06-Array Operations Part 1
text: Chapter 3.1-3.3

ECEGR 101
Engineering Problem Solving with Matlab
Professor Henry Louie

Overview

- Addition and Subtraction
- Matrix Multiplication
- Matrix Inversion

Array Operations in MATLAB

There are two types of operations between arrays in MATLAB:

- Element-by-element operations
  - Operations carried out on each of the element of the array(s).
- Matrix operations
  - Operations that follow the rules of linear algebra.

Addition and Subtraction

- Sum two (or more) matrices element-by-element.
- The matrices have to be of the same size.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{bmatrix}$$

$$A + B = \begin{bmatrix} A_{11} + B_{11} & A_{12} + B_{12} \\ A_{21} + B_{21} & A_{22} + B_{22} \\ A_{31} + B_{31} & A_{32} + B_{32} \end{bmatrix}$$

Addition and Subtraction

```
>> x = 1:6;
>> y = [2 3 5 6 3 4];
>>
>> z = x+y

z =

    3    5    8   10    8   10
```

```
>> a1 = [2 -4 5 9; 1 1 1 1]
a1 =
    2   -4    5    9
    1    1    1    1
>>
>> a2 = [1 0 -1 1; 1 2 -4 0]
a2 =
    1    0   -1    1
    1    2   -4    0
>>
>> a3 = a1 - a2
a3 =
    1   -4    6    8
    0   -1    5    1
```

```
>> s = [1:5; 3 4 5 2 2; 3 -3 3 -3 0]
s =
    1    2    3    4    5
    3    4    5    2    2
    3   -3    3   -3    0
>>
>> s = s + 10
s =
   11   12   13   14   15
   13   14   15   12   12
   13    7   13    7   10
```


Array Multiplication (Dot Product)

- Let A and B be arrays. A*B is a valid expression if the number of columns in matrix A is equal to the number of rows in matrix B.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \end{bmatrix}$$

$$A = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} & A_{11}B_{13} + A_{12}B_{23} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} & A_{21}B_{13} + A_{22}B_{23} \\ A_{31}B_{11} + A_{32}B_{21} & A_{31}B_{12} + A_{32}B_{22} & A_{31}B_{13} + A_{32}B_{23} \\ A_{41}B_{11} + A_{42}B_{21} & A_{41}B_{12} + A_{42}B_{22} & A_{41}B_{13} + A_{42}B_{23} \end{bmatrix}$$

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Array Multiplication (Dot Product)

- Consider two matrices A and B of dimensions (p x q) and (r x s), respectively
  - p: number of rows in matrix A
  - q: number of columns in matrix A
  - r: number of rows in matrix B
  - S: number of columns in matrix B
- Dot product is valid if and only if q = r
- Result is matrix C with dimension (p x s)
  - (p x q) * (q x s), (p x s)

```
>> x = [1 2 3; 4 5 6; 7 8 9]
x =
     1     2     3
     4     5     6
     7     8     9
>>
>> y = [1 1; 2 2; 3 3]
y =
     1     1
     2     2
     3     3
>>
>> z = x*y
z =
    14    14
    32    32
    50    50
```

For matrices, $A * B \neq B * A$

```
>> y*x
??? Error using ==> mtimes
Inner matrix dimensions must agree.
```



```
>> a = 1:10
a =
     1     2     3     4     5     6     7     8     9    10
>>
>> b = ones(10,1)
b =
     1
     1
     1
     1
     1
     1
     1
     1
     1
     1
>>
>> c = a*b
c =
    55
```

```
>> c = b*a
c =
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
     1     2     3     4     5     6     7     8     9    10
```

```
>> x1 = ones(3,3)
x1 =
     1     1     1
     1     1     1
     1     1     1
>>
>> x2 = [1 1 2; 2 2 3; 3 3 4]
x2 =
     1     1     2
     2     2     3
     3     3     4
>>
>> y = x1*x2
y =
     6     6     9
     6     6     9
     6     6     9
>>
>> y = x2*x1
y =
     4     4     4
     7     7     7
    10    10    10
```

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Matrix Multiplication by a Scalar

Each element is multiplied by the scalar

```
>> w = [1 2 1; 3 4 3]
w =
     1     2     1
     3     4     3
>>
>> q = 10*w
q =
    10    20    10
    30    40    30
```

Exercise

If matrices a and b are defined as shown below, calculate matrix c = a*b.

```
>> a = [1 0 1 0; 0 1 0 1];
>>
>> b = 4*ones(4,4);
```


Exercise

If matrices a and b are defined as shown below, calculate matrix c = a*b.

```
>> a = [1 0 1 0; 0 1 0 1];
>>
>> b = 4*ones(4,4);
```

```
>> a = [1 0 1 0; 0 1 0 1]
a =
     1     0     1     0
     0     1     0     1
>>
>> b = 4*ones(4,4)
b =
     4     4     4     4
     4     4     4     4
     4     4     4     4
     4     4     4     4
```

Exercise

If matrices a and b are defined as shown below, calculate matrix c = a*b.

```
>> a = [1 0 1 0; 0 1 0 1];
>>
>> b = 4*ones(4,4);
```

```
>> a * b
ans =
     8     8     8     8
     8     8     8     8
```

```
>> a = [1 0 1 0; 0 1 0 1]
a =
     1     0     1     0
     0     1     0     1
>>
>> b = 4*ones(4,4)
b =
     4     4     4     4
     4     4     4     4
     4     4     4     4
     4     4     4     4
```


Exercise

Create the following arrays:

x = [1 2 3 4 5];

and y = [10 10 10 10 10];

a) Why is the operation x*y not permitted?

b) Will the result of the operation (x')*y be different from x*(y')? What is the result of each operation?

Exercise

Create the two matrices:

```
>> x = [1 2 3 4 5]
x =
     1     2     3     4     5
>>
>> y = [10 10 10 10 10]
y =
    10    10    10    10    10
```


Exercise

x = [1 2 3 4 5] y = [10 10 10 10 10]

a) Why is the following operation not permitted:
x*y?

```
>> x*y
??? Error using ==> mtimes
Inner matrix dimensions must agree.
```

Exercise

x = [1 2 3 4 5] y = [10 10 10 10 10]

b) Why is the result of the operation (x')*y different from x*(y')?

```
>> (x')*y
ans =
    10    10    10    10    10
    20    20    20    20    20
    30    30    30    30    30
    40    40    40    40    40
    50    50    50    50    50
```

```
>> x*(y')
ans =
   150
```