07-Array Operations Part 2

text: Chapter 3.4-3.7

ECEGR 101
Engineering Problem Solving with Matlab
Professor Henry Louie

Overview

- Identity Matrix
- Matrix Inversion
- Linear Systems of Equations
- Element-by-Element Operations

Identity Matrix

- Identity matrix, I, is the matrix equivalent of scalar multiplication by 1

$$AI = A = IA$$

- I is simply a square matrix (number of rows = number of columns) with 1s on the diagonal

Identity Matrix

```
>> x = [1 10 100 1000; 2 30 400 5000]
x =
        1       10      100     1000
        2       30      400     5000
>>
>> I = eye(4)
I =
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
>>
>> z = x*I
z =
        1       10      100     1000
        2       30      400     5000
```

Built-in function to create the Identify Matrix
4 is the size of the matrix

Inverse of a Matrix

- Matrix B is an inverse of A if A*B=B*A=I.
- To find an inverse of A, use the inv function.

A and B have to be square.
Notation: A^{-1}.

```
>> A = [ 4 5 6 1;2 3 3 5; -1 0 2 3; -2 -2 9 0]
A =
    4    5    6    1
    2    3    3    5
   -1    0    2    3
   -2   -2    9    0
>>
>> B = inv(A)
B =
   -0.48171    1.0183   -1.5366    0.32317
    0.59146   -0.90854    1.3171   -0.38415
    0.02439    0.02439   -0.04878   0.097561
   -0.17683    0.32317   -0.14634   0.042683
```


Inverse of a Matrix

```
>> A*B
ans =
           1 -1.6653e-016  3.3307e-016 -2.7756e-017
 1.1102e-016           1 -2.2204e-016  8.3267e-017
           0           0           1 -2.7756e-017
-2.7756e-017  5.5511e-017 -1.1102e-016           1
```

```
>> format short
>> A*B
ans =
    1.0000   -0.0000    0.0000   -0.0000
    0.0000    1.0000   -0.0000    0.0000
         0         0    1.0000   -0.0000
   -0.0000    0.0000   -0.0000    1.0000
```

Inverse of a Matrix: Determinant

Conditions for the existence of an inverse:

- A has to be square,
- Its determinant must be nonzero.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \det(A) = a_{11}a_{22} - a_{12}a_{21}$$

To calculate the determinant, use the MATLAB function det.

```
>> x = [1 4; 5 3]
x =
     1     4
     5     3
>>
>> det(x)
ans =
   -17
```


Inverse of a Matrix: Determinant

```
>> det(x)
ans =
     0
>>
>> inv(x)
Warning: Matrix is singular to working precision.
ans =
   Inf   Inf
   Inf   Inf
```

```
>> g = [ 1 3 -4; 2 9 0; 3 3 5]
g =
     1     3    -4
     2     9     0
     3     3     5
>>
>> inv(g)
ans =
    0.4545   -0.2727    0.3636
   -0.1010    0.1717   -0.0808
   -0.2121    0.0606    0.0303
```

System of Linear Equations

$$A_{11}x_1 + A_{12}x_2 + A_{12}x_3 + A_{14}x_4 = B_1$$
$$A_{21}x_1 + A_{22}x_2 + A_{22}x_3 + A_{24}x_4 = B_2$$
$$A_{31}x_1 + A_{32}x_2 + A_{32}x_3 + A_{34}x_4 = B_3$$
$$A_{41}x_1 + A_{42}x_2 + A_{42}x_3 + A_{44}x_4 = B_4$$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \qquad AX = B$$

System of Linear Eqs.

How can we solve AX=B ?

$A^{-1}AX=A^{-1}B$
$IX=A^{-1}B$
$X=A^{-1}B$ ← solution

equivalent to X = A\B

```
>> X = inv(A)*B
X =
     2
    -2
     1
```

```
>> A = [1 3 4; 2 4 8; 4 4 8]
A =
     1     3     4
     2     4     8
     4     4     8
>>
>> B = [0 4 8]'
B =
     0
     4
     8
>>
>> X = A\B
X =
     2
    -2
     1
>>
>> A*X
ans =
     0
     4
     8
```

System of Linear Eqs.

How can we solve XC=D ?

