05-Basic Laws Part 2

Text: Chapter 2.4

ECEGR 210

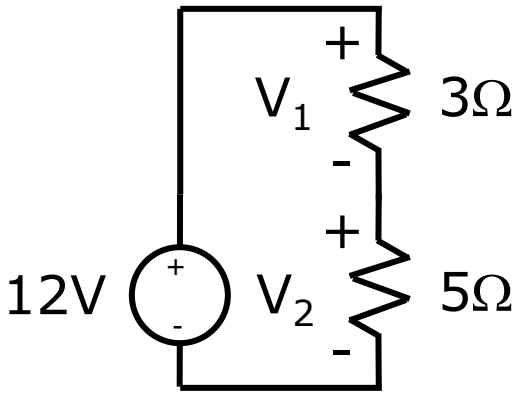
Electric Circuits I

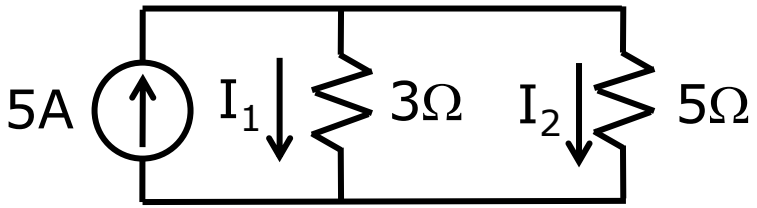
Overview

- Kirchhoff's Voltage Law
- Kirchhoff's Current Law

Introduction

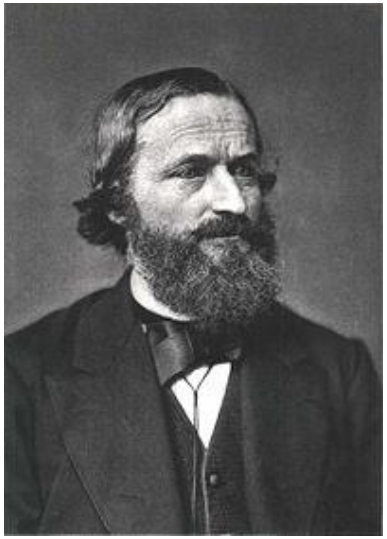
- Last lecture left off with trying to analyze circuits with two resistors
- To solve, apply Kirchhoff's Law(s)





Kirchhoff's Laws

- Named after Gustav Robert Kirchhoff
- Two laws:
  - Kirchhoff's Voltage Law (KVL)
  - Kirchhoff's Current Law (KCL)
- Both can be derived from Maxwell's Equations

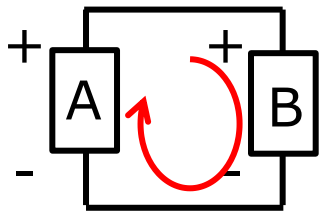


Kirchhoff's Laws

- Start with KVL
- KVL: sum of all voltages around a loop (closed path) is zero

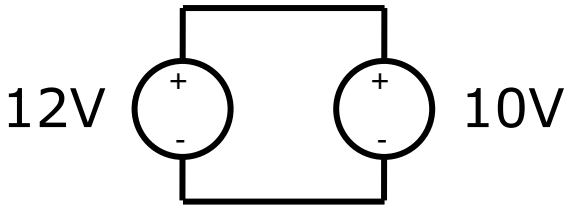
$$\sum_{m=1}^{M} v_m = 0$$

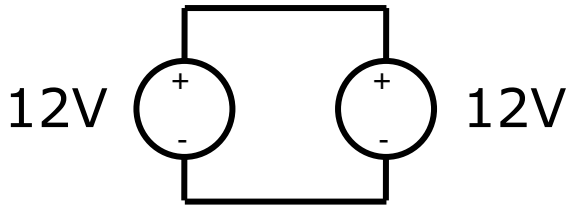
  - M: number of voltages in the loop

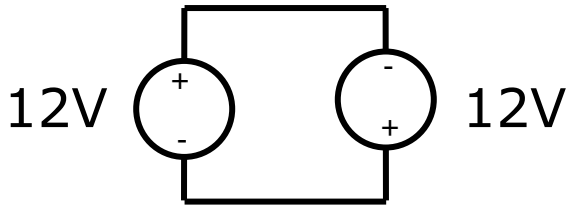


KVL

- Sum of voltage around a loop must equal zero
- Pay careful attention to polarity!
- Which circuit(s) obey Kirchhoff's Voltage Law?







