07-Measurements

ECEGR 450
Electromechanical Energy Conversion

Overview

- Power Measurements
- Two Wattmeter Method
- Impedance Measurements

Questions

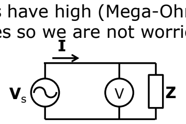
- How can you measure the complex power consumed by a load (or supplied by a generator)?
- How can total power be measured in systems with no neutral?
- How can you measure complex impedance?

Power Measurements

- Next we examine how to measure the power in a circuit
  - Single phase power
  - Three phase power
- Interested in measuring P and PF (power factor)
  - **S**, Q can be computed from P, PF

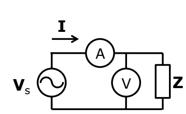
Power Measurements

- Consider a single phase circuit
- To measure the P and PF of the load, we need to know **V** and **I**
- Consider a voltmeter connected across the load
- Voltmeters measure the RMS voltage
  - RMS voltage is |**V**|
  - Voltmeters have high (Mega-Ohm) internal impedances so we are not worried about shorting the source



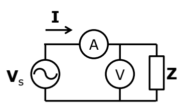
Power Measurements

- We can measure |**I**| with an Ammeter
- Where should we connect the Ammeter?



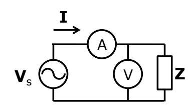
Power Measurements

- Ammeters measure the RMS current
  - RMS current is |**I**|
  - Ammeters have low (milli-Ohm) internal impedances so we can put it in series



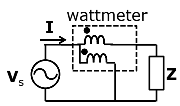
Power Measurements

- We now have |**V**| and |**I**|, but this is not enough information to determine P and PF
- We need to know the phase difference between **V** and **I**
- Possible to use a single meter (or oscilloscope) to measure current, voltage and phase angle



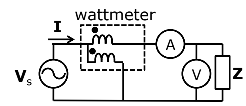
Power Measurements

- Lets assume you do not have such an instrument, but you do have an Wattmeter
- Wattmeters measure average power P
  - Contains a current coil and voltage coil
  - Polarity is important



Power Measurements

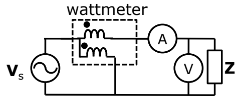
- If you use a wattmeter, voltmeter and ammeter you can determine P and PF
  - P from the Wattmeter
- How can you compute PF? Write the equation.
  - See text example 1.6 for a numerical example



Power Measurements

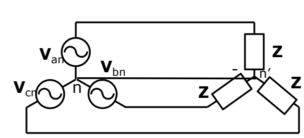
- How can you compute PF?

$$\frac{P}{|\mathbf{I}||\mathbf{V}|} = \frac{P}{|\mathbf{S}|} = PF$$



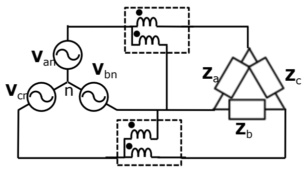
Three Phase Power Measurements

- If the neutral wire is available, simply set up 3 wattmeters (each connected to a different phase and to the neutral)
  - Add the readings to find the total P
  - If the system is balanced, all wattmeters will have the same reading



Three Phase Power Measurements

- If the neutral is not available or if either the source or load are Delta connected the <u>two-wattmeter method</u> can be used
- Note: line voltages are connected to the wattmeters



Three Phase Power Measurements

Wattmeter 1 reads:

$P_1 = \mathrm{Re}\{\mathbf{V}_{ab}\mathbf{I}^*_{na}\}$

$= \mathrm{Re}\{(|\mathbf{V}_{ab}|\angle 30^\circ)(|\mathbf{I}_{na}|\angle\theta_{I_{na}})^*\}$

$= \mathrm{Re}\{(\sqrt{3}|\mathbf{V}_{an}|\angle 30^\circ)(|\mathbf{I}_{na}|\angle\phi)\}$

$= \sqrt{3}|\mathbf{V}_{an}||\mathbf{I}_{na}|\cos(30+\phi)$

Setting θ_{an} as reference angle

$\phi = \theta_{V_{na}} - \theta_{I_{na}}$

$\theta^*_{I_{na}} = -\theta_{I_{na}} = \phi$

Note: this is not the a-phase power

Power Measurements

Wattmeter 2 reads:

$P_2 = \mathrm{Re}\{\mathbf{V}_{cb}\mathbf{I}^*_{nc}\}$

$= \mathrm{Re}\{(|\mathbf{V}_{cb}|\angle 90^\circ)(|\mathbf{I}_{nc}|\angle\theta_{I_{na}}+120^\circ)^*\}$

$= \mathrm{Re}\{(\sqrt{3}|\mathbf{V}_{cn}|\angle 90^\circ)(|\mathbf{I}_{nc}|\angle\phi-120^\circ)\}$

