    - Usually 0.2, but can be up to 0.8 for snow, ice
- We assume that this is proportional to the  $G_{\text{GHI}}$ 
  - $G_{\text{gnd}} = \rho G_{\text{GHI}}$
- Amount received depends on the tilt of the surface

# Ground Reflectance





# Ground Reflectance

- Accounting for the tilting of a surface
  - When  $\beta = 0^\circ$ , no of  $G_{\text{gnd}}$  is received
  - When  $\beta = 90^\circ$ , half of  $G_{\text{gnd}}$  is received
  - When  $\beta = 180^\circ$ , all  $G_{\text{gnd}}$  is received (surface is facing the ground)
- $G_{\text{gnd},T} = \frac{1}{2} \rho G_{\text{GHI}}(1 - \cos\beta)$



# Computing Beam Irradiance

- We now have solved for A, B and C
- $G_T = A + B + C$ 
  - $G_{bT} = G_b R_b$
  - $G_{dT} = \frac{1}{2} G_d (1 + \cos\beta)$
  - $G_{gnd,T} = \frac{1}{2} \rho G_{GHI} (1 - \cos\beta)$



# Calculating Irradiance

- Consider a surface that is tilted at  $30^\circ$ . The measured GHI is  $250 \text{ W/m}^2$ , and  $G_0 = 404 \text{ W/m}^2$ . Compute  $G_b$ ,  $G_d$  and  $G_T$ . Let  $\cos(\theta) = 0.693$  and  $\cos(\theta_z) = 0.286$ . Assume  $\rho = 0.2$ .





## Calculating Irradiance

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  - $k_t = 250/404 = 0.622$



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  - $k_t = 250/404 = 0.622$
  - $G_d/G_{\text{GHI}} = 1 - 1.13k_t$ 
    - $G_d = (1 - 1.13k_t)G_{\text{GHI}} = (0.297)G_{\text{GHI}} = 74.3 \text{ W/m}^2$



## Calculating Irradiance

- Consider a surface that is tilted at  $30^\circ$ . The measured GHI is  $250 \text{ W/m}^2$ , and  $G_0 = 404 \text{ W/m}^2$ . Compute  $G_b$ ,  $G_d$  and  $G_T$ . Let  $\cos(\theta) = 0.693$  and  $\cos(\theta_z) = 0.286$ . Assume  $\rho = 0.2$ .
  - $k_t = 250/404 = 0.622$
  - $G_d = 74.3 \text{ W/m}^2$
  - $G_b = 250 - 74.3 = 175.7 \text{ W/m}^2$



## Calculating Irradiance

- Consider a surface that is tilted at  $30^\circ$ . The measured GHI is  $250 \text{ W/m}^2$ , and  $G_0 = 404 \text{ W/m}^2$ . Compute  $G_b$ ,  $G_d$  and  $G_T$ . Let  $\cos(\theta) = 0.693$  and  $\cos(\theta_z) = 0.286$ . Assume  $\rho = 0.2$ .
  - $k_t = 250/404 = 0.622$
  - $G_d = 74.3 \text{ W/m}^2$
  - $G_b = 175.7 \text{ W/m}^2$
  - $R_b = \cos\theta / \cos\theta_z = 2.42$



