08-Mesh Analysis

Text: 3.4 – 3.6

ECEGR 210

Electric Circuits I

Overview

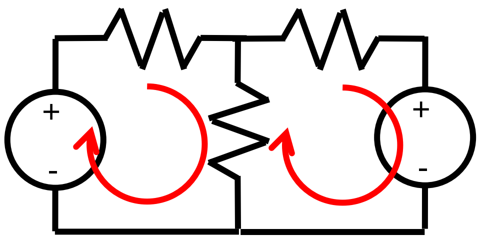
- Introduction
- Mesh Analysis
- Mesh Analysis with Current Sources
- Analysis by Inspection
- Mesh Analysis vs. Nodal Analysis

Introduction

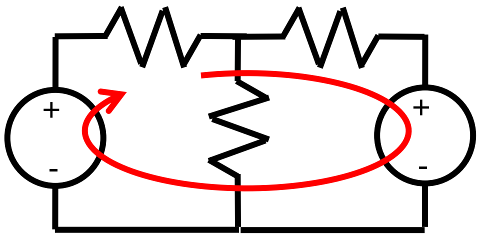
- Last lecture considered Nodal Analysis
  - Use KCL at each node
  - Solve for node voltage
  - Special treatment (supernode) for voltage sources
- Next consider mesh analysis
  - Use KVL at each node
  - Solve for current through meshs
  - Special treatment (supermesh) for current sources

Introduction

- Mesh analysis can only be applied to planar circuits (considered in this class)
  - Non-planar use nodal analysis
- Mesh: a loop that does not contain any other loop



two meshes



a loop, not a mesh

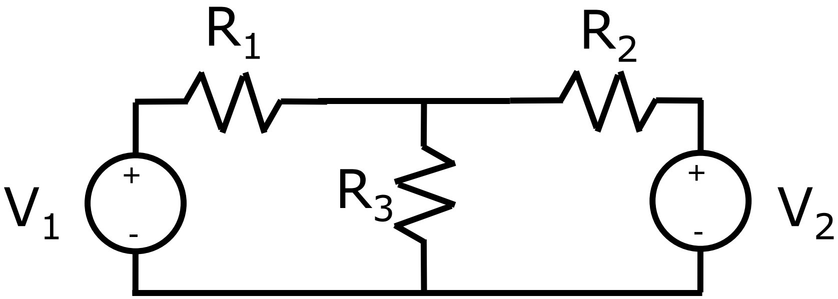
Mesh Analysis

Steps to solve a circuit with N meshes

1. Assign a mesh current to N meshes
2. Apply KVL to each of the N meshes to generate N equations
3. Use Ohm's Law to express voltages as functions of mesh currents (may be combined with step 2)
4. Solve resulting simultaneous linear equations

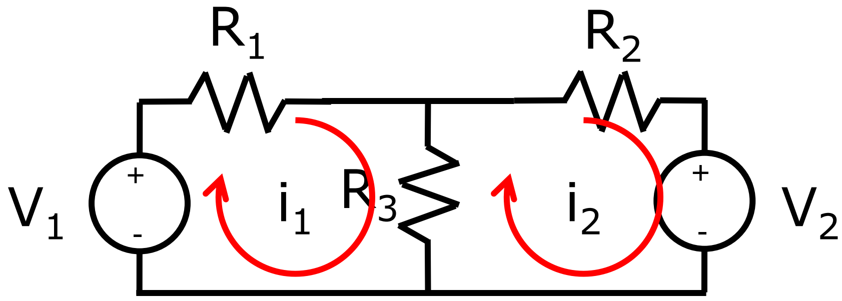
Mesh Analysis

- Consider the circuit below



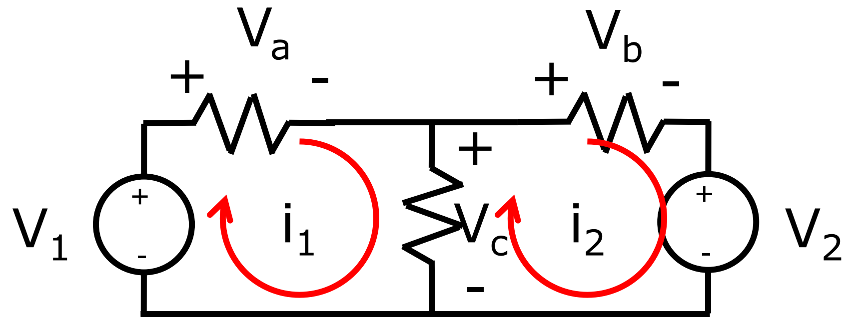
Step 1: Assign Mesh Currents

- Mesh: a loop (closed path) that does not contain another loop
- Two meshes: i_1, i_2



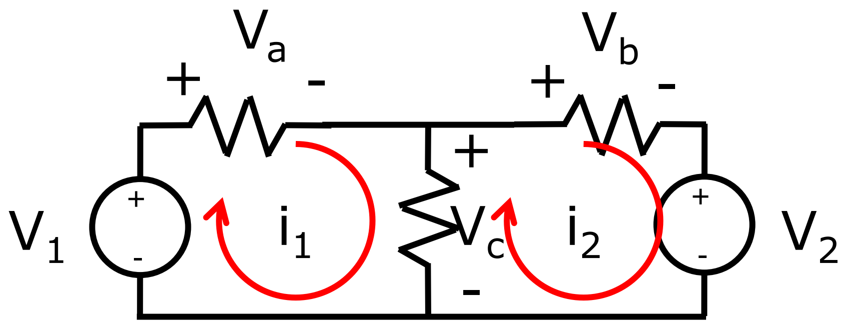
Step 2: Apply KVL

- KVL gives one equation per mesh (two equations)
  - $V_1 = V_a + V_c$
  - $-V_2 = V_b - V_c$



Step 3: Apply Ohm's Law

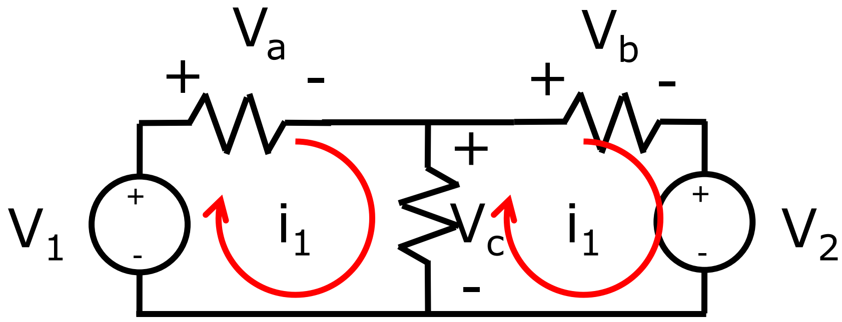
- Need to write unknown voltages as function of mesh currents
  - $V_a = R_1 i_1$
  - $V_b = R_2 i_2$
  - $V_c = R_3(i_1 - i_2)$



- Via substitution into KVL equations
  - $V_1 = R_1 i_1 + R_3(i_1 - i_2)$
  - $-V_2 = R_2 i_2 + R_3(i_2 - i_1)$

Step 4: Solve Equations

- Recap
  - two variables (i_1, i_2)
  - two KVL equations
    - $V_1 = R_1 i_1 + R_3(i_1 - i_2)$
    - $-V_2 = R_2 i_2 + R_3(i_2 - i_1)$



