06-Basic Laws Part 3

Text: Chapter 2.5-2.8

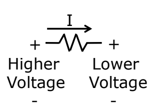
ECEGR 210
Electric Circuits I

Overview

- Voltage Drop
- Voltage Divider
- Series Resistors
- Current Divider
- Parallel Resistors
- Delta-Wye Conversion

Voltage Drop

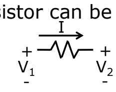
- The voltage difference across the terminals of a resistor is called the "voltage drop"
  - If the current is negative, then it is a voltage rise
- Voltage drop is caused by the current flowing the resistor and by Ohm's Law is equal to I x R



Voltage Drop

- The current through a resistor can be quickly found by:

$$I = \frac{V_1 - V_2}{R}$$



  - Where V_1 and V_2 are referenced to the same node, and $V_1 > V_2$

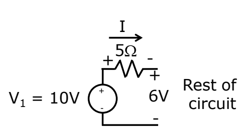
Voltage Drop

- For example, the current through the resistor can be computed as:

$$0 = -V_1 + V_R + V_2$$

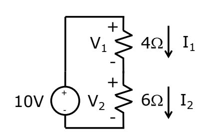
$$0 = -V_1 + IR + V_2$$

$$I = \frac{V_1 - V_2}{R} = \frac{4}{5} = 0.8A$$



Multi-Resistor Circuits

- Now back to multi-resistor circuits
- What is the voltage across each resistor?
  - KVL: $10 = V_1 + V_2$
  - KCL: $I_1 = I_2$
  - Ohm's Law:
    - $V_1 = 4I_1$
    - $V_2 = 6I_2$



Multi-Resistor Circuits

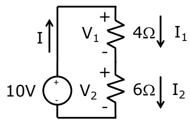
- Using
  - $10 = V_1 + V_2$
  - $I_1 = I_2 = I$
  - $V_1 = 4I_1$
  - $V_2 = 6I_2$
- Gives

$$10 = 4I + 6I$$

$$I = \frac{10}{4+6}$$

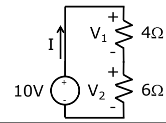
$$V_1 = 10\frac{4}{4+6} = 4V$$

$$V_2 = 10\frac{6}{4+6} = 6V$$



Multi-Resistor Circuits

- From this simple example, note:
  - Resistors are in series
  - Proportion of applied voltage across a resistor, is the same proportion of the resistor to the total resistance
  - R_1: <u>40%</u> of total resistance and <u>40%</u> of the voltage (V_1 = 4V)
  - R_2: <u>60%</u> of total resistance and <u>60%</u> of the voltage (V_2 =6V)



Voltage Divider

- The voltage across each resistor is

$$V_1 = \frac{R_1}{R_1 + R_2}V$$

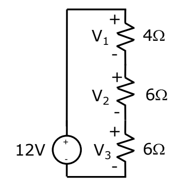
$$V_2 = \frac{R_2}{R_1 + R_2}V$$

- More generally, the voltage across the nth resistor of N resistors in series is:

$$V_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N}V$$

Example

- Find V_1, V_2, V_3



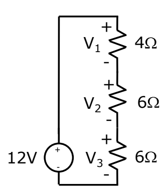
Example

- Find V_1, V_2, V_3

$$V_1 = 12\frac{4}{4+6+6} = 3V$$

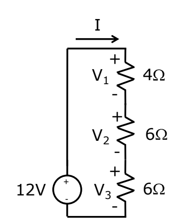
$$V_2 = 12\frac{6}{16} = 4.5V$$

$$V_3 = 12\frac{6}{16} = 4.5V$$



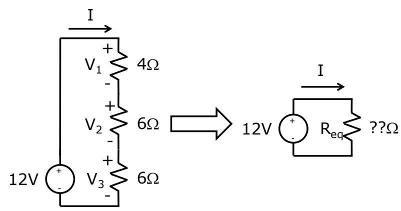
Series Resistances

- What is the current?
  - V_1 = 3V = 4I
    - I = 0.75A
  - V_2 = 4.5V = 6I
    - I = 0.75A
  - V_3 = 4.5V = 6I
    - I = 0.75A