KVL

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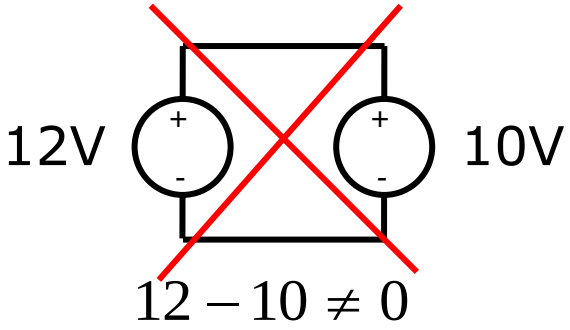
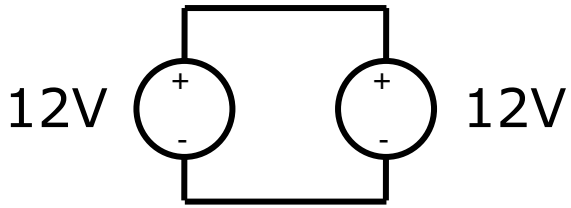
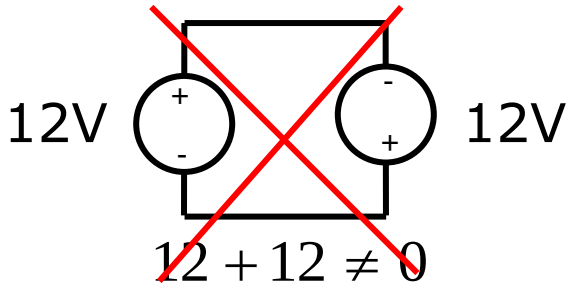


$12 - 10 \neq 0$



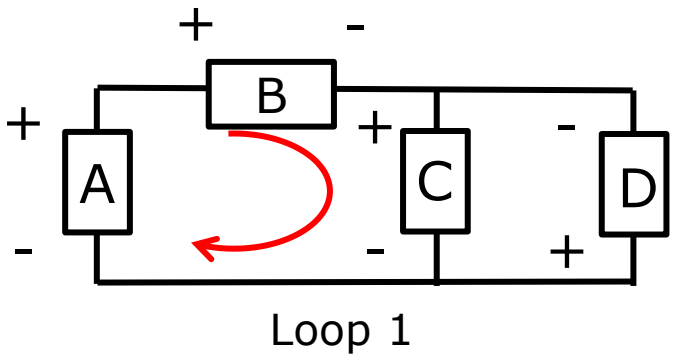
$12 - 12 = 0$



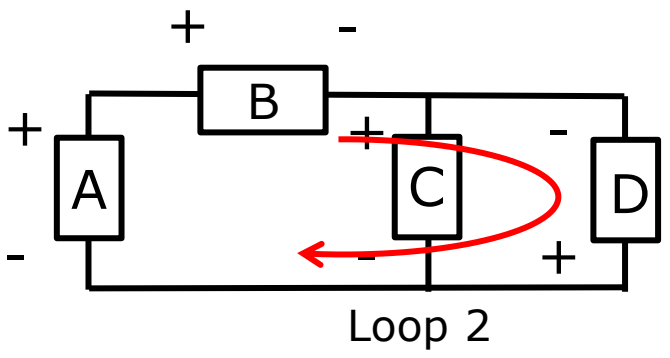
$12 + 12 \neq 0$

KVL

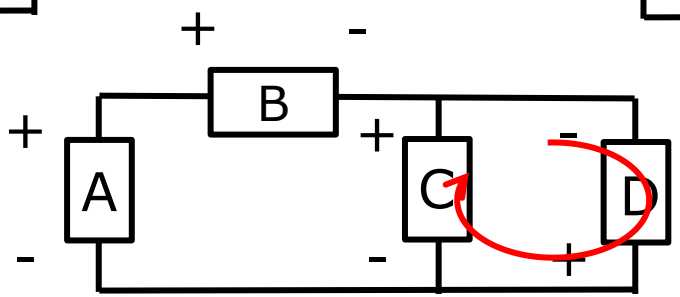
- Example
  - Loop 1: $0 = -V_A + V_B + V_C$ or $V_A = V_B + V_C$
  - Loop 2: $0 = -V_A + V_B - V_D$ or $V_A = V_B - V_D$
  - Loop 3: $0 = -V_C - V_D$ or $V_C = V_D$