Step 4: Solve Equations

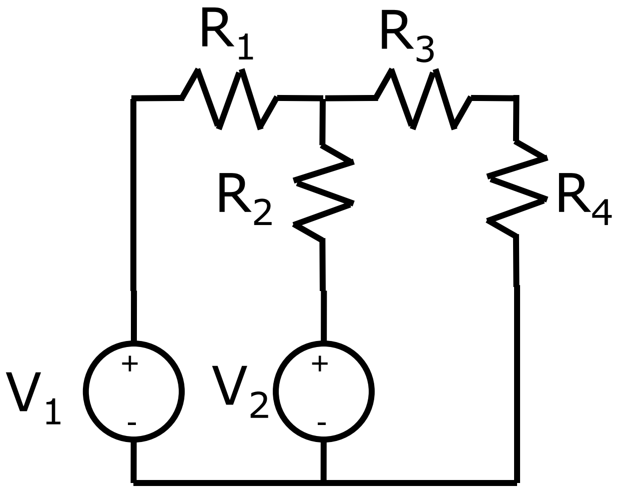
- In matrix form
  - $V_1 = R_1 i_1 + R_3(i_1 - i_2)$
  - $-V_2 = R_2 i_2 + R_3(i_2 - i_1)$

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

- Many methods of solving linear equations

Example

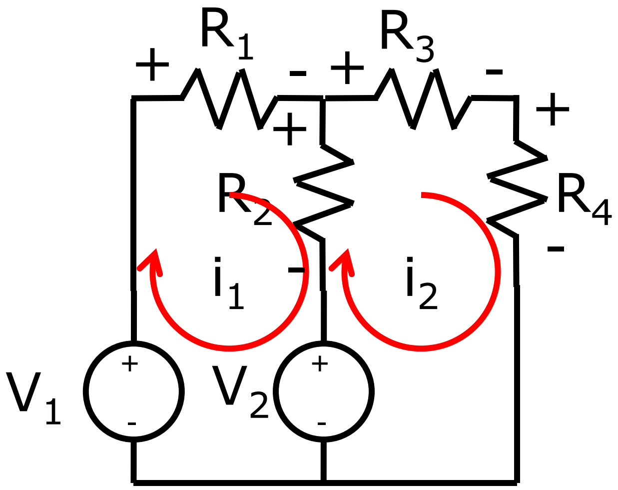
- Let
  - $V_1 = 15V$
  - $V_2 = 10V$
  - $R_1 = 5\Omega$
  - $R_2 = 10\Omega$
  - $R_3 = 6\Omega$
  - $R_4 = 4\Omega$
- Find all mesh currents



Example

- Equations:
  - $V_1 = i_1R_1 + R_2(i_1 - i_2) + V_2$
  - $V_2 = -R_2(i_1 - i_2) + i_2R_3 + i_2R_4$
  - Note: the second equation can also be written as:

    $V_2 = R_2(i_2 - i_1) + i_2R_3 + i_2R_4$
- Using circuit values:
  - $15 = i_1 5 + 10(i_1 - i_2) + 10$
  - $10 = 10(i_2 - i_1) + 6i_2 + 4i_2$



Example

- Equations
  - $15 = i_1 5 + 10(i_1 - i_2) + 10$

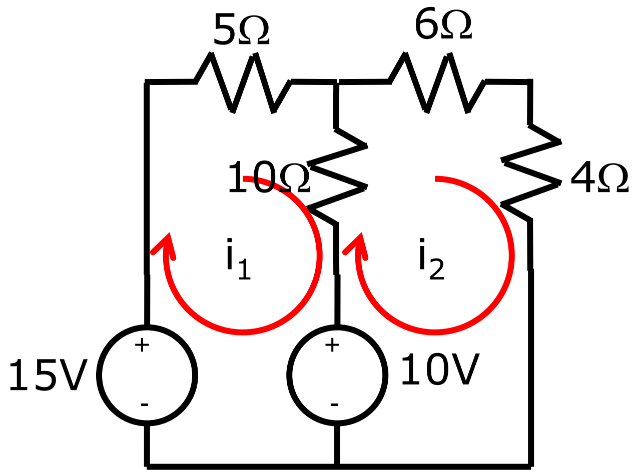
    $5 = 15i_1 - 10i_2$

  - $10 = 10(i_2 - i_1) + 6i_2 + 4i_2$

    $10 = -10i_1 + 20i_2$

- In matrix form:

$$\begin{bmatrix} 15 & -10 \\ -10 & 20 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$



Example

- Solving via matrix inversion

$$\begin{bmatrix} 15 & -10 \\ -10 & 20 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

R (matrix)

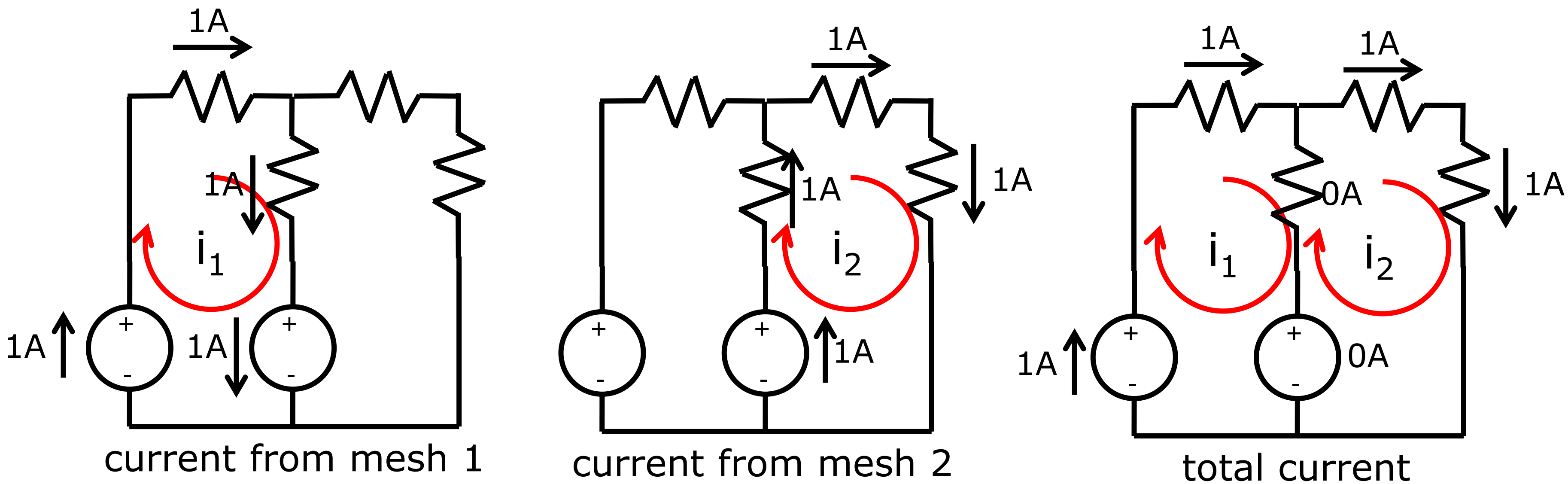
$$\mathbf{R} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \mathbf{R}^{-1} \begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{where } \mathbf{R}^{-1} = \begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.075 \end{bmatrix}$$

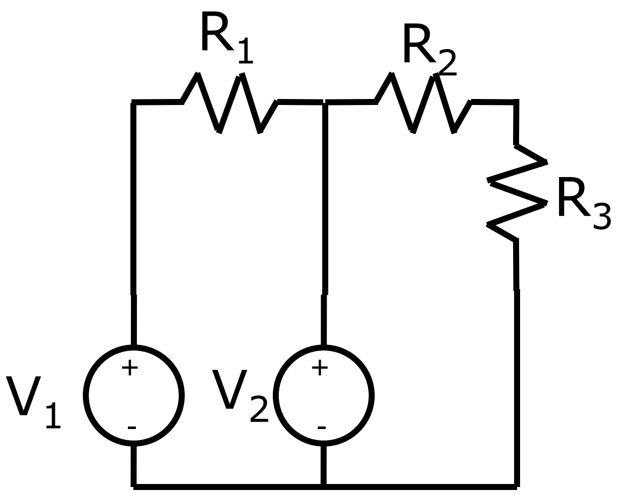
Branch Currents

- With mesh currents known, the current through each branch can be found
  - $i_1 = 1A$
  - $i_2 = 1A$



Example

- Let
  - $V_1 = 12V$
  - $V_2 = 6V$
  - $R_1 = 1\Omega$
  - $R_2 = 2\Omega$
  - $R_3 = 3\Omega$
- How many meshes?
- What are the equations?



