06-Three Phase Analysis

ECEGR 450
Electromechanical Energy Conversion

Overview

- Three Phase Power
- Three Phase Analysis
- Per Phase Analysis
- Practical Considerations

Questions

- How does three phase power compare to single phase power?
- What is the "neutral" connection in a three phase system?

Conventions & Assumptions

- Voltage: line-to-line in rms
- Current: line in rms
- Current direction: source to load
- Balanced three phase

Three Phase Power

Benefits of three phase power:

- efficient use of conductors over three, single phases
- rotating field is needed for some loads (eg three phase motors)
- effective for power transfer
- per-phase analysis can be used in many cases
- Constant power delivery to three phase loads

Three Phase Power

- Total power delivered is the sum of power delivered by each phase

$\mathbf{S}_{3\phi} = \mathbf{V}_{an}\mathbf{I}^{*}_{na} + \mathbf{V}_{bn}\mathbf{I}^{*}_{nb} + \mathbf{V}_{cn}\mathbf{I}^{*}_{nc}$

$= 3\mathbf{V}_{an}\mathbf{I}^{*}_{na}$ (due to symmetry)

$= 3\mathbf{S}$

- where
  - $\mathbf{S}_{3\phi}$: total three phase complex power (VA)
  - $\mathbf{S}$: single phase complex power (VA)

Three Phase Power

Similarly

$$P_{3\phi} = \mathrm{Re}\{\mathbf{V}_{an}\mathbf{I}^*_{na}\} + \mathrm{Re}\{\mathbf{V}_{bn}\mathbf{I}^*_{nb}\} + \mathrm{Re}\{\mathbf{V}_{cn}\mathbf{I}^*_{nc}\}$$
$$= 3\mathrm{Re}\{\mathbf{V}_{an}\mathbf{I}^*_{na}\}$$
$$= 3P$$

and

$$Q_{3\phi} = \mathrm{Im}\{\mathbf{V}_{an}\mathbf{I}^*_{na}\} + \mathrm{Im}\{\mathbf{V}_{bn}\mathbf{I}^*_{nb}\} + \mathrm{Im}\{\mathbf{V}_{cn}\mathbf{I}^*_{nc}\}$$
$$= 3\mathrm{Im}\{\mathbf{V}_{an}\mathbf{I}^*_{na}\}$$
$$= 3Q$$

Three Phase Power

What about instantaneous power?

$$p_{3\phi}(t) = i_{max}v_{max}\cos(\omega t + 0^\circ)\cos(\omega t + \theta_{ia}{}^\circ) + i_{max}v_{max}\cos(\omega t - 120^\circ)\cos(\omega t + \theta_{ia}{}^\circ - 120^\circ) + i_{max}v_{max}\cos(\omega t + 120^\circ)\cos(\omega t + \theta_{ia}{}^\circ + 120^\circ)$$

$$p_{3\phi}(t) = \frac{1}{2}v_{max}i_{max}[\cos(-\theta_{ia}) + \cos(2\omega t + \theta_i) + \cos(-\theta_{ia}) + \cos(2\omega t + \theta_i - 120^\circ) + \cos(-\theta_{ia}) + \cos(2\omega t + \theta_i + 120^\circ)]$$

Next use: $\cos(x + y) = \cos x \cos y - \sin x \sin y$

Three Phase Power

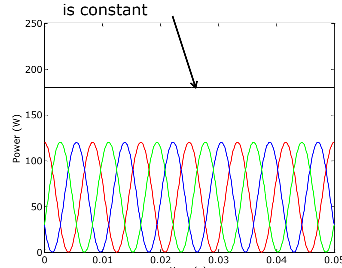
$$p_{3\phi}(t) = \frac{1}{2}v_{max}i_{max}[3\cos(-\theta_{ia}) + \cos(2\omega t + \theta_i) + \cos(2\omega t + \theta_i)\cos(-120^\circ) - \sin(2\omega t + \theta_i)\sin(-120^\circ) + \cos(2\omega t + \theta_i)\cos(120^\circ) - \sin(2\omega t + \theta_i)\sin(120^\circ)]$$