$= \sqrt{3}|\mathbf{V}_{cn}||\mathbf{I}_{nc}|\cos(-30+\phi)$

$= \sqrt{3}|\mathbf{V}_{cn}||\mathbf{I}_{nc}|\cos(30-\phi)$

Note: this is not the c-phase power

Power Measurements

- Adding the readings:

$P_{3\Phi} = P_1 + P_2$

$= \sqrt{3}|\mathbf{V}_{an}||\mathbf{I}_{na}|\cos(30+\phi) + \sqrt{3}|\mathbf{V}_{cn}||\mathbf{I}_{nc}|\cos(30-\phi)$

$= \sqrt{3}|\mathbf{V}_{cn}||\mathbf{I}_{nc}|(\cos(30+\phi)+\cos(30-\phi))$

using: $\cos(30+\phi) = \cos(30)\cos(\phi) - \sin(30)\sin(\phi)$

$\cos(30-\phi) = \cos(30)\cos(\phi) + \sin(30)\sin(\phi)$

$\Rightarrow \cos(30+\phi)+\cos(30-\phi) = 2\cos(30)\cos(\phi)$

$P_{3\Phi} = \sqrt{3}|\mathbf{V}_{an}||\mathbf{I}_{na}|\,2\frac{\sqrt{3}}{2}\cos(\phi)$

$= 3|\mathbf{V}_{an}||\mathbf{I}_{na}|\cos(\phi)$ This is the 3 times the per-phase power so the two wattmeter method works

Power Measurements

- Adding the two readings on the wattmeters equals the total power due to:
  - $P_{3\Phi} = 3|\mathbf{V}_{an}||\mathbf{I}_{na}|\cos(\phi)$
  - $P_{3\Phi} = \sqrt{3}|\mathbf{V}_\ell||\mathbf{I}_\ell|\cos(\phi)$
  - V_ℓ: line-line voltage
  - I_ℓ: line current
- Depending on the source/load configuration either

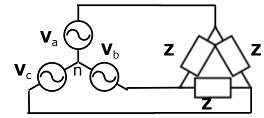
$|\mathbf{V}_\ell| = \sqrt{3}|V_a|$ and $|\mathbf{I}_\ell| = |\mathbf{I}_a|$

or

$|\mathbf{V}_\ell| = |\mathbf{V}_a|$ and $|\mathbf{I}_\ell| = \sqrt{3}|\mathbf{I}_a|$

Example

- Let the line-line voltage be 480
- Let **Z** = 6 + 12j
- The voltage coil of wattmeter 1 is place from a phase to b phase and its current coil is connected to a phase
- The voltage coil of wattmeter 2 is place from c phase to b phase and its current coil is connected to c phase
- Find P_1, P_2 and the total power to the load



Example

- First find the line currents for a-phase and c-phase (let $\mathbf{V}_{an}$ be the reference)

$$\mathbf{I}_{na} = \frac{480/\sqrt{3}}{2+4j} = 27.7 - j55.42 = 61.97\angle -63.4^\circ$$

$$\mathbf{I}_{nc} = 61.97\angle 56.6^\circ$$

Example

- Power read by wattmeter 1 is:

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 480\angle 30^\circ$$

$$P_1 = \text{Re}\{(480\angle 30^\circ)(61.97\angle -63.4^\circ)^*\} = -1782\text{W}$$

- Power read by wattmeter 2 is:

$$\mathbf{V}_{cb} = \mathbf{V}_{cn} - \mathbf{V}_{bn} = 480\angle 90^\circ$$

$$P_2 = \text{Re}\{(480\angle 90^\circ)(61.97\angle 56.6^\circ)^*\} = -24,822\text{W}$$

- Total power is:

$$P_{3\Phi} = P_1 + P_2 = 23,040\text{W}$$

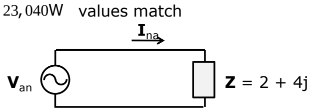
Example

- We can verify by examining the per-phase equivalent:

$$\mathbf{I}_a = \frac{277\angle 0^\circ}{2+4j} = 61.9\angle -63.4^\circ$$

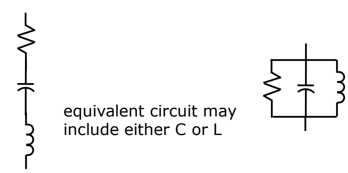
$$P_a = \text{Re}\{(277\angle 0^\circ)(61.9\angle -63.4^\circ)^*\} = 7680\text{W}$$

$P_{3\Phi} = 3 \times 7680 = 23,040\text{W}$ values match



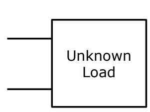
Impedance Measurements

- We are interested in modeling the impedance of unknown loads as well as machines
- Want to develop an equivalent circuit model
  - Series model
  - Parallel model



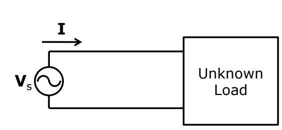
Impedance Measurements

- Consider a single-phase load with unknown components
  - Assume we know the rated voltage and frequency
- How should be determine its equivalent series or parallel equivalent?