Example

- Two meshes

  i_1: $V_1 = i_1R_1 + V_2$

  i_2: $V_2 = i_2R_2 + i_2R_3$

- Via substitution

  $12 = i_1 1 + 6$

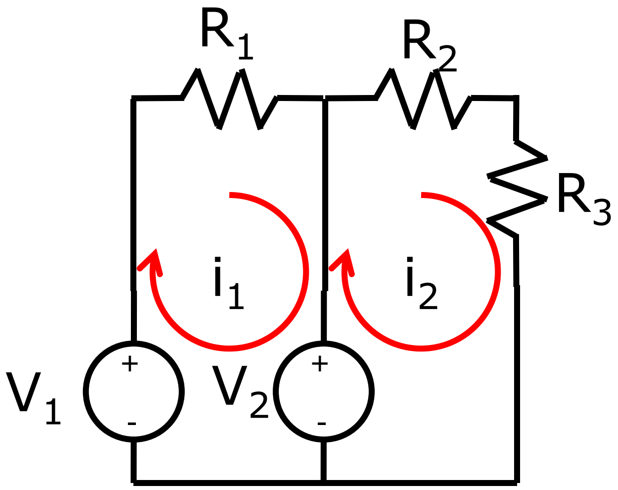
  $6 = i_2 5$

- Solving:

  $i_1 = 6A$

  $i_2 = 1.2A$

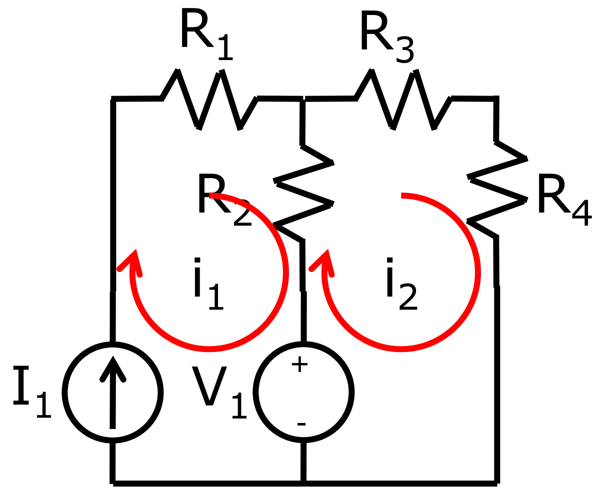
In this example, the equations are decoupled



Mesh Analysis with Current Sources

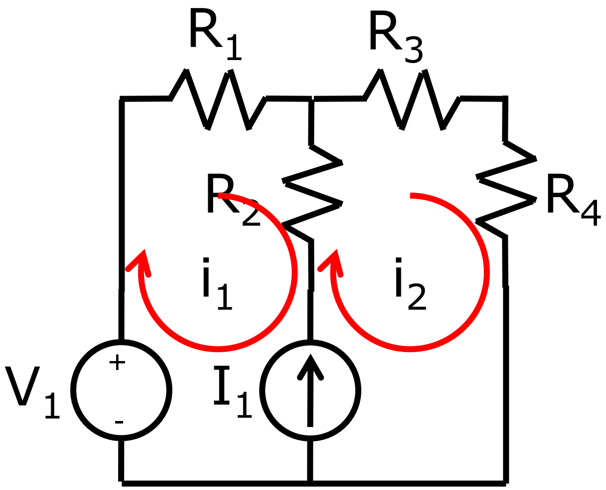
- Presence of current sources reduces the number of equations (unknowns) mesh analysis
- If current source (independent or dependent) exists in one loop only
  - Mesh current = current source

$(I_1 = i_1)$



Mesh Analysis with Current Sources

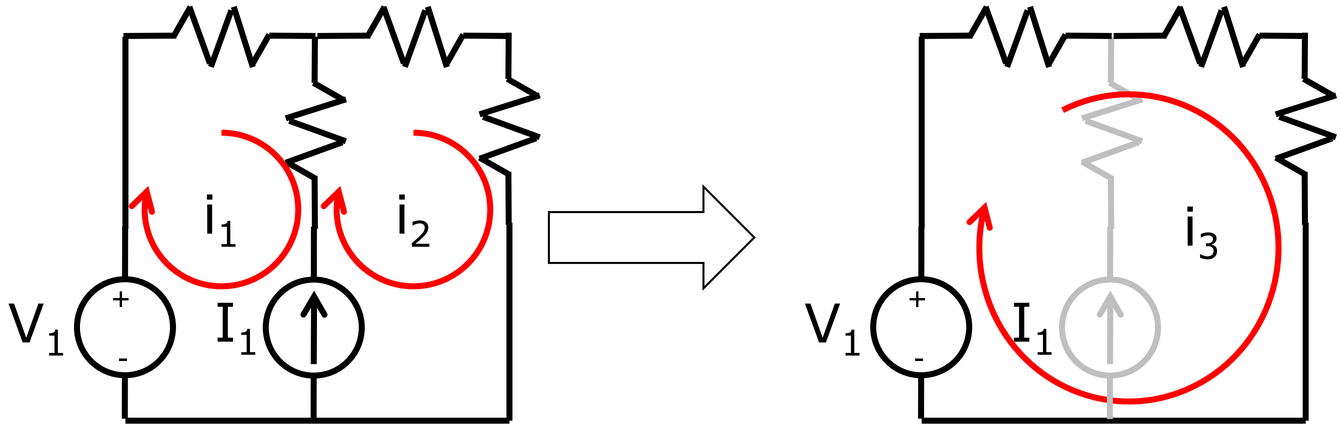
- If the <u>current</u> source exists between two meshes, then create a supermesh



$(I_1 = i_1 - i_2)$

Mesh Analysis with Current Sources

- Supermesh: a closed loop created from combining meshes by ignoring current sources AND <u>any elements in series with them</u>



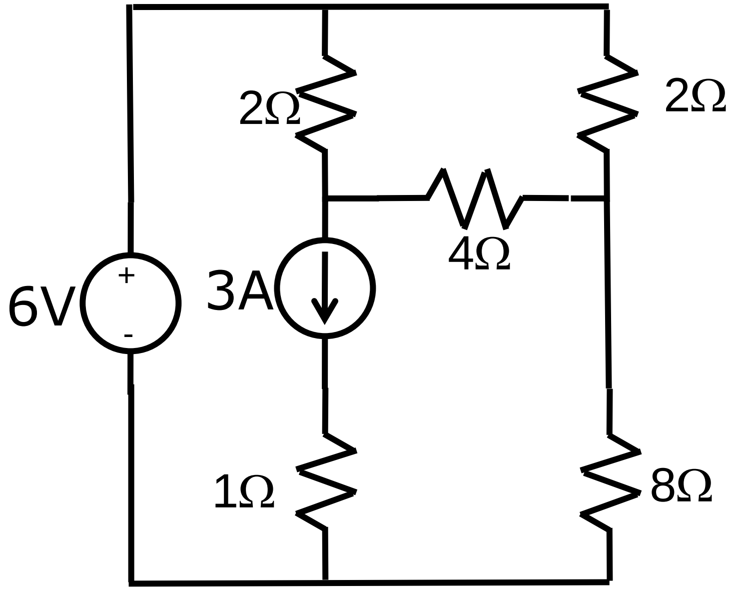
- Sum of voltages around supermesh = 0
  - Use mesh currents as variables
- Also applies to dependent current sources

Mesh Analysis with Current Sources

- Two or more supermeshes that intersect can be combined into a larger supermesh
- Properties of a supermesh
  - Current source inside the supermesh provides a constraint equation needed to solve for the individual mesh currents
  - A supermesh has no current of its own
  - A supermesh reduces the number of KVL equations
  - KCL must be also applied for an additional independent equation

