

07-Minterm and Maxterm Expansions

Text: Unit 4

ECEGR/ISSC 201
Digital Operations and Computations
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Overview

- Minterm Expansions
- Maxterm Expansions
- Minterms and Maxterms from Switching Expressions

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Minterm Expansions

- A minterm of n variables is the product of n literals in which each variable appears once in either a complemented or uncomplemented form
- $F = A'BC + AB'C' + AB'C + ABC$ has four minterms
- $F = AC$ is not a minterm because B is not present; we can write this expression as
 - $F = AC(B'+B) = ACB' + ACB$; ACB' and ACB are minterms

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Minterm Expansions

- Each minterm will be equal to 1 for one combination of the variables (in this case, A, B, C)
- Consider:

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

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Minterm Expansions

- The minterms and associated A, B, C combinations:

A	B	C	minterms
0	0	0	$A'B'C'$
0	0	1	$A'B'C$
0	1	0	$A'BC'$
0	1	1	$A'BC$
1	0	0	$AB'C'$
1	0	1	$AB'C$
1	1	0	ABC'
1	1	1	ABC

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Minterm Expansions


- For our example:
 $F = A'BC + AB'C' + AB'C + ABC$

Four minterms,
four 1s

A	B	C	F	minterms
0	0	0	0	$A'B'C'$
0	0	1	0	$A'B'C$
0	1	0	0	$A'BC'$
0	1	1	1	$A'BC$
1	0	0	1	$AB'C'$
1	0	1	1	$AB'C$
1	1	0	0	ABC'
1	1	1	1	ABC

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Minterm Notation

- There is shorthand for writing expressions that have been expanded in minterms
- Note that each row in truth table corresponds to one minterm (which may or may not be present in an expansion of F)


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Minterm Notation

- If we order the truth table consistently such that the first row is always A=0, B=0, C=0, the second row is always A = 0, B =0, C =1... and the eight row is A=1, B=1, C=1 then we can correlate minterm with row number
- Example: row 4=> A=1, B =0, C=0 =>AB'C'

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Minterm Notation

- $F = A'BC + AB'C' + AB'C + ABC$
- Also written as: $F = m_3 + m_4 + m_5 + m_7$

Row	A	B	C	F	minterms
0	0	0	0	0	A'B'C'
1	0	0	1	0	A'B'C
2	0	1	0	0	A'BC'
3	0	1	1	1	A'BC
4	1	0	0	1	AB'C'
5	1	0	1	1	AB'C
6	1	1	0	0	ABC'
7	1	1	1	1	ABC

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Minterm Notation

- $F = m_3 + m_4 + m_5 + m_7$
- In more simplified form: $F(A,B,C) = \sum m(3,4,5,7)$

Row	A	B	C	F	minterms
0	0	0	0	0	A'B'C'
1	0	0	1	0	A'B'C
2	0	1	0	0	A'BC'
3	0	1	1	1	A'BC
4	1	0	0	1	AB'C'
5	1	0	1	1	AB'C
6	1	1	0	0	ABC'
7	1	1	1	1	ABC

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Example

- Write the minterms of the truth table

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

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Example

- Write the minterms of the truth table

$$F = A'B'C' + AB'C' + ABC \quad F(A,B,C) = \sum m(0,4,7)$$

Row	A	B	C	F	minterms
0	0	0	0	1	A'B'C'
1	0	0	1	0	A'B'C
2	0	1	0	0	A'BC'
3	0	1	1	0	A'BC
4	1	0	0	1	AB'C'
5	1	0	1	0	AB'C
6	1	1	0	0	ABC'
7	1	1	1	1	ABC


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Maxterm Expansions

- Minterms correspond to a sum of products expression
- Maxterms correspond to a product of sums
- A maxterm of n variables is a sum of n literals in which each literal appears exactly once in true or complemented form, but not both


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Maxterm Expansions

- $F = (A+B+C)(A'+B'+C)$ has two maxterms
- $F = (A+C)(A' + B' + C)$ is not in maxterm form
- $F = (AB+C)(A'+B'+C)$ is not in maxterm form


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Max Expansions

- Each maxterm will be equal to 0 for one combination of the variables
- From our example:
 $F = (A+B+C)(A+B+C')(A+B'+C)(A'+B'+C)$

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Maxterm Expansions

- $F = (A+B+C)(A+B+C')(A+B'+C)(A'+B'+C)$

Four maxterms, four 0s

A	B	C	F	maxterms
0	0	0	0	A+B+C
0	0	1	0	A+B+C'
0	1	0	0	A+B'+C
0	1	1	1	A+B'+C'
1	0	0	1	A'+B+C
1	0	1	1	A'+B+C'
1	1	0	0	A'+B'+C
1	1	1	1	A'+B'+C'