Loop 1



Loop 2



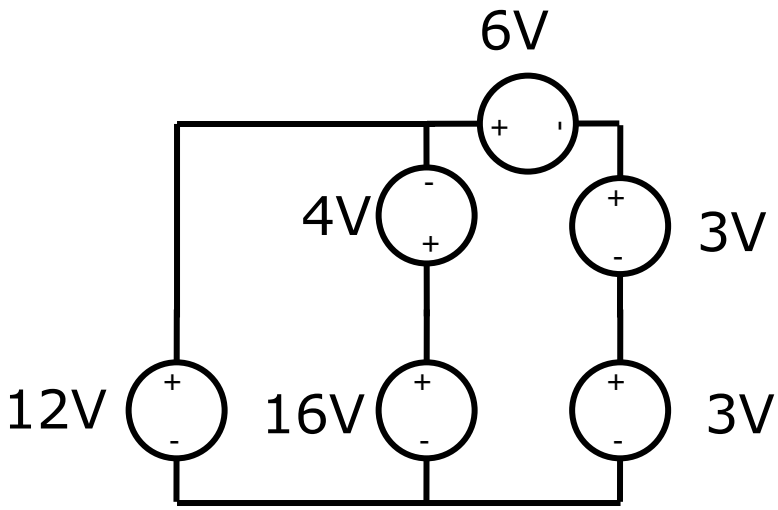
Loop 3

KVL

- Identify which loop to check
- Start at a node in the loop
- Sum voltages in loop accounting for polarity
  - If a "+" is encountered, add the voltage
  - If a "-" is encountered, subtract the voltage
- Check sum
  - If equal to 0, then KVL is obeyed
  - If not equal to 0, then there is an error in the circuit
- Repeat for additional loops until all elements considered

KVL

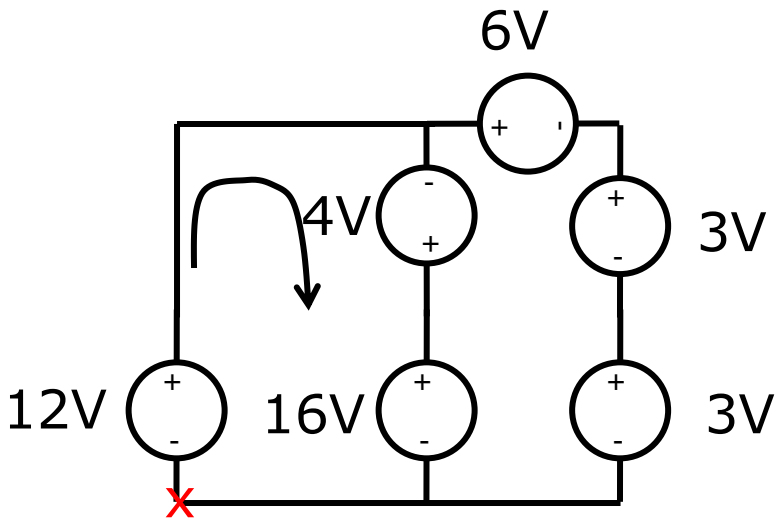
- Does this circuit obey KVL?



KVL

- Consider loop on the left

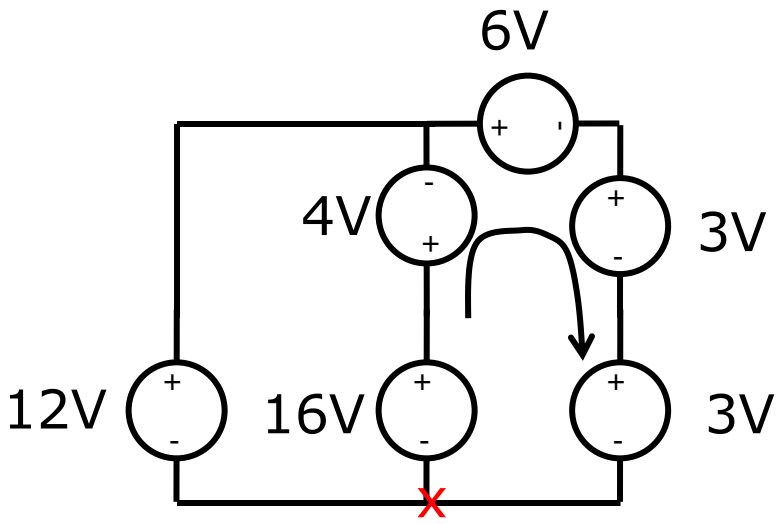
  -12 – 4 + 16 = 0 (KVL is obeyed)