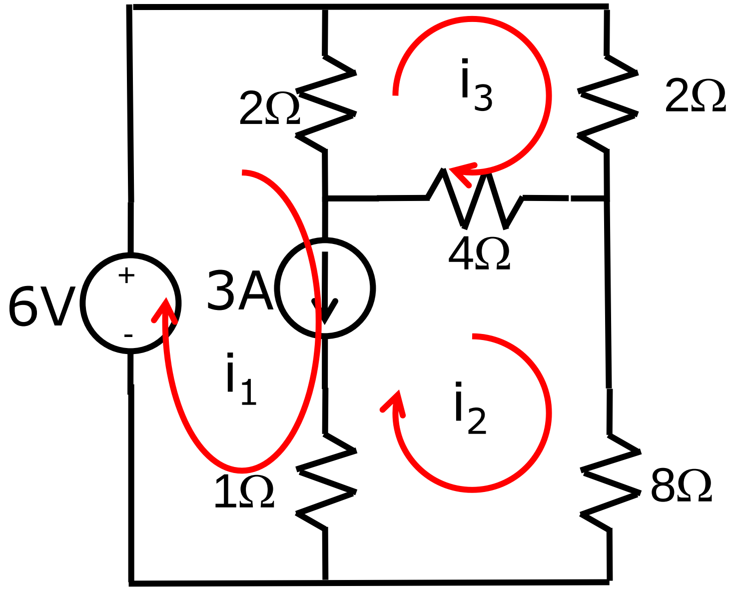
Mesh Analysis with Current Sources

- Draw the mesh currents (assume clockwise rotation)



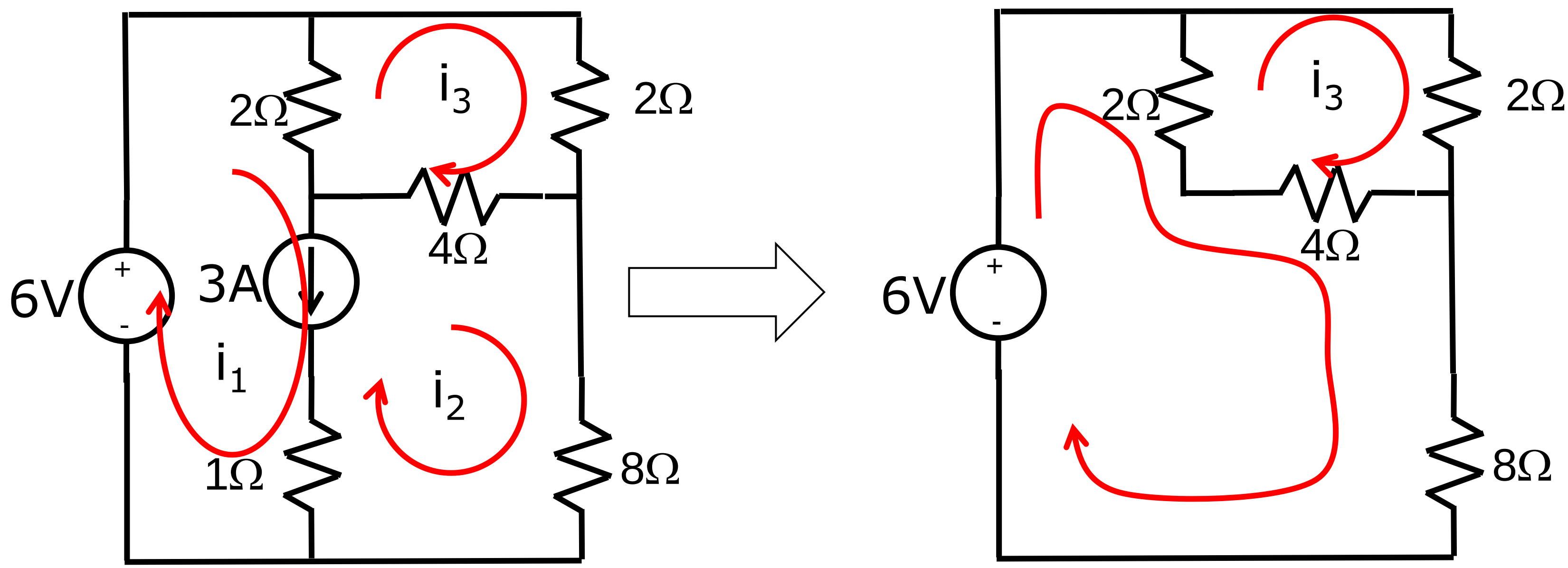
Mesh Analysis with Current Sources

- Find the mesh currents



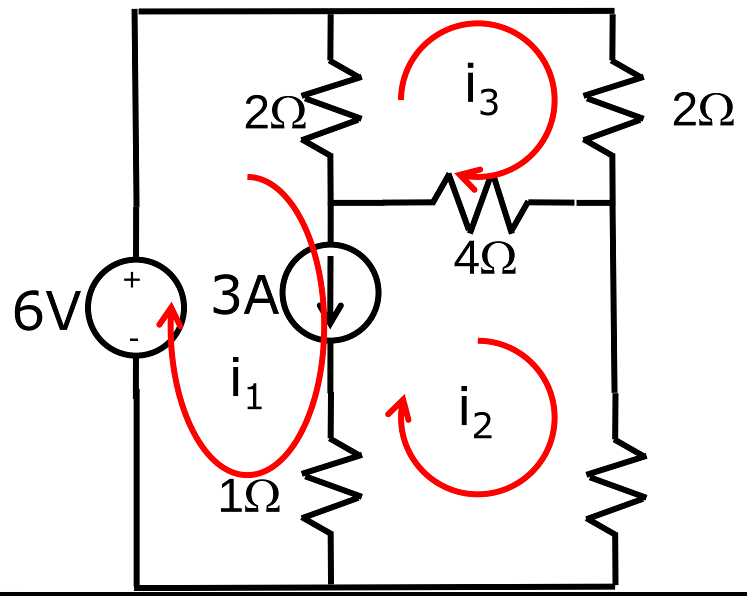
Mesh Analysis with Current Sources

- Make a supermesh out of the current source and series resistor
- Write KVL for supermesh (using i_1, i_2 and i_3)
  - $6 = 2(i_1 - i_3) + 4(i_2 - i_3) + 8i_2$



Mesh Analysis with Current Sources

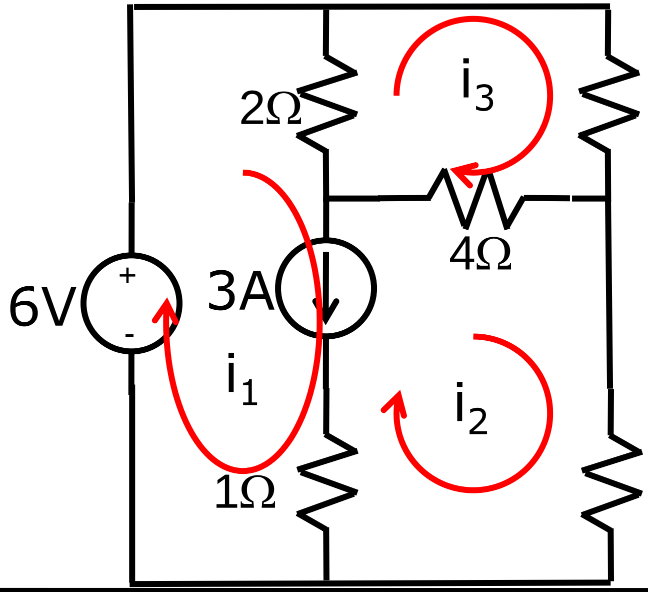
- Write KVL for i_3 (cannot write for $i_{1,}$ i_2)
  - $0 = 2i_1 + 4i_2 - (2 + 4 + 2)i_3$
- Current source provides a constraint equation
  - $3 = i_1 - i_2$