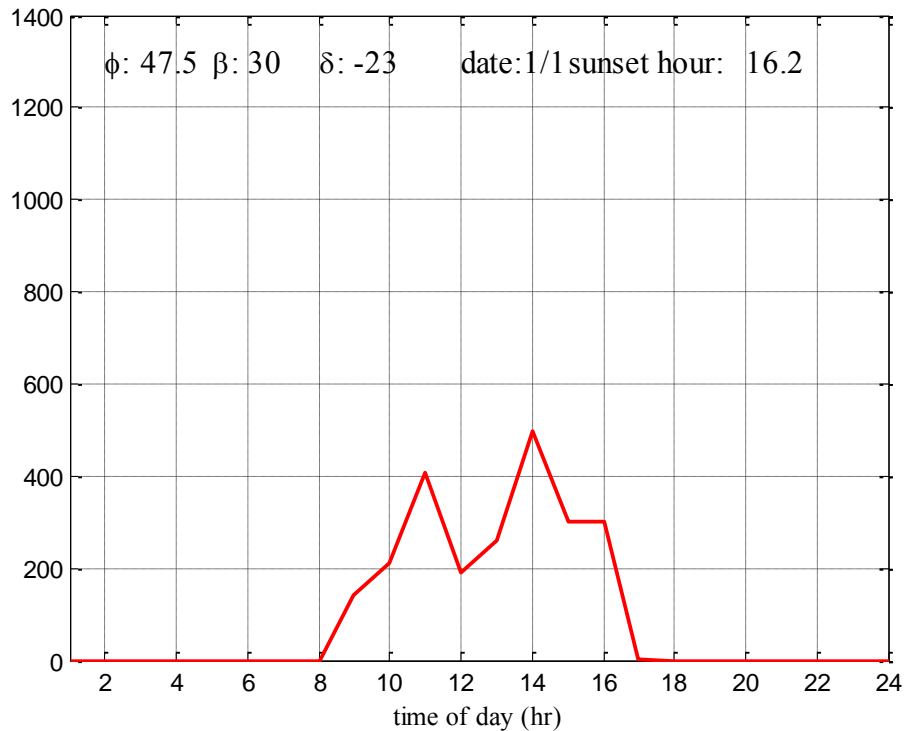
## Calculating Irradiance

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  - $k_t = 250/404 = 0.622$
  - $G_d = 74.3 \text{ W/m}^2$
  - $G_b = 175.7 \text{ W/m}^2$
  - $R_b = \cos\theta / \cos\theta_z = 2.42$
  - $G_T = G_b R_b + \frac{1}{2} G_d (1 + \cos\beta) + \frac{1}{2} \rho G_{\text{GHI}} (1 - \cos\beta)$
  - $175.7(2.42) + \frac{1}{2}(74.3)(1 + \cos 30) + \frac{1}{2}(0.2)(250)(1 - \cos 30)$
  - $G_T = 425.6 + 69.3 + 3.3 = 498.3 \text{ W/m}^2$

