05-Three Phase Analysis

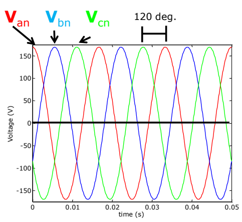
Text: 2.4 – 2.7

ECEGR 451
Power Systems

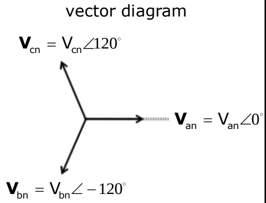
Overview

- Three-Phase Sources
- Delta and Y Connections
- Three-Phase Loads
- Three-Phase Power
- Three-Phase Analysis
- Per-Phase Analysis

Three-Phase Voltages

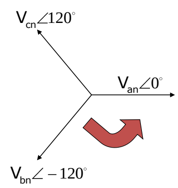


Note: instantaneous values sum to 0



Three Phase

- Power systems use 3-phase
- We are concerned with balanced 3-phase
- Balanced circuit conditions:
  - impedances are equal for each phase
  - voltage source phasors have equal magnitude and have a 120 deg. phase shift
  - a, b, c phase rotation

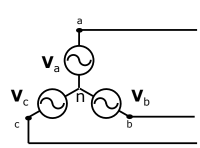


Benefits of Three-Phase Systems

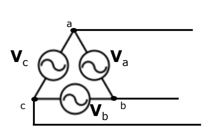
- Efficient use of conductors over three, single phases
- Rotating field is needed for some loads (eg three phase motors)
- Effective for power transfer
- Per-phase analysis can be used in many cases
- Constant power delivery to three phase loads

Three Phase

Three phase generators connected as Y or Δ



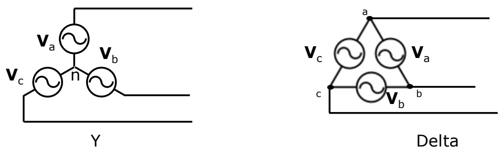
Y (Wye)



Delta

Three Phase Voltage

- Phase voltage: the voltage across each source ($\mathbf{V}_a$, $\mathbf{V}_b$ and $\mathbf{V}_c$)
- Line voltage: the voltage across the lines of the sources (also called "Line-to-Line Voltage")



Three Phase Voltage

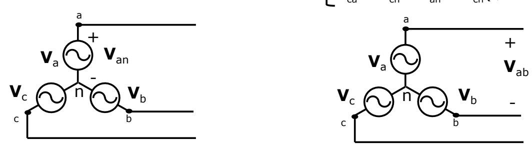
For Y-connected sources:

Phase Voltages

$$\mathbf{V}_{an} = \mathbf{V}_a$$
$$\mathbf{V}_{bn} = \mathbf{V}_b$$
$$\mathbf{V}_{cn} = \mathbf{V}_c$$

Line Voltages

by KVL
$$\begin{cases} \mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = \mathbf{V}_{an}(\sqrt{3}\angle 30^\circ) \\ \mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \mathbf{V}_{bn}(\sqrt{3}\angle 30^\circ) \\ \mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \mathbf{V}_{cn}(\sqrt{3}\angle 30^\circ) \end{cases}$$



Three Phase Voltage

For Delta-connected sources:

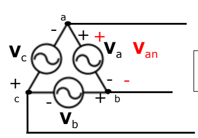
Phase Voltages

$$\mathbf{V}_{ab} = \mathbf{V}_a$$
$$\mathbf{V}_{bc} = \mathbf{V}_b$$
$$\mathbf{V}_{ca} = \mathbf{V}_c$$

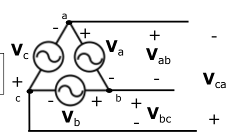
Do not confuse phase voltage with "per-phase voltage" as discussed later

Line Voltages

$$\mathbf{V}_{ab} = \mathbf{V}_a$$
$$\mathbf{V}_{bc} = \mathbf{V}_b$$
$$\mathbf{V}_{ca} = \mathbf{V}_c$$



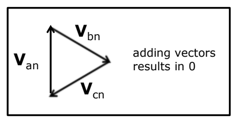
Delta-connections: phase voltages = line voltages

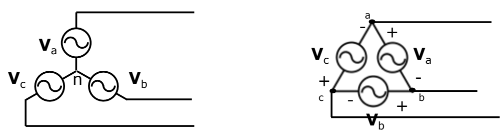


Three Phase Voltage

Voltages (line or phase) sum to zero

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0$$
$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$
$$\mathbf{V}_{ab} + \mathbf{V}_{bc} + \mathbf{V}_{ca} = 0$$



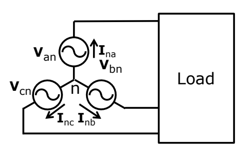


Three Phase Current

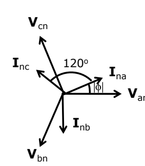
- Phase current: current flowing through the voltage sources in a three-phase circuit
- Line current: current flowing from a three-phase source to a load
- Each set of phase currents and sets of line currents are balanced:
  - Sum to zero
  - Equal in magnitude
  - Displaced by 120 degrees