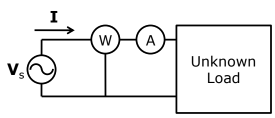
Impedance Measurements

- Connect an AC source of known rated voltage and frequency
- Measure the current and power (we already know voltage)



Impedance Measurements

- Need to know if current leads or lags the voltage
  - Use a two-channel oscilloscope (not shown)



Impedance Measurements

- We know P, |**V**|, |**I**|, ω and if the load is leading or lagging
- We can compute:

  $|\mathbf{S}| = |\mathbf{V}||\mathbf{I}|$

  $\phi = \cos^{-1}(P/|\mathbf{S}|)$ (if current lags voltage)

  $\phi = -\cos^{-1}(P/|\mathbf{S}|)$ (if current leads voltage)
- Assuming voltage is taken as reference

  $\mathbf{V} = |\mathbf{V}_s|\angle 0^\circ$

Impedance Measurements

- To determine the equivalent series impedance

  $\mathbf{Z}_s = \frac{\mathbf{V}}{\mathbf{I}} = R_S \pm jX_S$ (need to know **I**)

  $\mathbf{Z}_s$: equivalent series impedance (Ohms)

  R_s: equivalent series resistance (Ohms)

  X_s: equivalent series reactance (Ohms)
- |**I**| is measured by the ammeter, how do we find **I**?

Impedance Measurements

- Note that the current phasor:

  $\mathbf{I} = |\mathbf{I}|\angle \pm\phi = |\mathbf{I}|\cos\phi \pm j|\mathbf{I}|\sin\phi$

  can be resolved into

  $\mathbf{I} = i_r \pm j i_x$

  through substitution:

  $i_r = |\mathbf{I}|\cos\phi$

  $i_x = |\mathbf{I}|\sin\phi$

  Note: i_r and i_x are not phasors

Impedance Measurements

- The plus/minus in the previous slide is set based upon the observed phase between voltage and current from the oscilloscope
- If the current leads the voltage, what should the sign be?

  $\mathbf{I} = |\mathbf{I}|\cos\phi \pm j|\mathbf{I}|\sin\phi$

Impedance Measurements

- The plus/minus in the previous slide is set based upon the observed phase between voltage and current from the oscilloscope
- If the current leads the voltage, what should the sign be?

  $\mathbf{I} = |\mathbf{I}|\cos\phi \pm j|\mathbf{I}|\sin\phi$

  Positive.

  A capacitive circuit

  ELI the ICE man

Impedance Measurements

- Comparing:

  $\mathbf{Z}_s = \frac{\mathbf{V}}{\mathbf{I}} = R_S \pm jX_S$ and $\mathbf{I} = i_r \pm ji_x$
- We can solve:

$$\mathbf{Z}_s = \frac{\mathbf{V}}{\mathbf{I}} = R_S \pm jX_S$$
$$R_S = \text{Re}\{\mathbf{V} / \mathbf{I}\}$$
$$X_S = \text{Im}\{\mathbf{V} / \mathbf{I}\}$$
$$L = \frac{X_S}{\omega}$$

Summary

- $|\mathbf{V}|$: given by applied voltage source
- ω: given by applied voltage source
- θ_V: set to 0 degrees
- P: measured by wattmeter
- $|\mathbf{I}|$: measured by ammeter
- $|\mathbf{S}|$: computed
- ϕ: magnitude computed, sign determined by oscilloscope
- $\mathbf{Z}_s = \frac{\mathbf{V}}{\mathbf{I}} = R_S \pm jX_S$

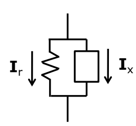
$$R_S = \text{Re}\{\mathbf{V} / \mathbf{I}\}$$
$$X_S = \text{Im}\{\mathbf{V} / \mathbf{I}\}$$
$$L = \frac{X_S}{\omega}$$

$$\begin{cases} |\mathbf{S}| = |\mathbf{V}||\mathbf{I}| \\ \phi = \cos^{-1}(P/|\mathbf{S}|) \\ \phi = -\cos^{-1}(P/|\mathbf{S}|) \end{cases}$$

Impedance Measurements

- In a parallel circuit the real portion of the current must flow through R, and the reactive through X
- Since **V** is applied in parallel to both:

$$R_p = \frac{|\mathbf{V}|}{i_r}$$
$$X_p = \frac{|\mathbf{V}|}{i_X}$$
$$C = \frac{1}{X_p \omega}$$



Impedance Measurements

- See text Example 1.9 for how these concepts are applied to three phase loads

Summary

- Voltage and current magnitude and phase information is needed to compute complex power
- Measuring and computing impedances are useful in testing electrical machines