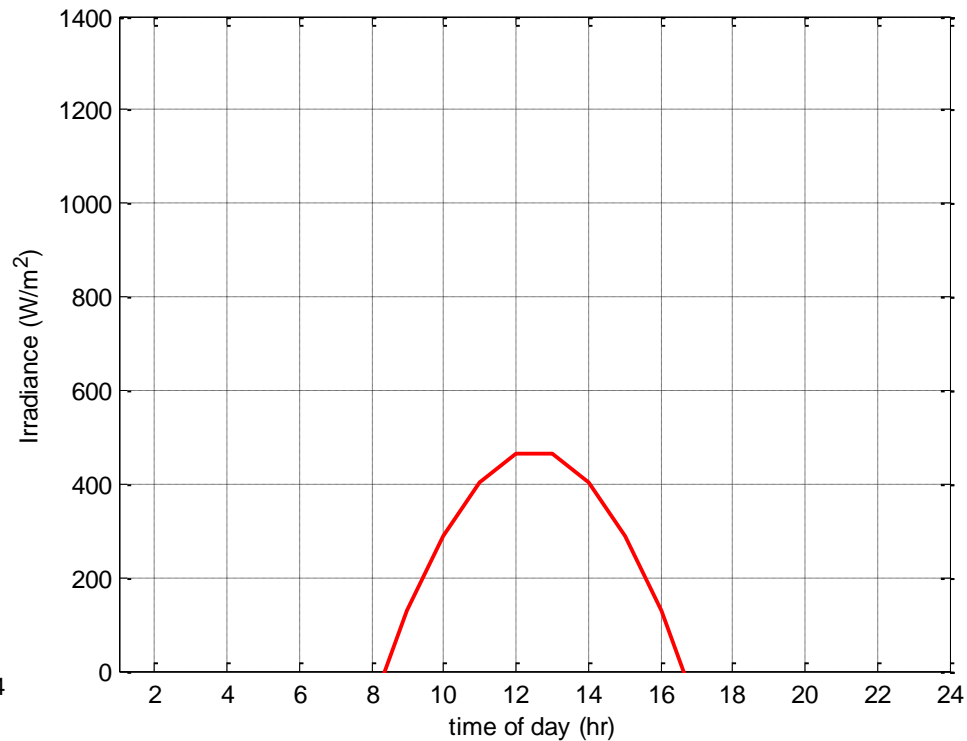


# Calculating Irradiance

Irradiance on the Surface

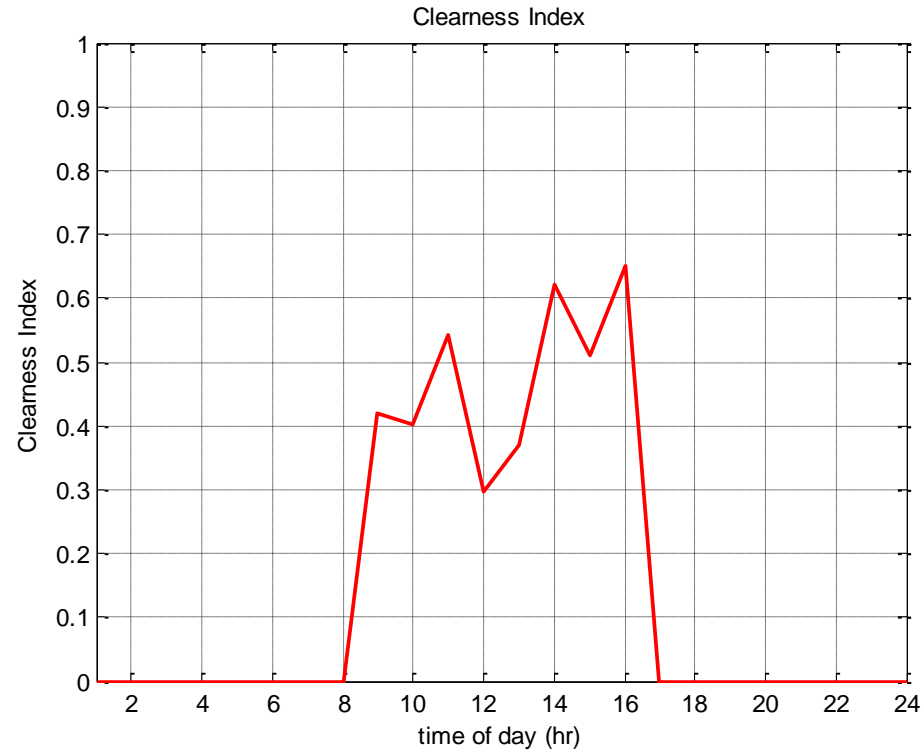
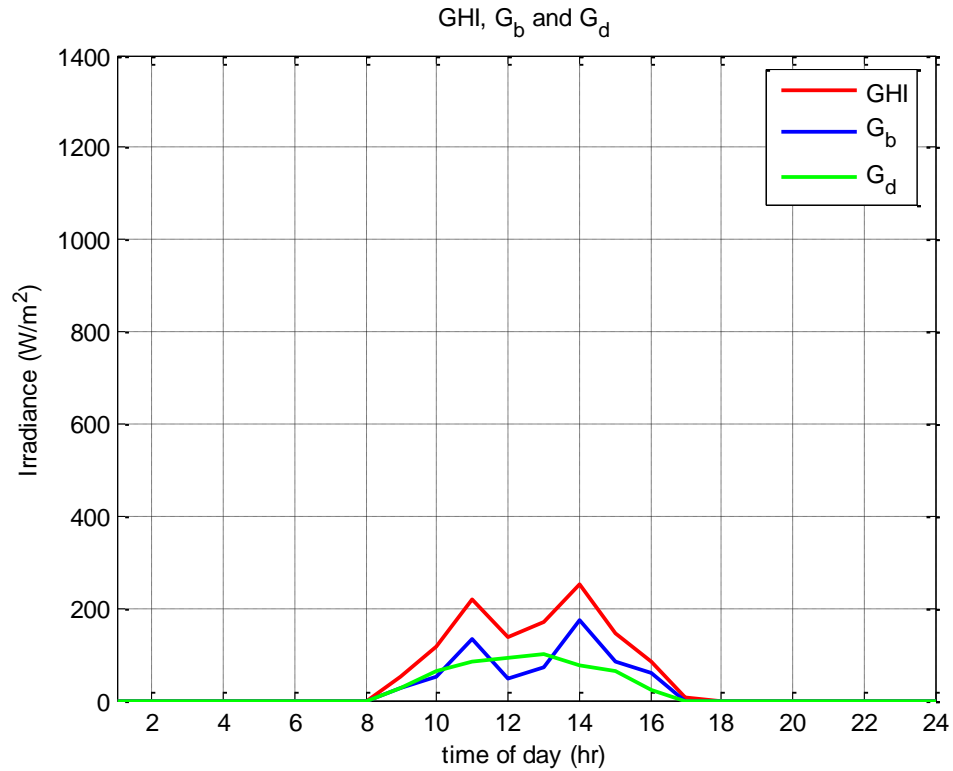