$$p_{3\phi}(t) = \frac{1}{2}v_{max}i_{max}[3\cos(-\theta_{ia}) + \cos(2\omega t + \theta_i) + \frac{-1}{2}\cos(2\omega t + \theta_i) - \frac{-\sqrt{3}}{2}\sin(2\omega t + \theta_i) + \frac{-1}{2}\cos(2\omega t + \theta_i) - \frac{\sqrt{3}}{2}\sin(2\omega t + \theta_i)]$$

$$p_{3\phi}(t) = \frac{3}{2}v_{max}i_{max}\cos(\phi)$$

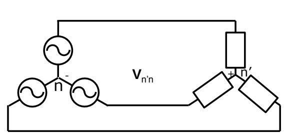
What does this say about the real power?

Three Phase Power



Three-Phase Analysis

- Consider the Y-connected source and load
- Determine $\mathbf{V}_{n'n}$



Three-Phase Analysis

- Analysis is easier using admittance, **Y**

$$\mathbf{Y} = \frac{1}{\mathbf{Z}}$$

- Line current is equal to phase current

$$\mathbf{I}_{na} = \mathbf{Y}(\mathbf{V}_{an} - \mathbf{V}_{nn'})$$
$$\mathbf{I}_{nb} = \mathbf{Y}(\mathbf{V}_{bn} - \mathbf{V}_{nn'})$$
$$\mathbf{I}_{nc} = \mathbf{Y}(\mathbf{V}_{cn} - \mathbf{V}_{nn'})$$

due to wye connection

Three-Phase Analysis

- Summing the line current

$$\mathbf{I}_{na} + \mathbf{I}_{nb} + \mathbf{I}_{nc} = \mathbf{Y}(\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn}) - 3\mathbf{Y}\mathbf{V}_{nn'}$$

- Previously we showed that

$$\mathbf{I}_{na} + \mathbf{I}_{nb} + \mathbf{I}_{nc} = 0$$

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

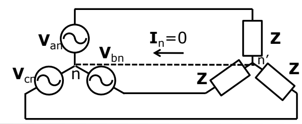
- Therefore

$$0 = \mathbf{Y}(0) - 3\mathbf{Y}\mathbf{V}_{nn'}$$

$$\Rightarrow \mathbf{V}_{nn'} = 0$$

Three-Phase Analysis

- Since $\mathbf{V}_{nn}' = 0$, we can make a hypothetical connection without affecting the circuit
- Called the Neutral Conductor
- For balanced sources and loads, no current flows on the neutral conductor

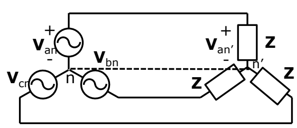


Three-Phase Analysis

Let's now analyze the current through the a-phase load

$$\mathbf{V}_{an} = \mathbf{V}_{an'}$$

$$\mathbf{I}_{na} = \frac{\mathbf{V}_{an'}}{\mathbf{Z}}$$

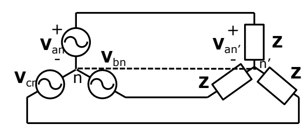


Three-Phase Analysis

Let's now analyze the current through the b-phase load

$$\mathbf{V}_{bn} = \mathbf{V}_{bn'}$$

$$\mathbf{I}_{nb} = \frac{\mathbf{V}_{bn'}}{\mathbf{Z}}$$



But

$$\mathbf{V}_{bn} = \mathbf{V}_{an}(1\angle -120^\circ)$$

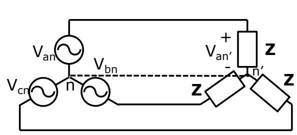
$$\mathbf{I}_{nb} = \frac{\mathbf{V}_{bn'}}{\mathbf{Z}} = \frac{\mathbf{V}_{an}(1\angle -120^\circ)}{\mathbf{Z}} = \mathbf{I}_{na}(1\angle -120^\circ)$$ (b-phase current in terms of a-phase current)