KVL

- Now try right loop

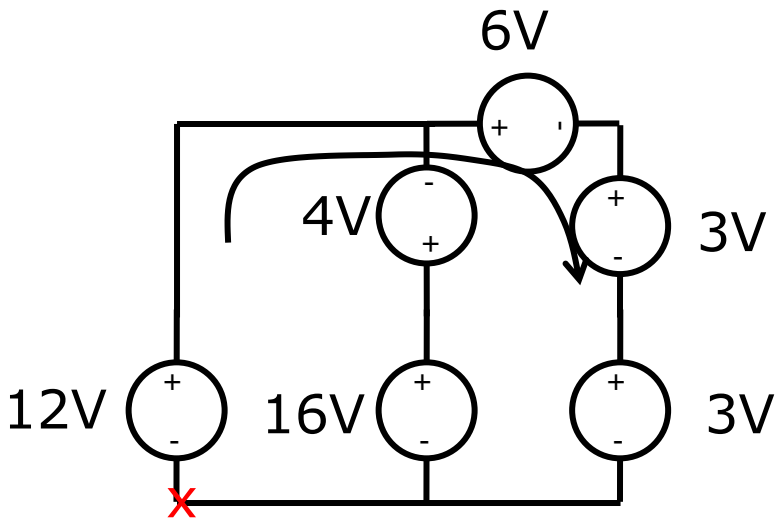
  -16 + 4 + 6 +3 + 3= 0 (KVL is obeyed)



KVL

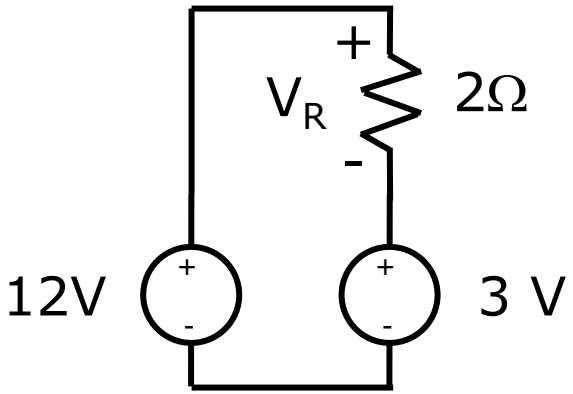
- We could have considered

  -12 + 6 + 3 + 3= 0 (KVL is obeyed)



KVL

- What must V_R be?



KVL

- What must V_R be?

  $-12 + V_R + 3 = 0$

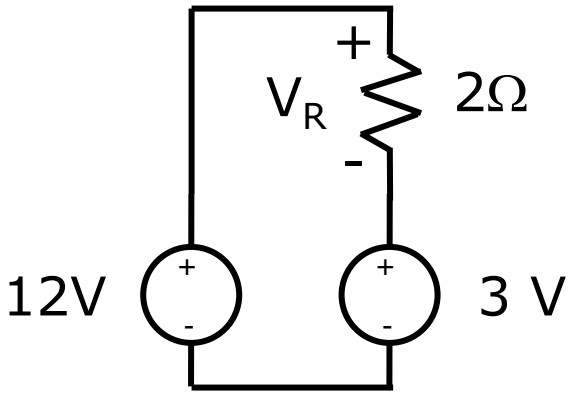
  $V_R = 9V$

- We could also compute the current using Ohm's Law (but it has added steps)