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Maxterm Notation

- $F(A,B,C) = M_0M_1M_2M_6$
- Also written as: $F(A,B,C) = \prod M(0,1,2,6)$

Row	A	B	C	F	maxterms
0	0	0	0	0	A+B+C
1	0	0	1	0	A+B+C'
2	0	1	0	0	A+B'+C
3	0	1	1	1	A+B'+C'
4	1	0	0	1	A'+B+C
5	1	0	1	1	A'+B+C'
6	1	1	0	0	A'+B'+C
7	1	1	1	1	A'+B'+C'


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Minterms and Maxterms from Switching Expressions

- Can easily find minterm or maxterm from a truth table
- Truth tables can be cumbersome to create if the number of variables is large
- Algebraic way:
 - Write the expression as a sum of products and introduce missing variables using $(X + X') = 1$ (for minterm) or XX' (for maxterm)


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Minterms and Maxterms from Switching Expressions

- For example, given:
 - $F = A'B' + AC$
- Introduce the missing variable in each product using $(X + X') = 1$
 - $F = A'B'(C + C') + A(B + B')C$
- Expand (with reordering) and find decimal notation:
 - $F = A'B'C' + A'B'C + AB'C + ABC$
 - (000) (001) (101) (111)
- So $F_{A,B,C} = \sum m_{0,1,5,7}$


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Minterms and Maxterms from Switching Expressions

- To write as Maxterm:
 - $F = A'B' + AC$
- Factor into product of sums (second distributive law)
 - $F = (A'B' + A)(A'B' + C)$
 - $F = (\cancel{A} + \cancel{A})(A+B')(C+A)(C+B')$
- Using $XX' = 0$
 - $F = (A+B'+CC')(C+A' + BB')(AA' + C+B')$
- Using $(X + Y)(X + Y') = X$
 - $F = (A+B'+C)(A+B'+C')(A'+C+B)(A'+C+B')(A+B'+C)(A'+B'+C)$


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Minterms and Maxterms from Switching Expressions

- Removing redundant terms:
 - $F = (A+B'+C)(A+B'+C')(A'+B+C)(A'+B'+C)(\cancel{A+B+C})(\cancel{A+B+C})$
- Reordering
 - $F = (A+B'+C)(A+B'+C')(A'+B+C)(A'+B'+C)$
- Into decimal notation (complements are 1, uncomplemented are 0):
 - 010 011 100 110
- Gives: $F_{A,B,C} = \prod M(2,3,4,6)$

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Minterms and Maxterms from Switching Expressions

- $F = A'B' + AC$
- Minterms should match $F = 1: F_{A,B,C} = \sum m_{0,1,5,7}$

Row	A	B	C	F	minterms	maxterms
0	0	0	0	1	A'B'C'	A+B+C
1	0	0	1	1	A'B'C	A+B+C'
2	0	1	0	0	A'BC'	A+B'+C
3	0	1	1	0	A'BC	A+B'+C'
4	1	0	0	0	AB'C'	A'+B+C
5	1	0	1	1	AB'C	A'+B+C'
6	1	1	0	0	ABC'	A'+B'+C
7	1	1	1	1	ABC	A'+B'+C'

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Minterms and Maxterms from Switching Expressions

- $F = A'B' + AC$
- Maxterms should match $F = 0: F_{A,B,C} = \prod M(2,3,4,6)$

Row	A	B	C	F	minterms	maxterms
0	0	0	0	1	A'B'C'	A+B+C
1	0	0	1	1	A'B'C	A+B+C'
2	0	1	0	0	A'BC'	A+B'+C
3	0	1	1	0	A'BC	A+B'+C'
4	1	0	0	0	AB'C'	A'+B+C
5	1	0	1	1	AB'C	A'+B+C'
6	1	1	0	0	ABC'	A'+B'+C
7	1	1	1	1	ABC	A'+B'+C'

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Example

- Find the minterm expansion of
 - $F = A'(B'+D)+ACD'$
- Basic steps:
 - Expand into SoP form (if necessary)
 - Use $(X + X') = 1$ to add missing variables to each product
 - Multiply out
 - Remove redundant terms
 - Convert to decimal notation

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Minterms and Maxterms from Switching Expressions

- If given the minterm expansion for F , then the maxterm expansion of F' are the same numbers as the minterms of F
- If given the maxterm expansion for F , then the minterm expansion of F' are the same numbers as the maxterms of F

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Minterms and Maxterms from Switching Expressions

- If $F_{A,B,C} = \prod M(2,3,4,6)$
Then $F'_{A,B,C} = \sum m_{0,1,5,7}$
- If $F_{A,B} = \sum m_{0,3}$
Then $F'_{A,B} = \prod M(1,2,3)$
- If $F'_{A,B} = \sum m_{0,3}$
Then $F_{A,B} = \sum m_{1,2}$
- If $F_{A,B,C} = \sum m_{3,4,5,6,7}$
Then $F'_{A,B,C} = \prod M(3,4,5,6,7)$

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