Three Phase Current

Phase currents in Y-connected sources:



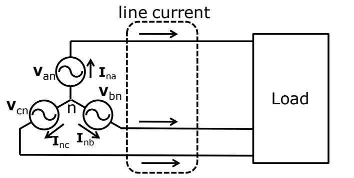
Vector Diagram



$\mathbf{V}_{an}$ is used as reference

Three Phase Current

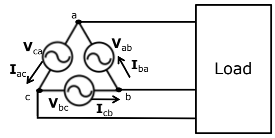
Phase currents in Y-connected sources:



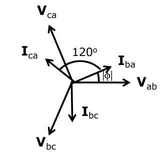
Y-connections: phase current = line current

Three Phase Current

Phase currents in Delta-connected sources:



Vector Diagram

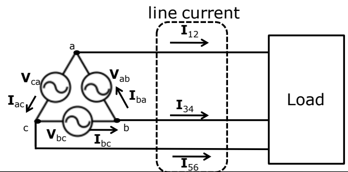


V_{ab} is used as reference

Three Phase Current

Line currents in Delta-connected sources:

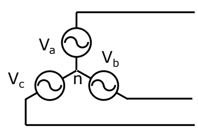
$$\left.\begin{aligned} \mathbf{I}_{12} &= \mathbf{I}_{ba} - \mathbf{I}_{ac} = \mathbf{I}_{ba}(\sqrt{3}\angle -30^\circ) \\ \mathbf{I}_{34} &= \mathbf{I}_{cb} - \mathbf{I}_{ba} = \mathbf{I}_{cb}(\sqrt{3}\angle -30^\circ) \\ \mathbf{I}_{56} &= \mathbf{I}_{ac} - \mathbf{I}_{cb} = \mathbf{I}_{ac}(\sqrt{3}\angle -30^\circ) \end{aligned}\right\} \text{KCL at nodes a, b, c}$$



Summary

Line Current = Phase Current

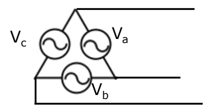
Line Voltage = Phase Voltage x $\sqrt{3}\angle 30^\circ$



Y (Wye)

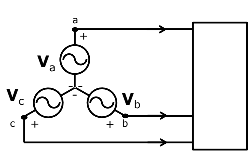
Line Current = Phase Current x $\sqrt{3}\angle -30^\circ$

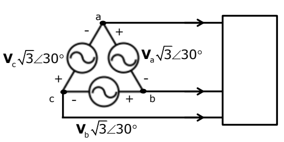
Line Voltage = Phase Voltage



Delta

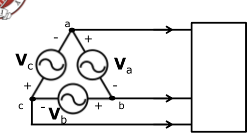
Y-Δ Source Conversion

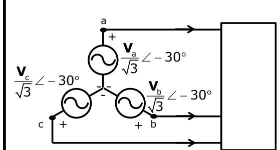




Each configuration has identical line-line voltages, line current, and complex power delivered to the load

Δ-Y Source Conversion



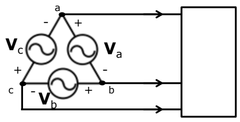


Each configuration has identical line-line voltages, line current, and complex power delivered to the load

"per-phase" voltage of a delta is the line-line voltage divided by sqrt(3)

Example

Convert the shown three-phase source to its Y equivalent given: $\mathbf{V}_a = 480\angle 0°$



Example

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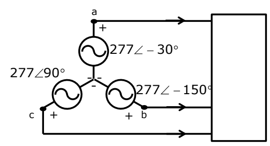

Note: usually we redefine the reference Voltage to be $\mathbf{V}_{an}$ so that $\mathbf{V}_{an} = 277\angle 0°$

Three Phase Loads

- Three phase sources are connected to three phase loads in two common configurations
  - Y (wye)
  - Delta
- Y sources can be connected to delta and/or Y loads
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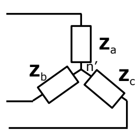
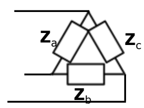


Y



Delta

Three Phase Loads

- Circuit analysis is easier if loads are connected as Y
- We can transform balanced Delta connected loads into balanced Y connected loads mathematically by

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

  - $\mathbf{Z}_Y$: complex impedance of Y-connected load (Ohms)
  - $\mathbf{Z}_\Delta$: complex impedance of a Delta-connected load (Ohms)
- Results only apply to terminal conditions

Conventions & Assumptions

- Voltage: line-to-line in rms
- Current: line in rms
- Current direction: source to load
- Balanced three phase

Three Phase Power

Total power delivered is the sum of power delivered by each phase

$$\mathbf{S}_{3\phi} = \mathbf{V}_{an}\mathbf{I}^*_{na} + \mathbf{V}_{bn}\mathbf{I}^*_{nb} + \mathbf{V}_{cn}\mathbf{I}^*_{nc}$$
$$= 3\mathbf{V}_{an}\mathbf{I}^*_{na} \quad \text{(due to symmetry)}$$
$$= 3\mathbf{S}$$

where

- $\mathbf{S}_{3\phi}$: total three phase complex power (VA)
- $\mathbf{S}$: single phase complex power (VA)