  $9 = 2I$

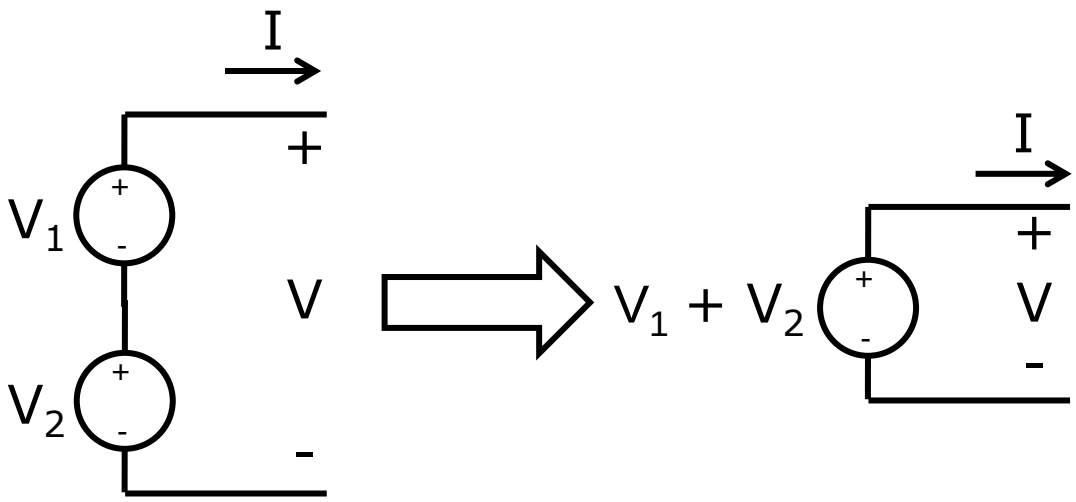
  $I = 4.5A$

  $2x4.5 = 9V$



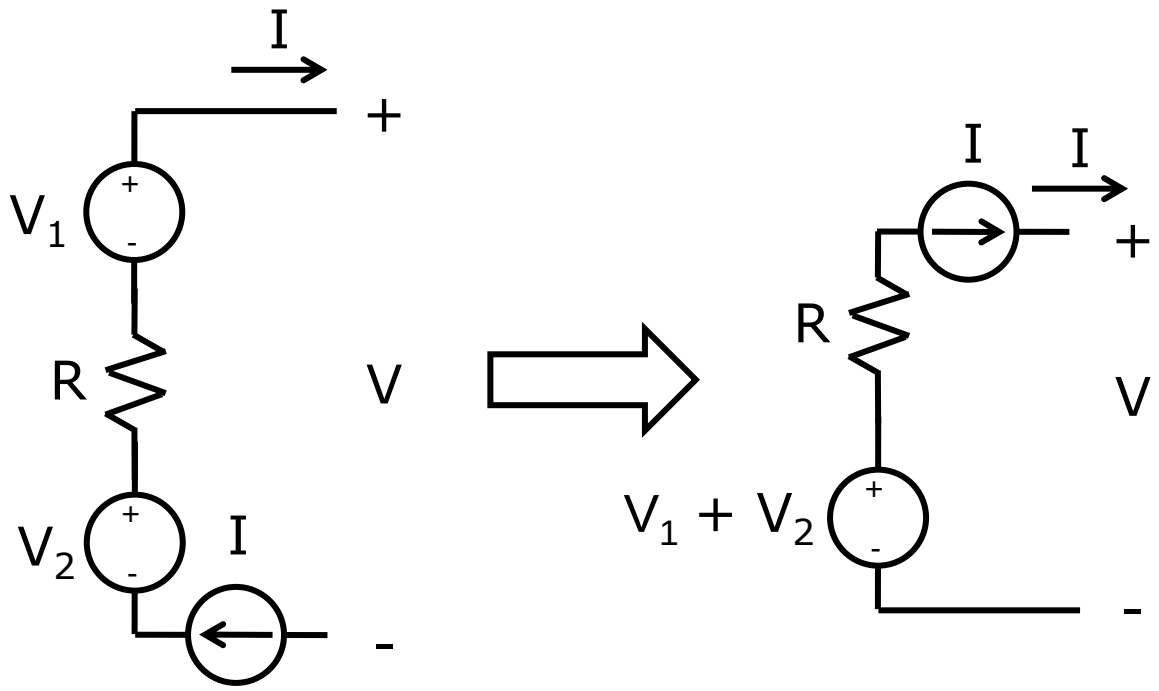
Combining Voltages

- KVL suggests that voltages in series can be simply combined



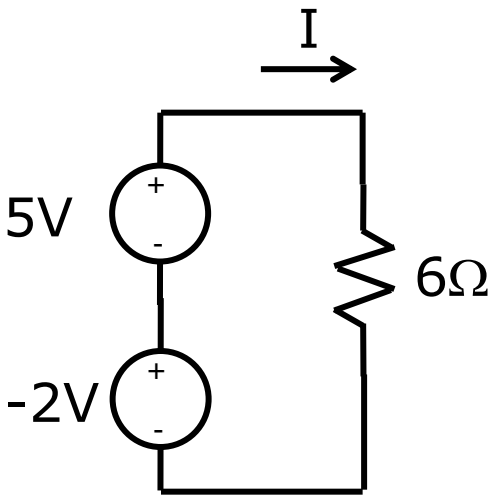
Combining Voltages

- In fact, any series elements (passive or active) can be arbitrarily rearranged
  - Polarity must not be changed