Mesh Analysis with Current Sources

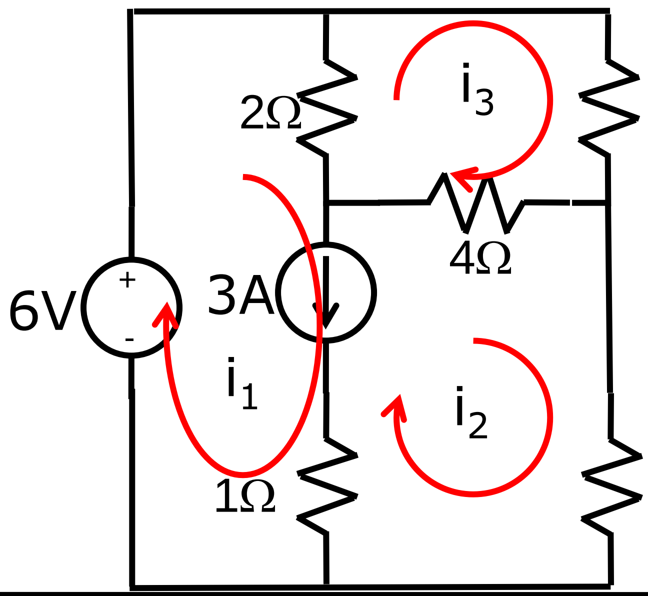
- How many independent equations?
  - $6 = 2(i_1 - i_3) + 4(i_2 - i_3) + 8i_2$ (supermesh)
  - $0 = 2i_1 + 4i_2 - 8i_3$ (mesh 3)
  - $3 = i_1 - i_2$ (current source constraint)
- How many unknowns?
  - i_1, i_2, i_3

  Solve!



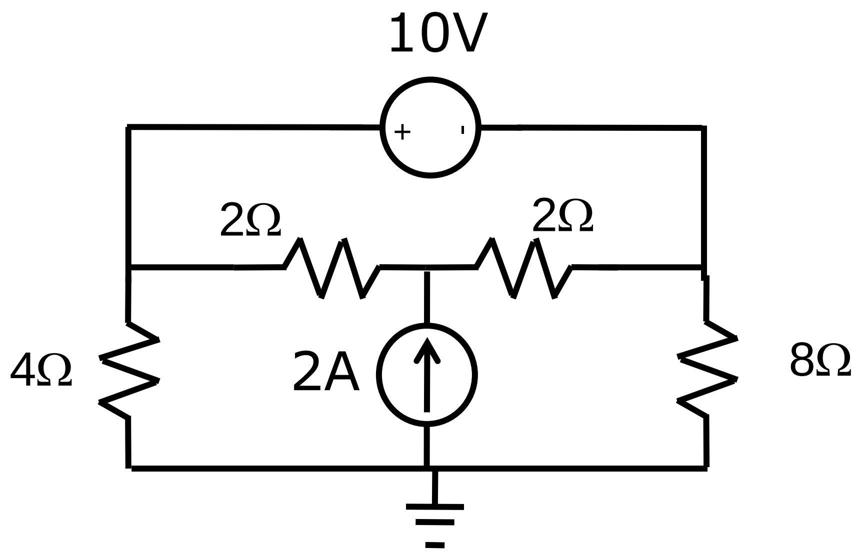
Mesh Analysis with Current Sources

- Solving…
  - $6 = 2(i_1 - i_3) + 4(i_2 - i_3) + 8i_2$
  - $0 = 2i_1 + 4i_2 - 8i_3$
  - $3 = i_1 - i_2$
- $i_1 = 3.474$ A
- $i_2 = 0.474$ A
- $i_3 = 1.105$ A



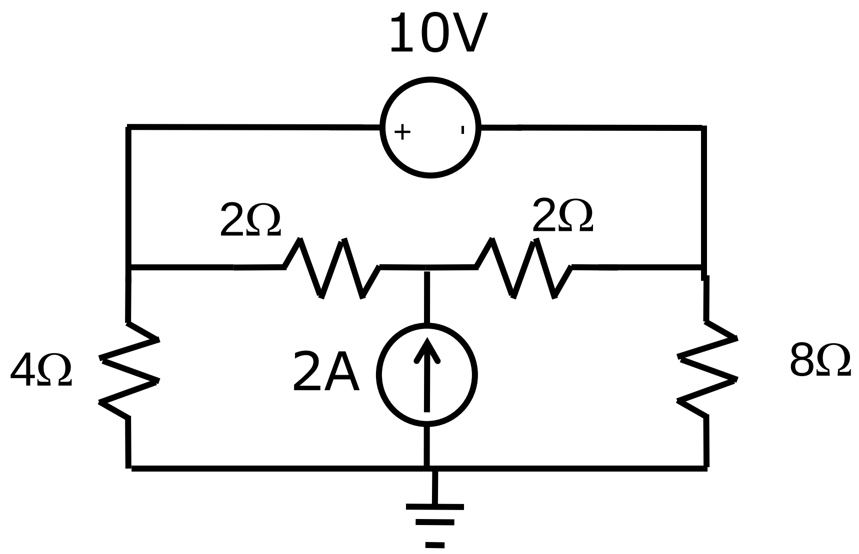
Example

- Find the mesh currents in the circuit shown. Find the voltage across the current source.