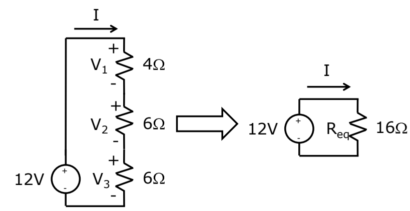
Series Resistances

- Find an equivalent circuit with one resistor (R_{eq}) so that the same current flows



Series Resistances

- By Ohm's Law: $12 = 0.75R_{eq}$
  - $R_{eq} = 16\Omega$
- Note $R_{eq} = 4 + 6 + 6 = 16\Omega$



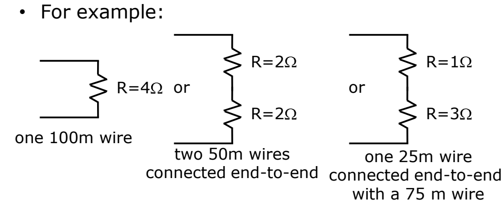
Series Resistances

- More generally: the equivalent resistance (R_{eq}) of any number (N) of resistors in series is the sum of the individual resistances (R_n)

$$R_{eq} = \sum_{n=1}^{N} R_n$$

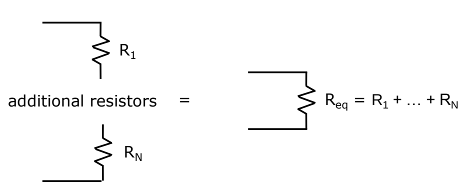
Series Resistances

- Consider a 100 m wire whose resistivity is 1 Ohm per 25m.
- We can model this in many ways
- For example:



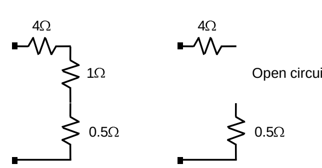
Series Resistances

- The equivalent resistance of series connected resistors is the sum of their resistances



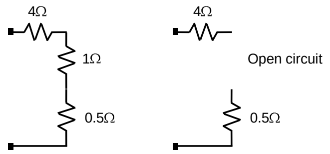
Example

- Compute the equivalent resistance between the terminals of the following circuits



Example

- Compute the equivalent resistance between the terminals of the following circuits



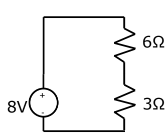
5.5Ω   infinite

Series Resistance

- When adding resistances in series the overall resistance increases
- Less current flows than if there is only one resistance

Example

- Find the voltage across each resistor, and the power supplied by the voltage source.

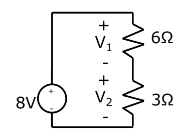


Example

- Find the voltage across each resistor, and the power supplied by the voltage source.

$$V_1 = 8\frac{3}{9} = 5.33V$$

$$V_2 = 8\frac{6}{9} = 2.67V$$



Example

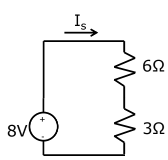
- Find the voltage across each resistor, and the power supplied by the voltage source.

$$P_s = \frac{V_1^2}{R_1} + \frac{V_2^2}{R_2} = 7.12W$$

or

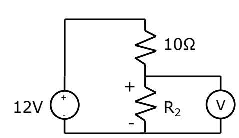
$$I_s = \frac{8}{9} = 0.89A$$

$$P_s = V_s I = 7.12W$$



Example

- The voltmeter reads 2V. What is R_2?



Example

- The voltmeter reads 2V. What is R_2?

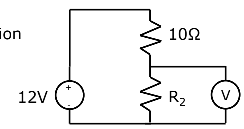
$V = 2$

$2 = 12\frac{R_2}{10 + R_2}$

$10 + R_2 = 6R_2$

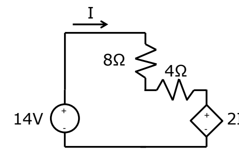
$R_2 = 2\Omega$

or by inspection



Example

- Find I, and the voltage across the dependent voltage source.