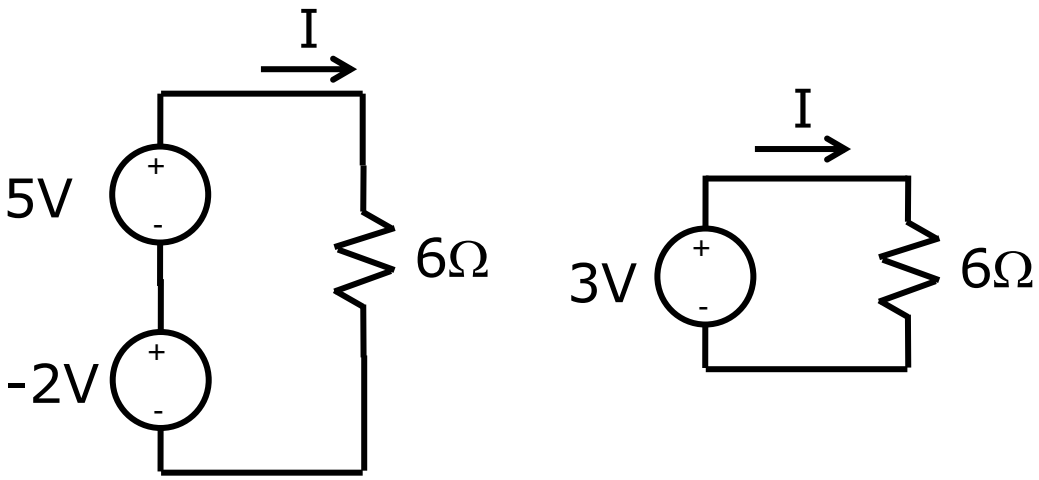
Example

- Find the current through the resistor



Example

- Find the current through the resistor
  - Combine voltage sources: 5 – 2 = 3V
  - Apply Ohm's Law: I = 3/6 = 0.5A

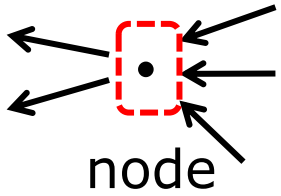


Kirchhoff's Laws

- Kirchhoff's Current Law (KCL) is analogous to Kirchhoff's Voltage Law

- KCL: sum of all currents a node (closed boundary) is zero

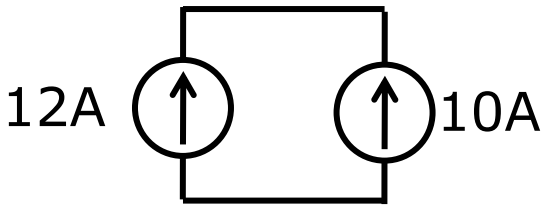
$$\sum_{n=1}^{N} i_n = 0$$

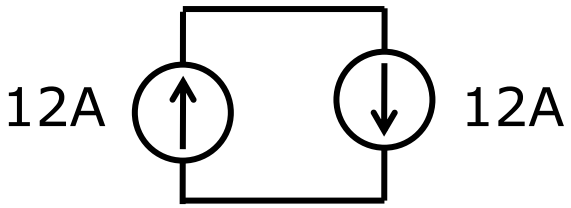


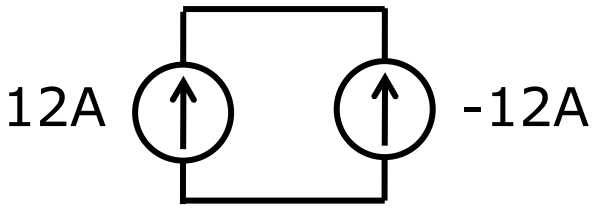
  - N: number of branches connected to the node

KCL

- Sum of currents into a node must equal zero
- Pay careful attention to polarity!
- Which circuit(s) obey KCL?







KCL

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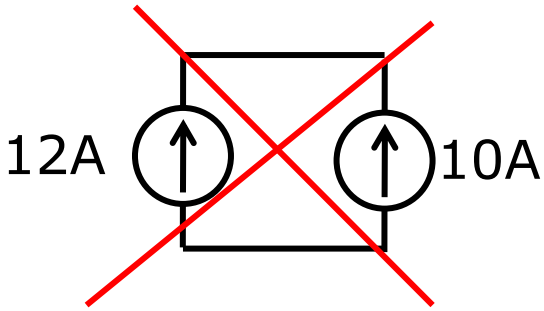
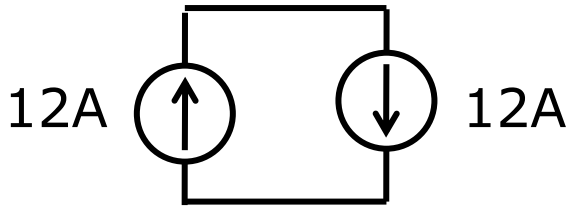
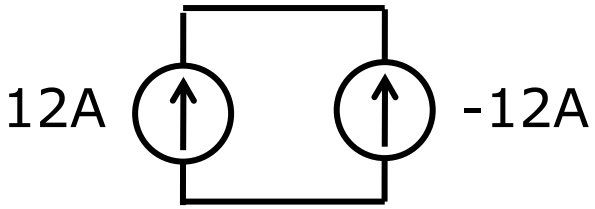
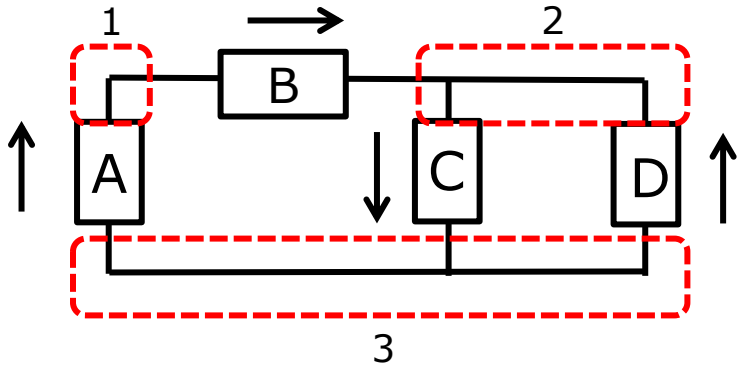