Three Phase Power

Similarly

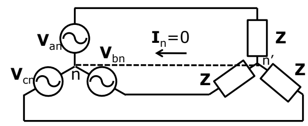
$$P_{3\phi} = \text{Re}\{\mathbf{V}_{an}\mathbf{I}^*_{na}\} + \text{Re}\{\mathbf{V}_{bn}\mathbf{I}^*_{nb}\} + \text{Re}\{\mathbf{V}_{cn}\mathbf{I}^*_{nc}\}$$
$$= 3\text{Re}\{\mathbf{V}_{an}\mathbf{I}^*_{na}\}$$
$$= 3P$$

and

$$Q_{3\phi} = \text{Im}\ \{\mathbf{V}_{an}\mathbf{I}^*_{na}\} + \text{Im}\ \{\mathbf{V}_{bn}\mathbf{I}^*_{nb}\} + \text{Im}\ \{\mathbf{V}_{cn}\mathbf{I}^*_{nc}\}$$
$$= 3\text{Im}\{\mathbf{V}_{an}\mathbf{I}^*_{na}\}$$
$$= 3Q$$

Three-Phase Analysis

- Recall that we can make a hypothetical connection between neutrals without affecting the circuit
  - Neutral Conductor
- For balanced sources and loads, no current flows on the neutral conductor



Three-Phase Analysis

- Phases can be conceptually decoupled
- No need to solve all three phases
  - Solve for a-phase (current or voltage)
  - Shift +120º for c-phase, and -120º for b-phase
- We can therefore do a per-phase analysis

Three-Phase Analysis

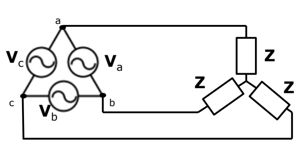
- Balanced Three-Phase Theorem
- Assume:
  - balanced three-phase system
  - all loads and sources are Y-connected (or convert them to Y-equivalent by dividing delta loads by 3 using per-phase voltage of delta sources)
  - no mutual inductances between phases

then

  - all neutrals have the same voltage
  - the phases are completely decoupled
  - all corresponding network variables occur in balanced sets of the same sequence as the sources

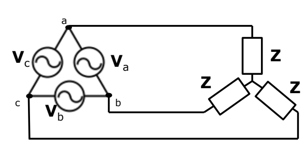
Example

Consider the following circuit where the line-line voltage is 12.47kV, and the load impedance is 1200 + 300j Ω. Determine the total complex power consumed by the load.



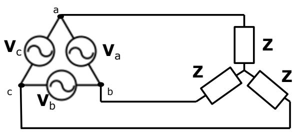
Example

1. Verify that the circuit is balanced



Example

2. Convert sources and loads to Y-connections



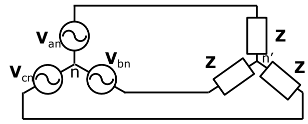
Example

2. Convert sources and loads to Y-connections

$$\mathbf{V}_{an} = \frac{12.47k}{\sqrt{3}} = 7200\angle -30^\circ V$$
$$\mathbf{V}_{bn} = \frac{12.47k}{\sqrt{3}} = 7200\angle -150^\circ V$$
$$\mathbf{V}_{cn} = \frac{12.47k}{\sqrt{3}} = 7200\angle 90^\circ V$$

shift by +30 degrees for convenience

$$\mathbf{V}_{an} = 7200\angle 0^\circ V$$
$$\mathbf{V}_{bn} = 7200\angle -120^\circ V$$
$$\mathbf{V}_{cn} = 7200\angle 120^\circ V$$



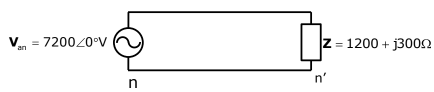
Load is already Y-connected

Example

3. Draw per-phase equivalent circuit

Example

- Virtual connection between n and n'



Example

4. Solve resulting circuit for single phase complex power and total complex power

Example

4. Solve resulting circuit for single phase complex power and total complex power

$$\mathbf{I} = \frac{\mathbf{V}_{an}}{\mathbf{Z}} = 5.82\angle -14^\circ$$
$$\mathbf{S} = \mathbf{V}_{an}\mathbf{I}^* = 40,654 + j10,163 \text{ VA}$$
$$\mathbf{S}_{3\phi} = 3\mathbf{S} = 121,960 + j30,490 \text{ VA}$$

alternatively

$$\mathbf{S} = \frac{\mathbf{V}_{an}^2}{1200} + j\frac{\mathbf{V}_{an}^2}{300} = 40,654 + j10,163 \text{ VA}$$
$$\mathbf{S}_{3\phi} = 3\mathbf{S} = 121,960 + j30,490 \text{ VA}$$