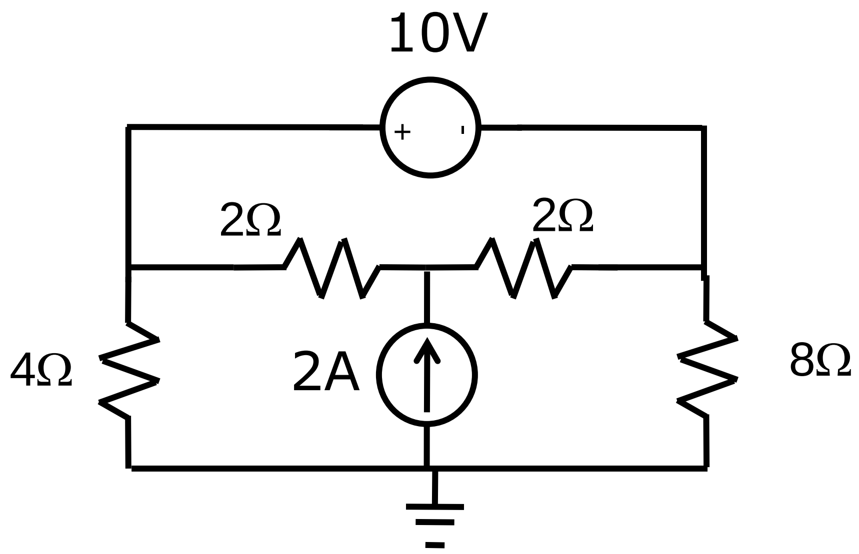
Example

- Identify meshes and supermesh



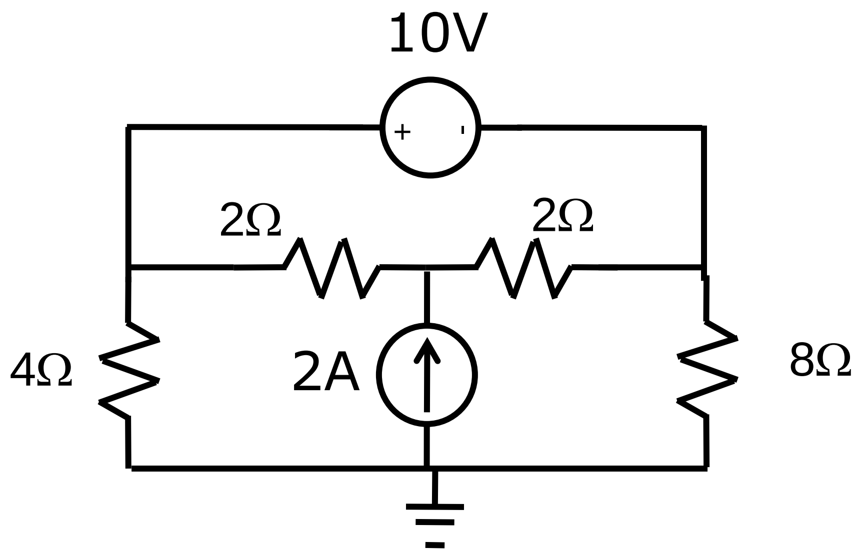
Example

- Write mesh and supermesh equations



Example

- Add constraint equation using KCL around supermesh

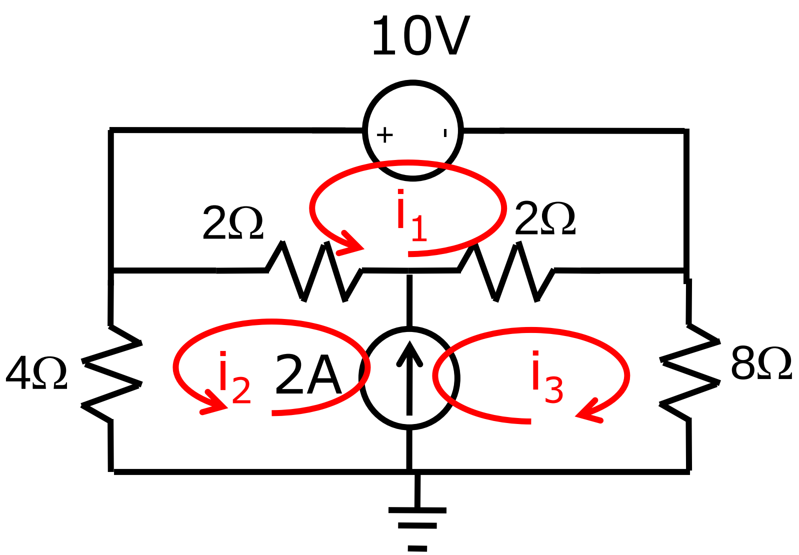


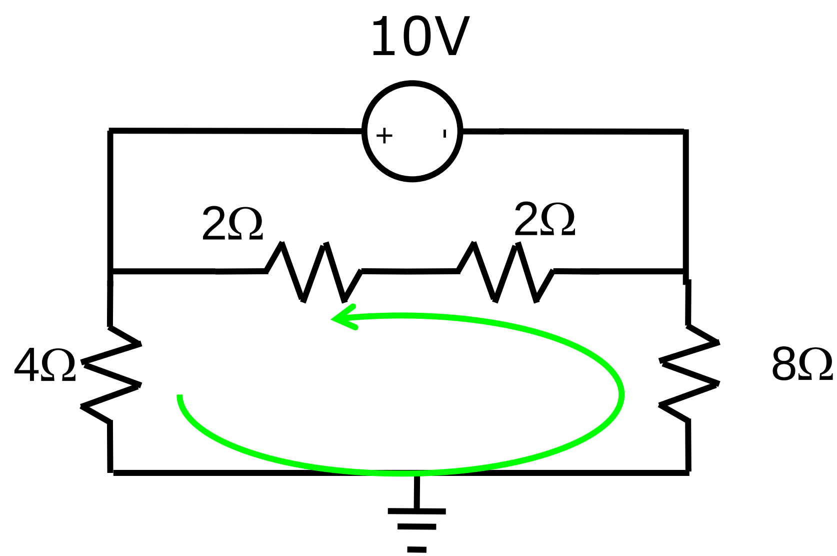
Example

- 3 equations, 3 unknowns. Solve.

Example

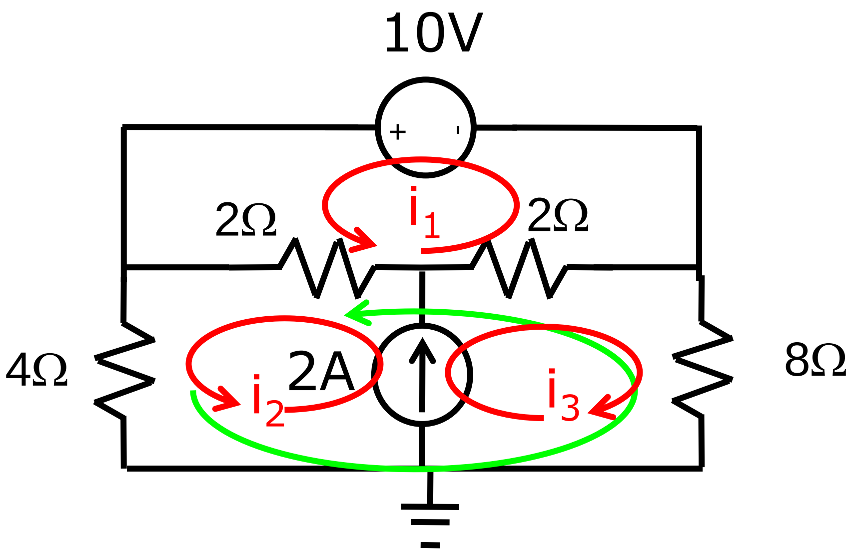
- Identify meshes and supermesh





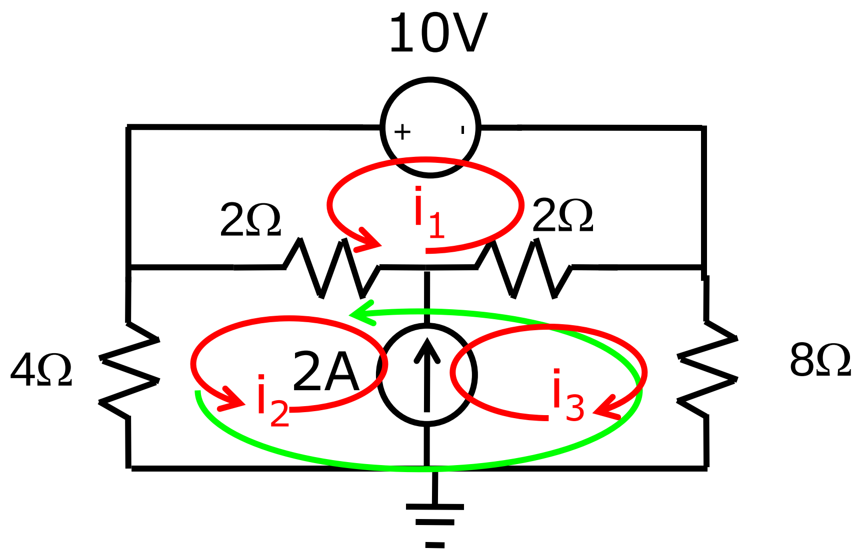
Example

- Write mesh and supermesh equations
  - $0 = -10 + (2+2)i_1 - 2i_2 + 2i_3$ (i_1 mesh)
  - $0 = -i_1(2+2) + (2+4)i_2 - (2+8)i_3$ (supermesh)