KCL

- Example
  - Node 1: $0 = I_A - I_B$ or $I_A = I_B$
  - Node 2: $0 = I_B - I_C + I_D$ or $I_C = I_B + I_D$
  - Node 3: $0 = I_C - I_A - I_D$ or $I_C = I_A + I_D$

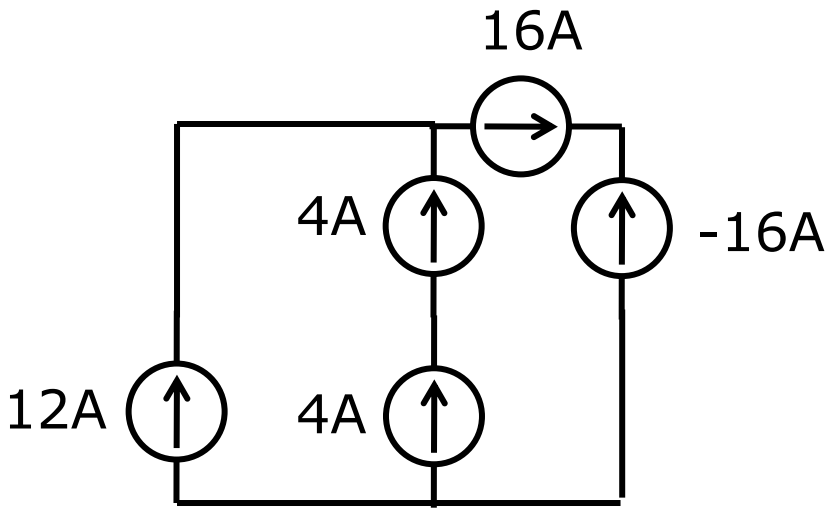


KCL

- Identify which node to check
- Sum currents into/out of node, accounting for polarity
  - If the current enters the node, the current is positive
  - If the current leaves the node, the current is negative
- Check sum
  - If equal to 0, then KCL is obeyed
  - If not equal to 0, then there is an error in the circuit
- Repeat for additional loops until all elements considered

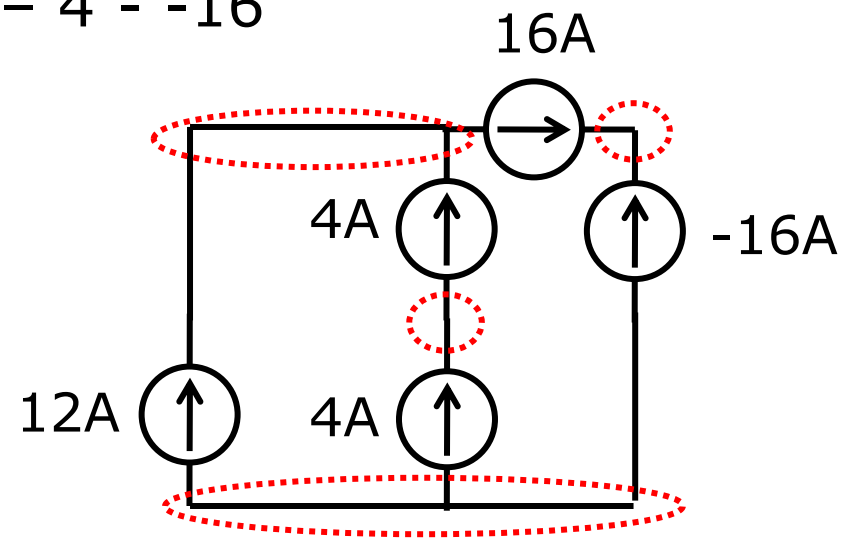
KCL

- Does this circuit obey KCL?