$G_0$





# Calculating Irradiance





# Irradiation

- Irradiation: irradiance received per unit area
- Computed by integrating  $G$  over time
  - $I$ : irradiance received over one hour,  $\text{Wh/m}^2$
  - $H$ : irradiance received over one day,  $\text{Wh/m}^2$
- Multiply irradiation by the surface area to compute radiation





# Irradiation

- Consider a surface tilted at its latitude
  - $\cos(\theta) = \cos(\delta)\cos(\omega)$
- What is the extraterrestrial irradiation from 2 pm to 3 pm?
  - Let  $G_{0n}\cos(\delta) = 1000 \text{ W/m}^2$
- Integrating:

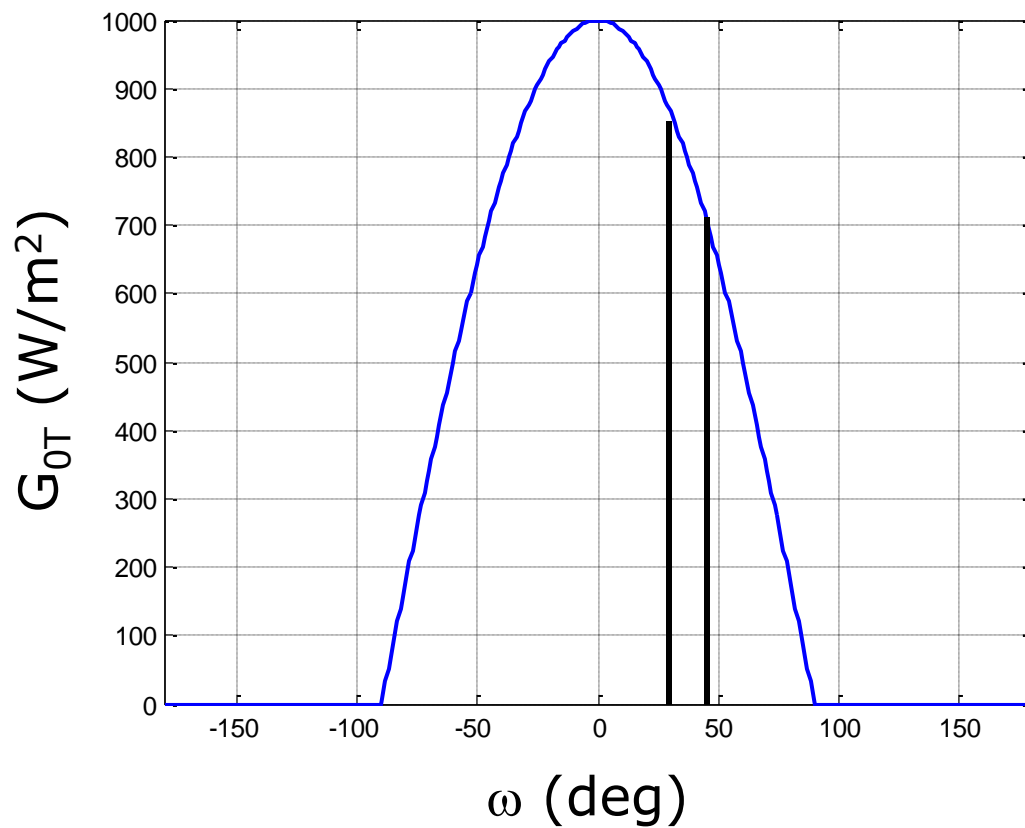
$$I_{0T} = 1000 \int_{30^\circ}^{45^\circ} \cos(\omega) d\omega$$

$$I_{0T} = 1000 \left( \sin\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right) \right) = 1000(0.207) = 207$$

What are the units?