Three-Phase Analysis

- Let's now analyze the current through the c-phase load

$$\mathbf{V}_{cn} = \mathbf{V}_{cn'}$$

$$\mathbf{I}_{nc} = \frac{\mathbf{V}_{cn'}}{\mathbf{Z}}$$



- But

$$\mathbf{V}_{cn} = \mathbf{V}_{an}(1\angle 120^\circ)$$

$$\mathbf{I}_{nc} = \frac{\mathbf{V}_{cn'}}{\mathbf{Z}} = \frac{\mathbf{V}_{an}(1\angle 120^\circ)}{\mathbf{Z}} = \mathbf{I}_{na}(1\angle 120^\circ)$$ (c-phase current in terms of a-phase current)

Three-Phase Analysis

- Once $\mathbf{I}_{na}$ is solved for, simply shift it by -/+120 degrees to find $\mathbf{I}_{nb}$ and $\mathbf{I}_{nc}$
- Phases can be conceptually decoupled
- No need to solve all three phases
  - Solve for a-phase (current or voltage)
  - Shift +120° for c-phase, and -120° for b-phase
- We can therefore do a per-phase analysis

Three-Phase Analysis

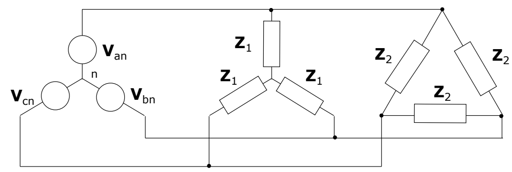
- Balanced Three-Phase Theorem
- Assume:
  - balanced three-phase system
  - all loads and sources are Y-connected (or convert them to Y-connected loads/sources)
  - no mutual inductances between phases

then

  - all neutrals have the same voltage
  - the phases are completely decoupled
  - all corresponding network variables occur in balanced sets of the same sequence as the sources

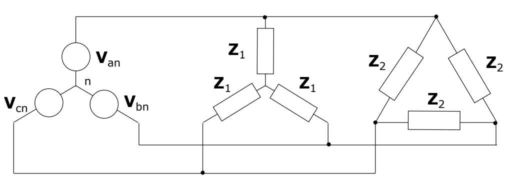
Per-Phase Analysis

- Consider the following circuit
- Is it balanced?



Per-Phase Analysis

- To analyze this circuit, we first need to find a per-phase equivalent
- Need to transform the delta load to Y load



Per-Phase Analysis

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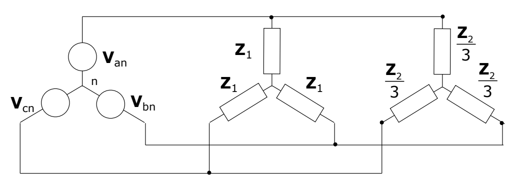
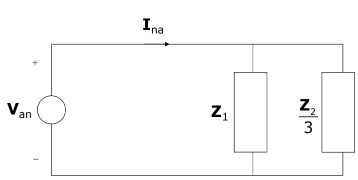
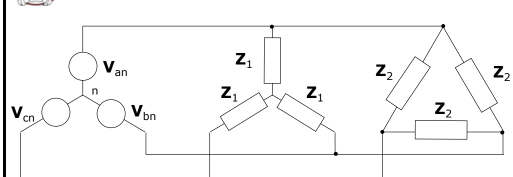


Per-Phase Analysis

- Analyze a single phase (arbitrarily a-phase)
- Analyze this phase to find the current, power, etc



Numerical Example



given:

balanced 3-phase source

$\mathbf{V}_{an} = 120\angle 0$ V

$\mathbf{Z}_1 = 10 + j2\ \Omega$

$\mathbf{Z}_2 = 12\angle 45^\circ\ \Omega$

find:

$\mathbf{I}_{na}$, $\mathbf{I}_{nb}$, $\mathbf{I}_{nc}$

total powered delivered to the loads

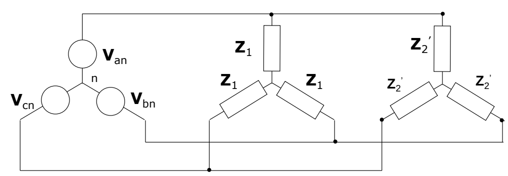
Example

Solution outline
- draw per-phase equivalent
- solve circuit for current
- compute single phase power
- translate per-phase values to 3-phase

Example

- Convert Δ to Y
- Redraw circuit

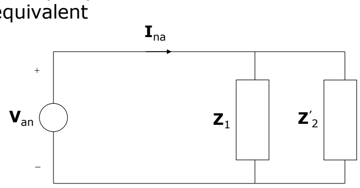
$\mathbf{Z}'_2 = 2.82 + j2.82\ \Omega$



Example

- Redraw circuit
- Draw per-phase equivalent

$\mathbf{Z}'_2 = 2.82 + j2.82\ \Omega$



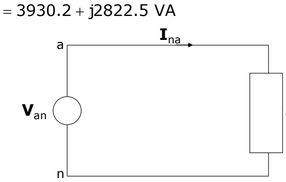
Example

- Draw per-phase equivalent
- Solve for $\mathbf{I}_{an}$ and $\mathbf{S}$

$\mathbf{V}_{an} = 120\angle 0^\circ$ V

$\mathbf{I}_{na} = 40.32\angle -35^\circ$ A

$\mathbf{S} = 3930.2 + j2822.5$ VA



$\mathbf{Z}_{\parallel} = \frac{\mathbf{Z}_1\mathbf{Z}'_2}{\mathbf{Z}_1 + \mathbf{Z}'_2} = 2.417 + j1.736\Omega$

Example

- Shifting and solving

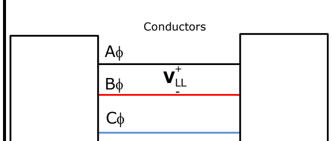
$\mathbf{I}_{nb} = 40.32\angle\ -35^\circ - 120^\circ\ = 40.32\angle\ -155^\circ$ A

$\mathbf{I}_{nc} = 40.32\angle\ -35^\circ + 120^\circ\ = 40.32\angle\ 85^\circ$ A

$\mathbf{I}_n = 0$A

$\mathbf{S}_{3\Phi} = 3\ \ 3930.19 + j2822.51\ \ = 11790.57 + j8467.53$VA

Practical Considerations



$\mathbf{V}_{LL} = \mathbf{V}_{phase}(\sqrt{3}\angle 30^\circ)$

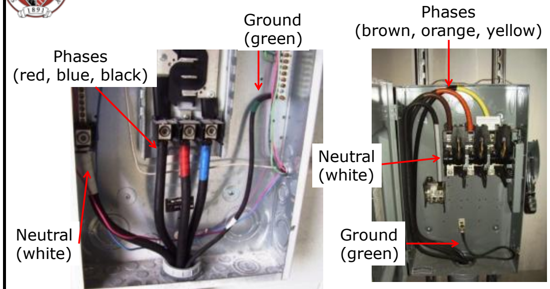
Three phase systems and components often referred to by RMS line-line voltages e.g. 500kV, 13.6kV, 480V

Three phase systems and components often referred to by $|\mathbf{S}_{3\phi}|$ e.g. 100 MVA transformer

Practical Considerations

- Neutral usually bonded to ground at service panel (see NEC for details)
- Balanced load assumption can be questionable
  - Neutral current nonzero
  - Neutral not at ground potential

Practical Considerations



Source: www.electrical-design-tutor.com/

Summary

- Current (voltages) sum to zero and no current flows on neutral conductor
- Total **S**, P, Q = three times single phase **S**, P, Q
- Three phase systems analysis: convert into wye connections, solve per-phase