Example

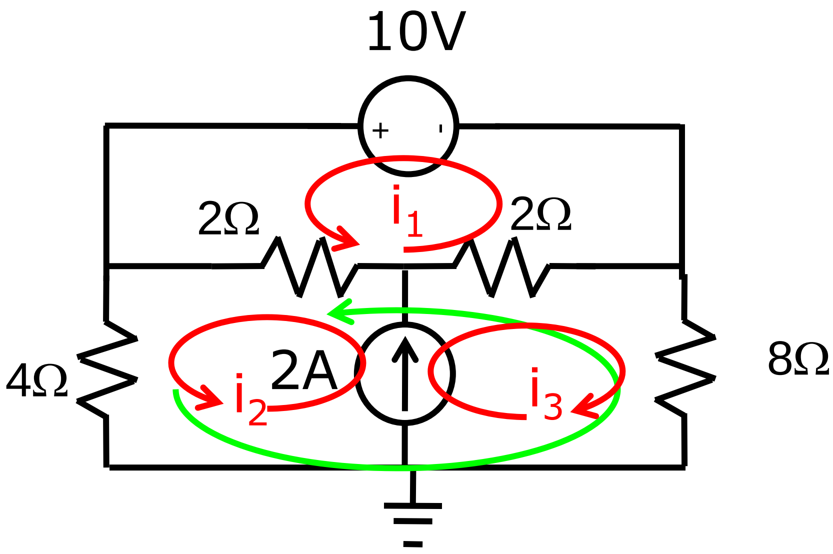
- Add constraint equation using KCL
  - $0 = -10 + (2+2)i_1 - 2i_2 + 2i_3$
  - $0 = -i_1(2+2) + (2+4)i_2 - (2+8)i_3$
  - $0 = 2 - i_2 - i_3$



Example

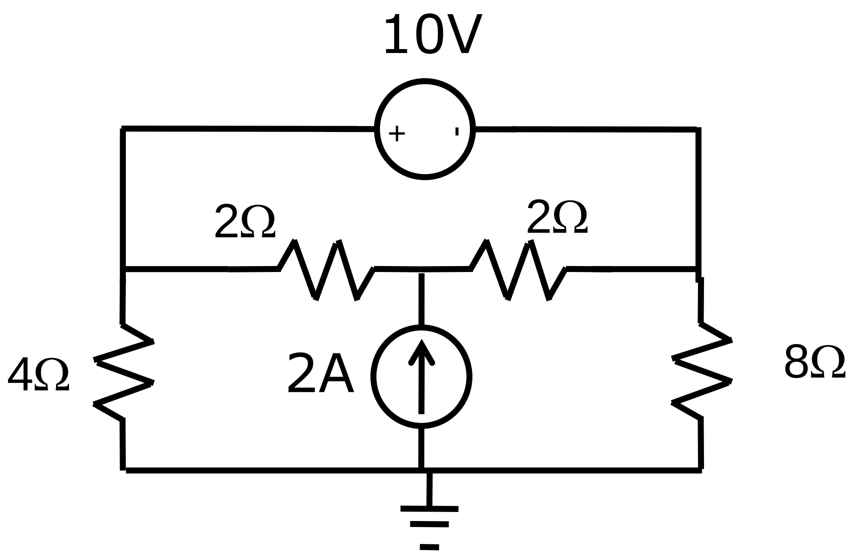
- Three equations, three unknowns. Solve.
  - $10 = 4i_1 - 2i_2 + 2i_3$
  - $0 = -4i_1 + 6i_2 - 10i_3$
  - $2 = i_2 + i_3$

- $i_1 = 3.667A$
- $i_2 = 2.167A$
- $i_3 = -0.167A$



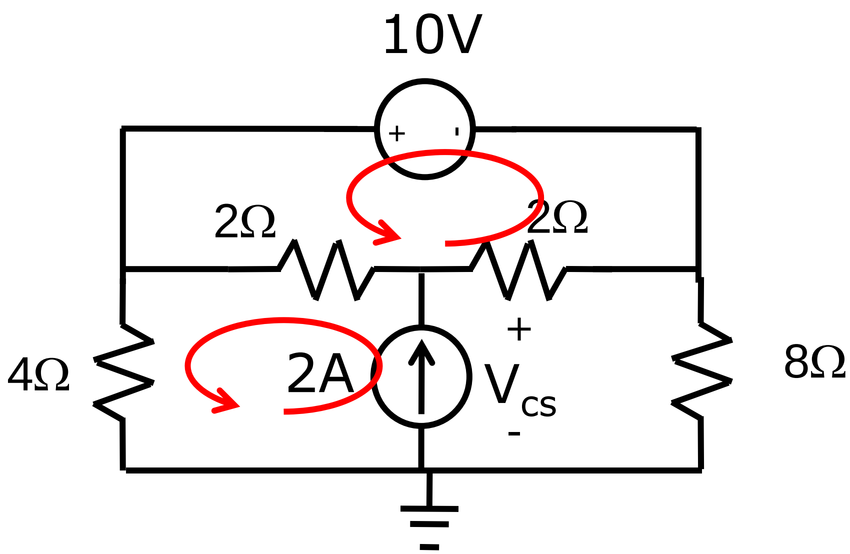
Example

- Now find the voltage across the current source



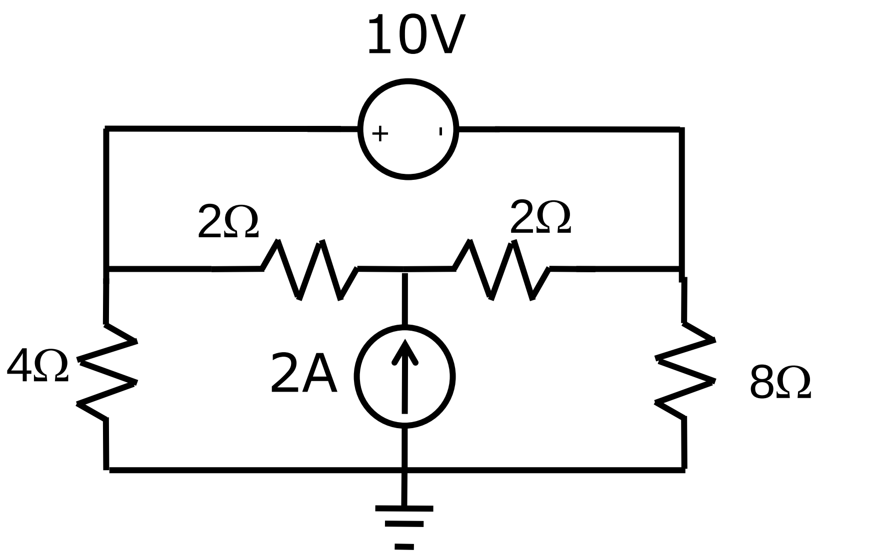
Example

- Voltage across current source is found by KVL:
  - $V_{CS} = (4+2)i_2 - 2i_1 = 13 - 7.333 = 5.667V$



Example

- The two circuits are therefore equivalent



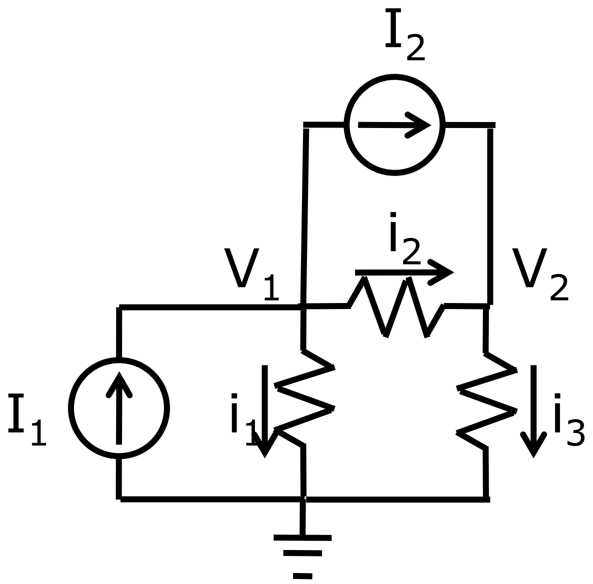
Nodal and Mesh Analyses by Inspection

- Under certain circumstances we can write nodal or mesh analysis equations simply by inspecting the circuit
  - Saves time
  - Fewer errors
- Inspection can be used in nodal analysis when all sources are independent current sources
- Inspection can be used in mesh analysis when all sources are independent voltage sources
- See text 3.6 for more details