# Irradiation





# Irradiation

- Clearly the  $I_{0T}$  does not equal 207 Wh
- We performed the integration using radians
  - Units:  $(\text{W}/\text{m}^2) \times \text{radians}$
- Expressed in Wh or kWh is more useful
  - 24 hours =  $2\pi$  radians
  - 1 rad =  $24/(2\pi) = 12/\pi$  hours
- Therefore:

$$207 \left( \frac{12}{\pi} \right) = 790.7 \text{ Wh}/\text{m}^2$$



# Irradiation

- GHI values are usually given as time-average values
  - 10 minutes
  - 1 Hour
  - 1 Day
  - 1 Month
- To compute the clearness index, average values of  $G_0$  must be used



# Irradiation

- Let  $\bar{G}_0$  be the hourly averaged value of  $G_0$  (average irradiance on an extraterrestrial horizontal surface  $\beta = 0$ )

$$\bar{G}_0 = G_{0n} \int_{\omega}^{\omega+15^\circ} \cos(\delta) \cos(\phi) \cos(\omega) + \sin(\delta) \sin(\phi) d\omega$$

$$\bar{G}_0 = G_{0n} \int_{\omega}^{\omega+15^\circ} \cos(\delta) \cos(\phi) \cos(\omega) d\omega + G_{0n} \int_{\omega}^{\omega+15^\circ} \sin(\delta) \sin(\phi) d\omega$$

$$\bar{G}_0 = \frac{12}{\pi} G_{0n} \left[ \cos(\delta) \cos(\phi) (\sin(\omega + 15^\circ) - \sin(\omega)) + \frac{\pi 15^\circ}{180^\circ} \sin(\delta) \sin(\phi) \right]$$

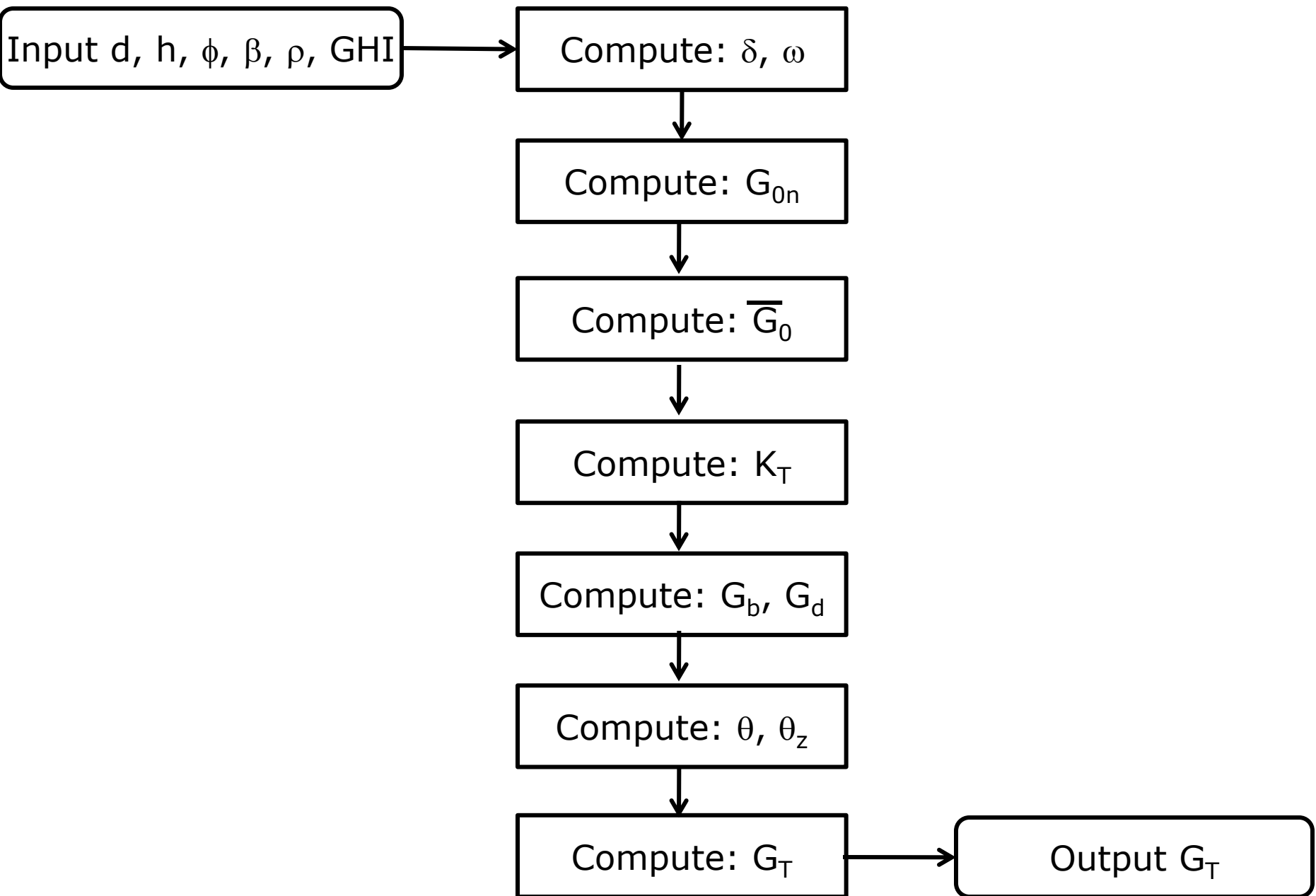
$$K_T = \frac{\bar{G}_{GHI}}{\bar{G}_0}$$