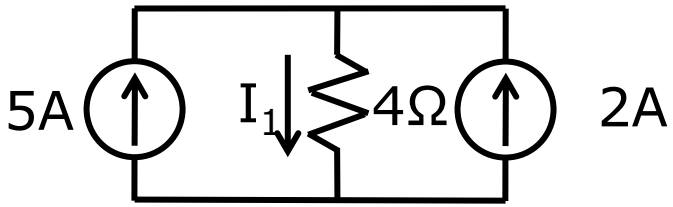
KCL

- Does this circuit obey KCL?
- Yes
  - 0 = 12 + 4 – 16
  - 0 = 16 + –16
  - 0 = 4 – 4
  - 0 = -12 – 4 - -16



KCL

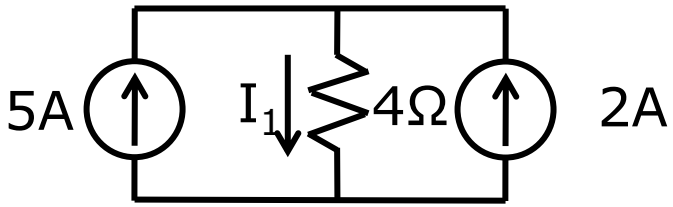
- What must I_1 be?



KCL

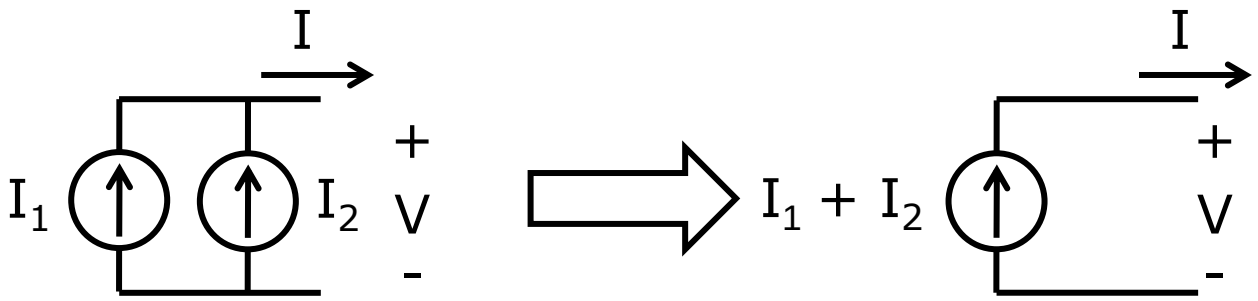
- What must I_1 be?

  $I_1 = 5 + 2 = 7A$



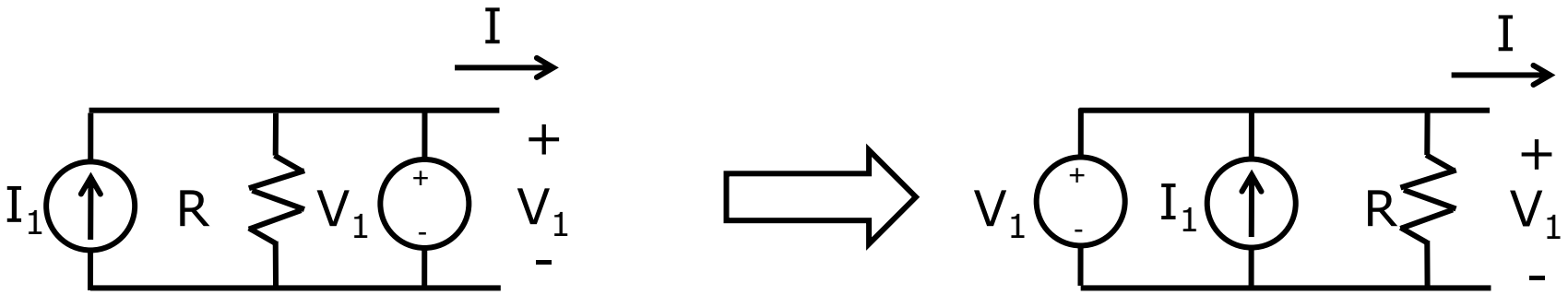
Combining Current

- KCL suggests that current sources in parallel can be simply combined



Combining Current

- In fact, parallel elements can be arbitrarily rearranged
  - Polarity must not be changed



Example

- A node has three branches. The current into the node from branch 1 is 1.5A; the current into the node from branch 2 is 3A. What is the current into the node from branch 3?

A: 1.5A

B: 4.5A

C: -1.5A

D: -4.5A

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