$XCC^{-1}=DC^{-1}$
$XI=DC^{-1}$
$X=DC^{-1}$ ← solution

equivalent to X = D/C

```
>> C = [1 3 4; 2 4 8; 4 4 8]
C =
     1     3     4
     2     4     8
     4     4     8
>>
>> D = [35 43 84]
D =
    35    43    84
>>
>> X = D/C
X =
     1     3     7
```

Used infrequently!

```
>> X = D*inv(C)
X =
     1     3     7
```


Exercise

Solve the following system of linear equations:

$x_1 + 2x_2 = 21$
$3x_1 + 5x_2 = 53$

Exercise

Solve the following system of linear equations:

$$x_1 + 2x_2 = 21$$
$$3x_1 + 5x_2 = 53$$

```
>> A = [1 2; 3 5]
A =
     1     2
     3     5
```

```
>> B = [21; 53]
B =
    21
    53
```

```
>> X = A\B
X =
    1.0000
   10.0000
```

```
>> X = inv(A)*B
X =
    1.0000
   10.0000
```


Element-By-Element Operations

Carried out on each array element separately:

multiplication: .*
exponent: .^
right division: ./
left division: .\

```
>> x = [1 3 7; 2 4 6]
x =
     1     3     7
     2     4     6
>>
>> y = x.^2
y =
     1     9    49
     4    16    36
```

```
>> x1 = [1 3 7; 2 4 6]
x1 =
     1     3     7
     2     4     6
>>
>> x2 = [1 2 3; 3 2 1]
x2 =
     1     2     3
     3     2     1
```

```
>> y = x1.*x2
y =
     1     6    21
     6     8     6
```

Element-By-Element Operations

```
>> x = [2 4 6; 8 10 12]
x =
    2    4    6
    8   10   12
>>
>> y = [2 2 3; 2 5 4]
y =
    2    2    3
    2    5    4
>>
>> z = x./y
z =
    1    2    2
    4    2    3
>>
>> w = x.\y
w =
   1.0000    0.5000    0.5000
   0.2500    0.5000    0.3333
```

```
>> x = 1:10
x =
    1    2    3    4    5    6    7    8    9   10
>>
>> y = x.^3-9
y =
   -8   -1   18   55  116  207  334  503  720  991
```

Calculates values of the function $y=x^3-9$ for integer x between 1 and 10.

Exercise

Create the following arrays:

x = [1 2 3 4 5]; and y = [10 10 10 10 10];

What operation do you have to perform to get as the result [10 20 30 40 50]?

Exercise

Create the following arrays:

x = [1 2 3 4 5]; and y = [10 10 10 10 10];

What operation do you have to perform to get as the result [10 20 30 40 50]?

```
>> x.*y
ans =
 10    20    30    40    50
```


Exercise

Assume that a, b, c, and d are defined as follows:

```
>> a = [1 0; 2 1]
a =
    1    0
    2    1
>>
>> b = [-1 2; 0 1]
b =
   -1    2
    0    1
>>
>> c = [3; 2]
c =
    3
    2
>>
>> d = 5
d =
    5
```

What is the result of the following expressions?

a) a + b
b) a .* b
c) a * b
d) a * c
e) a + c
f) a + d
g) a .* d
h) a * d

Exercise

Assume that a, b, c, and d are defined as follows:

```
>> a = [1 0; 2 1]
a =
    1    0
    2    1
>>
>> b = [-1 2; 0 1]
b =
   -1    2
    0    1
>>
>> c = [3; 2]
c =
    3
    2
>>
>> d = 5
d =
    5
```

What is the result of the following expressions?
a) a + b

```
>> a + b
ans =
    0    2
    2    2
```


Exercise

Assume that a, b, c, and d are defined as follows:

```
>> a = [1 0; 2 1]
a =
    1    0
    2    1
>>
>> b = [-1 2; 0 1]
b =
   -1    2
    0    1
>>
>> c = [3; 2]
c =
    3
    2
>>
>> d = 5
d =
    5
```

What is the result of the following expressions?
b) a .* b

```
>> a .* b
ans =
   -1    0
    0    1
```

Exercise

Assume that a, b, c, and d are defined as follows:

```
>> a = [1 0; 2 1]
a =
    1    0
    2    1
>>
>> b = [-1 2; 0 1]
b =
   -1    2
    0    1
>>
>> c = [3; 2]
c =
    3
    2
>>
>> d = 5
d =
    5
```

What is the result of the following expressions?
c) a * b

```
>> a * b
ans =
   -1    2
   -2    5
```


Exercise

Assume that a, b, c, and d are defined as follows:

```
>> a = [1 0; 2 1]
a =
    1    0
    2    1
>>
>> b = [-1 2; 0 1]
b =
   -1    2
    0    1
>>
>> c = [3; 2]
c =
    3
    2
>>
>> d = 5
d =
    5
```

What is the result of the following expressions?
d) a * c

```
>> a * c
ans =
    3
    8
```

Exercise

Assume that a, b, c, and d are defined as follows:

```
>> a = [1 0; 2 1]
a =
    1    0
    2    1
>>
>> b = [-1 2; 0 1]
b =
   -1    2
    0    1
>>
>> c = [3; 2]
c =
    3
    2
>>
>> d = 5
d =
    5
```