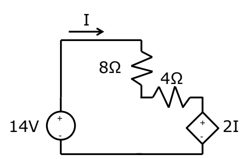
Example

- First find I, then the voltage across the independent source V_d

$14 = 12I + V_d = 14I$

$I = 1A$

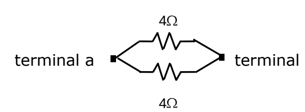
$V_d = 2V$



Parallel Resistance

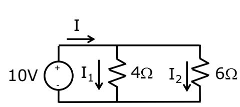
- Assume two 4Ω wires are connected as

- What is the circuit model of this?



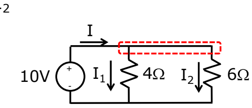
Current Divider

- Find the fraction of I that flows through each resistor



Current Divider

- By KCL
  - $I = I_1 + I_2$
- By KVL
  - $10 = V_1 = V_2$
- By Ohm's Law
  - $V_1 = 4I_1$
  - $V_2 = 6I_2$



Current Divider

- Combining
  - $I = I_1 + I_2$
  - $10 = V_1 = V_2$
  - $V_1 = 4I_1$
  - $V_2 = 6I_2$
- Yields

$$I_2 = I_1 \frac{4}{6}$$

$$I = I_1 + \frac{4}{6} I_1 = I_1 \frac{6+4}{6}$$

$$I_1 = \frac{6}{6+4} I$$

$$I_2 = \frac{4}{6+4} I$$

Describes how current divides

Current Divider

- Note:

$$I_1 = \frac{6}{6+4} I = \frac{R_2}{R_1+R_2} I = 0.6I$$

$$I_2 = \frac{4}{6+4} = \frac{R_1}{R_1+R_2} I = 0.4I$$

- If we divide the numerators and denominators by R_1R_2, then:

$$I_1 = \frac{1/R_1}{1/R_1 + 1/R_2} I = \frac{G_1}{G_1+G_2} I$$

$$I_2 = \frac{1/R_2}{1/R_1 + 1/R_2} I = \frac{G_2}{G_1+G_2} I$$

Current Divider

- From this simple example:
  - Resistors are in parallel
  - Greater current flows through the resistor with lower resistance
  - Proportion of current through a resistor is proportional to the conductance of that resistor to the total conductance
    - 60 percent of total through the 4 Ohm resistor
    - 40 percent of total through 6 Ohm resistor

Parallel Resistance

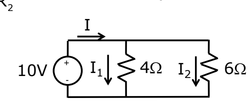
- We now know how the current divides, but how much current flows?

$$I = I_1 + I_2$$

$$I = 10\frac{1}{R_1} + 10\frac{1}{R_2} = 10(\frac{1}{R_1} + \frac{1}{R_2}) = 4.17A$$

Therefore

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = G_1 + G_2 = 2.4\Omega \quad \text{(sum of conductances)}$$

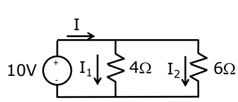


Parallel Resistance

- Note:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1R_2}{R_1+R_2}$$

- The equivalent resistance of <u>two</u> parallel resistors is their product divided by their sum



Current Divider

- The equivalent resistance of N resistors in parallel is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = G_1 + G_2 + \cdots + G_N = G_{eq}$$

- The current through the nth resistor of N resistors in parallel is:

$$I_n = \frac{1/R_n}{1/R_1 + 1/R_2 + \cdots + 1/R_N} I$$

$$= \frac{G_n}{G_1 + G_2 + \cdots + G_N} I$$

Parallel Resistance

- When adding resistances in parallel the equivalent resistance decreases
  - Resistance always be <u>less than the smallest individual resistance</u>
- More total current flows
- Symbol "||" is used in an equation to designate a parallel relationship between elements
- Example: $R_1||R_2$

Current Divider

- The equivalent resistance is always smaller than the smallest individual resistance
- Example: a 100 Ohm, 10 Ohm and 1 Ohm resistors are placed in parallel. What is the equivalent resistance?