- Clearness index using hourly-averaged values is



# Irradiance Algorithm

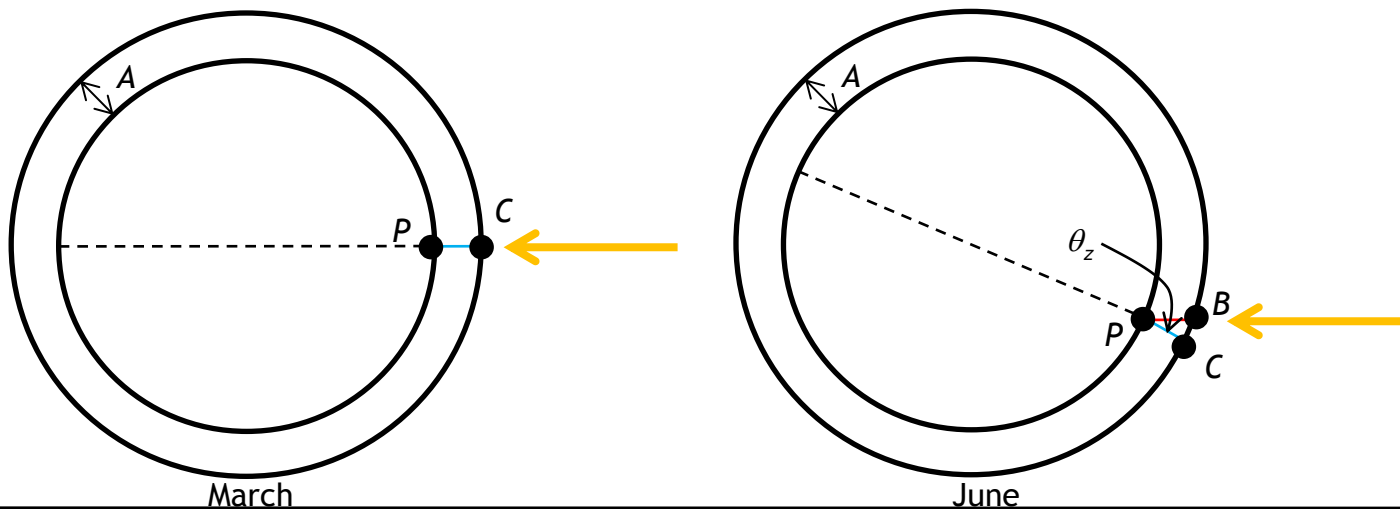
- Computer programs such as Homer and RETScreen compute irradiance for a surface under specified conditions
- Algorithms differ in detail, but in approach they are very similar
- Once irradiance is known, power output of any arrangement of PV panels can be computed





# Air Mass

- Effects of absorption depend on the mass of air the radiation travels through
- The mass of air that solar radiation travels before reaching the surface varies with zenith angle
- This mass is smallest at solar noon with the sun directly overhead







# Atmospheric Absorption

- Air Mass Ratio (AM) =  $PC/PB$
- for  $\theta_z < 70^\circ$ , this approximates to
$$AM \approx \frac{1}{\cos \theta_z}$$
- AM0 is at the top of the atmosphere
- AM is sometimes divided by direct irradiance (AMD XX) and global irradiance (AMG XX) at air mass XX
- AMG 1.5 ( $\theta_z = 48^\circ$ ) is the PV industry standard for the spectrum distribution, with  $1000 \text{ W/m}^2$  irradiance
- How common is  $G = 1000 \text{ W/m}^2$  ?