Nodal Analysis by Inspection

- Example: use nodal analysis to find the node voltages
  - $I_1 = I_2 + i_1 + i_2$ (KCL)
  - $I_2 = i_3 - i_2$ (KCL)
  - $i_1 = (V_1 - 0)/R_1$ (Ohm's Law)
  - $i_2 = (V_1 - V_2)/R_2$ (Ohm's Law)
  - $i_3 = (V_2 - 0)/R_3$ (Ohm's Law)

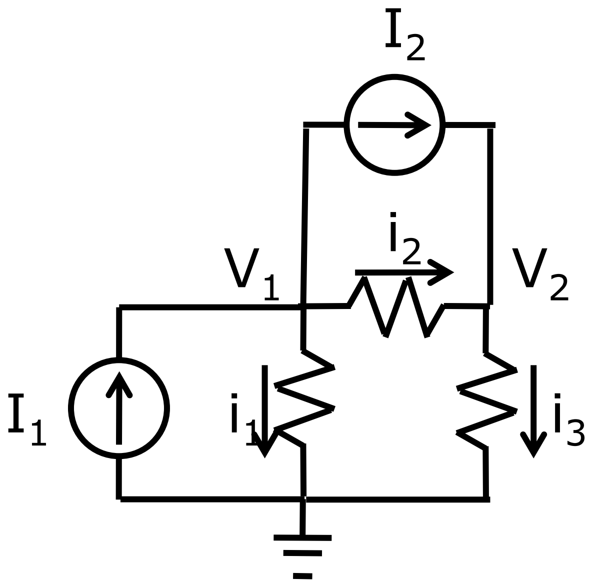
$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_3} + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$



Nodal Analysis by Inspection

- In terms of conductances:

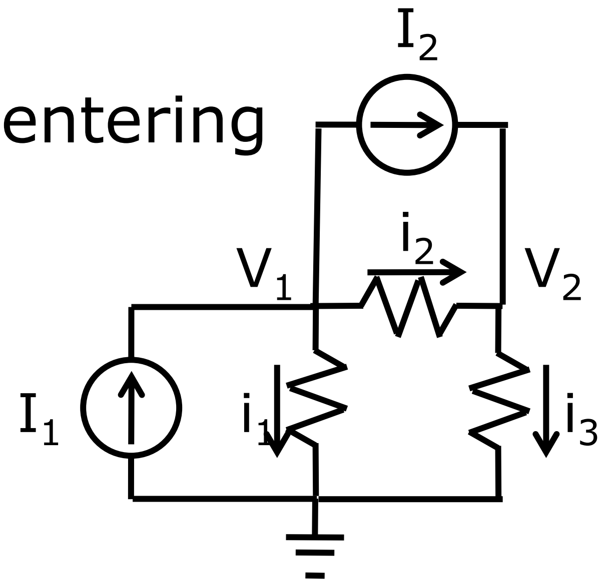
$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$



Nodal Analysis by Inspection

- **G** is symmetric
- Diagonal elements are sum of conductances connected to each node
- Off-diagonal elements are negative of conductances between nodes
- Current vector is sum of currents entering each node

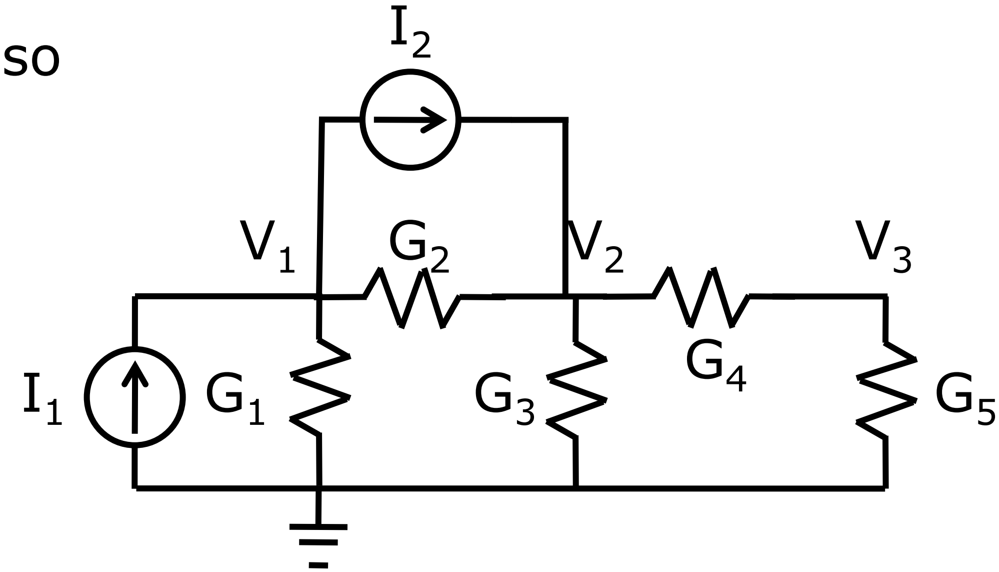
$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$



Nodal Analysis by Inspection

$$\begin{bmatrix} G_1 + G_2 & -G_2 & 0 \\ -G_2 & G_2 + G_3 + G_4 & -G_4 \\ \boxed{0} & -G_4 & G_4 + G_5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \\ 0 \end{bmatrix}$$

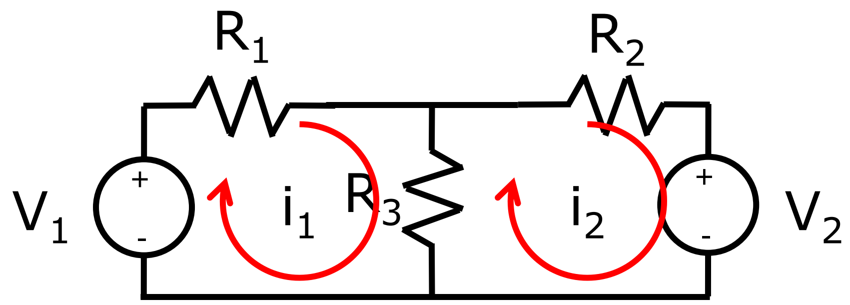
Node 1 is not directly connected to node 3, so there is a 0 here



Mesh Analysis by Inspection

- Similarly, for mesh analysis:

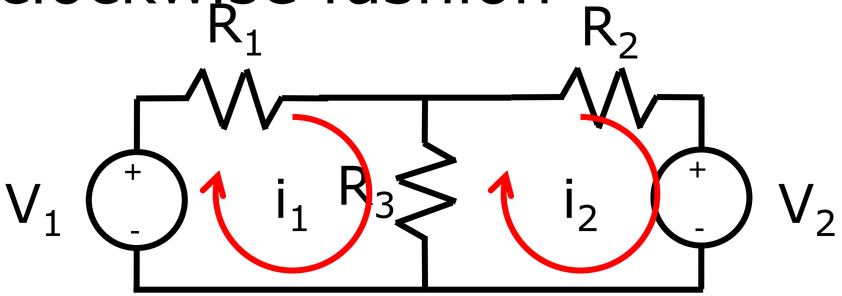
$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$



Mesh Analysis by Inspection

- **R** is symmetric
- Diagonal elements are sum of resistances in the mesh
- Off-diagonal elements are negative of resistances common to the meshes
- Voltage vector is sum of independent voltage sources in each mesh in a clockwise fashion

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$



Nodal Versus Mesh Analysis

- When should nodal analysis be used?
- When should mesh analysis be used?
- Factors to consider:
  - Nature of the network
  - What information is to be solved for

Nodal Versus Mesh Analysis

- Use nodal analysis when network has:
  - Many parallel connected elements
  - Many current sources
  - Supernodes
- Use mesh analysis when network has:
  - Many series connected elements
  - Many voltage sources
  - Supermeshes

Nodal Versus Mesh Analysis

- If node voltages are required:
  - Nodal analysis
- If branch or mesh currents are required:
  - Mesh analysis
- Nodal analysis is usually used in computer-aided analysis, such as in PSPICE