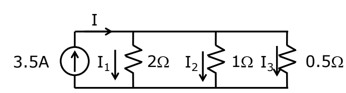
Current Divider

- The equivalent resistance is always smaller than the smallest individual resistance
- Example: a 100 Ohm, 10 Ohm and 1 Ohm resistors are placed in parallel. What is the equivalent resistance?

$$\frac{1}{R_{eq}} = 0.01 + 0.1 + 1$$
$$R_{eq} = 0.901\Omega$$

Example

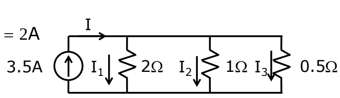
- Find the current through each resistor, and the power supplied by the current source



Example

- Find the current through each resistor, and the power supplied by the current source

$$I_1 = \frac{1/2}{1/2 + 1/1 + 1/0.5} I = \frac{0.5}{0.5 + 1 + 2} 3.5 = 0.5A$$
$$I_2 = \frac{1/1}{3.5} I = 1A$$
$$I_3 = \frac{1/0.5}{3.5} I = 2A$$



Example

- Find the current through each resistor, and the power supplied by the current source

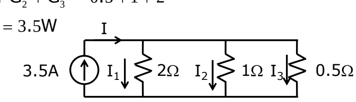
$V = I_1 R_1 = 0.5x2 = 1$

$P = IV = 3.5W$

or

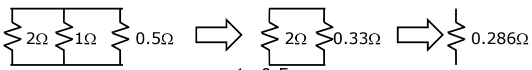
$$R_{eq} = \frac{1}{G_1 + G_2 + G_3} = \frac{1}{0.5 + 1 + 2} = 0.286\Omega$$

$P = I^2 R_{eq} = 3.5W$



Parallel Circuits

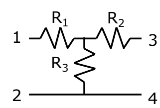
- It may be easier to combine parallel resistors two at a time (easy to compute product-over-sum)
- Example:



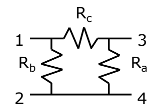
$\frac{1 \times 0.5}{1 + 1.5} = 0.33\Omega$ $\frac{2 \times 0.33}{2 + 0.33} = 0.286\Omega$

Delta-Wye Circuits

- It is possible for elements to be arranged such that they are not in series or parallel
- Examples



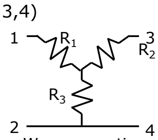
Tee connection



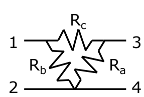
Π connection

Delta-Wye Circuits

- Often interested in converting a Delta (Π) to a Wye (Tee) or vice versa
- Interested in terminal characteristics
  - Convert from Delta to Wye (or vice versa) so that same resistance is seen from terminals (1,2) and (3,4)



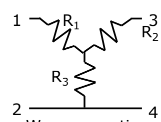
Wye connection



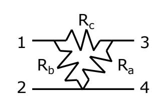
Delta connection

Delta-Wye Circuits

- Resistance between terminals 1,2 in Wye is:
  - $R_{12} = R_1 + R_3$
- In Delta it is:
  - $R_{12} = R_b || (R_a + R_b)$



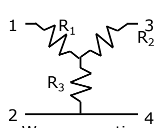
Wye connection



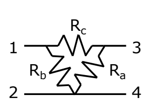
Delta connection

Delta-Wye Circuits

- Setting them equal and solving…
  - $R_b || (R_a + R_b) = R_1 + R_3$
- Repeating for terminals 1,3 and 3,4 gives
  - $R_c || (R_c + R_b) = R_1 + R_2$
  - $R_a || (R_c + R_b) = R_2 + R_3$



Wye connection



Delta connection

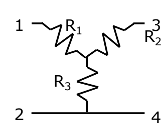
Delta-Wye Circuits

- Solving R_1, R_2 and R_3 in terms of R_a, R_b, R_c by combining equations

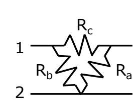
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



Wye connection



Delta connection

Product of adjacent divided by sum or resistors (see Fig. 2.49)

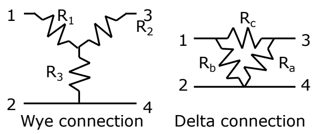
Delta-Wye Circuits

- Solving R_a, R_b and R_c in terms of R_1, R_2, R_3 by combining equations

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$



Sum of products of all combinations of resistors taken two at a time divided by the opposite Wye resistor (see Fig. 2.49)

Delta-Wye Circuits

- How are the equations reduced when $R_1 = R_2 = R_3$ or $R_a = R_b = R_c$?

$$R_Y = \frac{R_\Delta}{3}$$

$$R_\Delta = 3R_Y$$

- This occurs in balanced three phase networks