What is the result of the following expressions?

e) a + c

```
>> a + c
??? Error using ==> plus
Matrix dimensions must agree.
```


Exercise

Assume that a, b, c, and d are defined as follows:

```
>> a = [1 0; 2 1]
a =
    1    0
    2    1
>>
>> b = [-1 2; 0 1]
b =
   -1    2
    0    1
>>
>> c = [3; 2]
c =
    3
    2
>>
>> d = 5
d =
    5
```

What is the result of the following expressions?

f) a + d

```
>> a + d
ans =
    6    5
    7    6
```

Exercise

Assume that a, b, c, and d are defined as follows:

```
>> a = [1 0; 2 1]
a =
    1    0
    2    1
>>
>> b = [-1 2; 0 1]
b =
   -1    2
    0    1
>>
>> c = [3; 2]
c =
    3
    2
>>
>> d = 5
d =
    5
```

What is the result of the following expressions?

g) a .* d

```
>> a .* d
ans =
    5    0
   10    5
```


Exercise

Assume that a, b, c, and d are defined as follows:

```
>> a = [1 0; 2 1]
a =
    1    0
    2    1
>>
>> b = [-1 2; 0 1]
b =
   -1    2
    0    1
>>
>> c = [3; 2]
c =
    3
    2
>>
>> d = 5
d =
    5
```

What is the result of the following expressions?

h) a * d

Exercise

Assume that a, b, c, and d are defined as follows:

```
>> a = [1 0; 2 1]
a =
    1    0
    2    1
>>
>> b = [-1 2; 0 1]
b =
   -1    2
    0    1
>>
>> c = [3; 2]
c =
    3
    2
>>
>> d = 5
d =
    5
```

What is the result of the following expressions?

h) a * d

```
>> a * d
ans =
    5    0
   10    5
```


Exercise

If C and F are Celsius and Fahrenheit temperatures, respectively, the formula for conversion from Celsius to Fahrenheit is

F = 9C/5 + 32.

Use vector operations to compute and display the Fahrenheit equivalent of Celsius temperatures ranging from 20° to 30° in steps of 1°.

Exercise

```
>> C = 20:1:30
C =
   20   21   22   23   24   25   26   27   28   29   30


>> F = 9*C/5 + 32
F =
  Columns 1 through 7
       68       69.8       71.6       73.4       75.2
   77       78.8
  Columns 8 through 11
       80.6       82.4       84.2         86
```


Exercise

Compute z given the following values for b, c and n:

```
b = 2
c = 10
n = [1 2 3 4 5]
```

$$z = \frac{b^n}{c}$$

Exercise

Compute z given the following values for b, c and n:

```
b = 2
c = 10
n = [1 2 3 4 5]
```

$$z = \frac{b^n}{c}$$

```
>> b = 2;  c = 10;  n = 1:5;

>> z = b.^n
z =
   2    4    8   16   32

>> z = b.^n/c
z =
      0.2       0.4       0.8       1.6       3.2
```

$$b^n = 2^{[1\,2\,3\,4\,5]} = [2^1\ 2^2\ 2^3\ 2^4\ 2^5]$$

Exercise

Compute z given the following values for b, c and n:

```
b = 2
c = [1 2 3 4 5]
n = 3
```

$$z = \frac{b^n}{c}$$

Exercise

Compute z given the following values for b, c and n:

```
b = 2
c = [1 2 3 4 5]
n = 3
```

$$z = \frac{b^n}{c}$$

```
>> b = 2; c = 1:5; n = 3;

>> z = b^n./c
z =
    8.0000    4.0000    2.6667    2.0000    1.6320

>> z = b^n/c
??? Error using ==> mrdivide
Matrix dimensions must agree.
```

$$\frac{8}{[1\,2\,3\,4\,5]} = \left[\frac{8}{1}\ \frac{8}{2}\ \frac{8}{3}\ \frac{8}{4}\ \frac{8}{5}\right]$$

Exercise

Compute z given the following values for b, c and n:

```
b = [1 2 3 4 5]
c = 10
n = 3
```

$$z = \frac{b^n}{c}$$

Exercise

Compute z given the following values for b, c and n:

```
b = [1 2 3 4 5]
c = 10
n = 3
```

$$z = \frac{b^n}{c}$$

```
>> b = 1:5; c = 10; n = 3;

>> b.^n
ans =
   1    8   27   64  125

>> z = b.^n/c
z =
      0.1       0.8       2.7       6.4      12.5
```

$$[1\,2\,3\,4\,5]^3 = [1^3\ 2^3\ 3^3\ 4^3\ 5^3]$$