04-Power Factor

ECEGR 450
Electromechanical Energy Conversion

Overview

- Power Factor Definition
- Leading and Lagging Power Factor
- Power Factor Correction

Questions

- What is power factor?
- What are the real-world consequences of low power factor?

Power Factor

- Recall Power Factor:

$PF \triangleq \cos\phi$

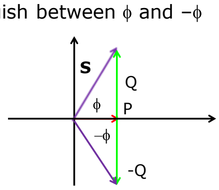
$\phi \triangleq \theta_v - \theta_i$

- Power Factor gives an indication of how much apparent power **S** is used for real work, P

$$\cos\phi = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{P}{|\mathbf{S}|}$$

Power Factor

- Power factor is non-negative
- $\cos(\phi) = \cos(-\phi)$
- Need to distinguish between ϕ and $-\phi$



Power Factor

- For example let $\theta_v = 0°$
- Case 1: $\theta_i = 30°$
  - Capacitive circuit
  - PF = 0.866
- Case 2: $\theta_i = -30°$
  - Inductive circuit
  - PF = 0.866

Same power factor

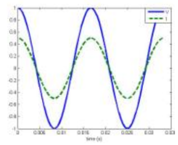
Leading/Lagging Power Factor

Must describe the PF value along with whether the current leads or lags voltage

- Lagging: current lags voltage (inductive)
- Leading: current leads voltage (capacitive)
- Useful mnemonic: ELI the ICE man

Identify as Leading, Lagging or Unity

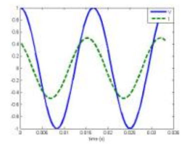
(a)



$\mathbf{V} = \frac{1}{\sqrt{2}}\angle 0^\circ$

$\mathbf{I} = \frac{0.5}{\sqrt{2}}\angle 0^\circ$

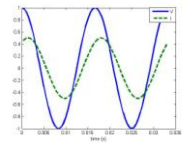
(b)



$\mathbf{V} = \frac{1}{\sqrt{2}}\angle 0^\circ$

$\mathbf{I} = \frac{0.5}{\sqrt{2}}\angle 30^\circ$

(c)



$\mathbf{V} = \frac{1}{\sqrt{2}}\angle 0^\circ$

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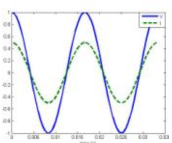
Identify as Leading, Lagging or Unity

(a)



$\mathbf{V} = \frac{1}{\sqrt{2}}\angle 0^\circ$

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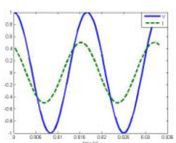
unity

(b)

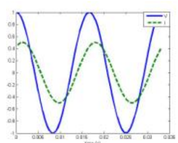


$\mathbf{V} = \frac{1}{\sqrt{2}}\angle 0^\circ$

$\mathbf{I} = \frac{0.5}{\sqrt{2}}\angle 30^\circ$

leading

(c)



$\mathbf{V} = \frac{1}{\sqrt{2}}\angle 0^\circ$

$\mathbf{I} = \frac{0.5}{\sqrt{2}}\angle -30^\circ$

lagging

Power Factor Example

- For this circuit

  $\mathbf{V} = 120\angle 0^\circ$ V

  $\mathbf{I} = 11.22\angle -20.7^\circ$ A

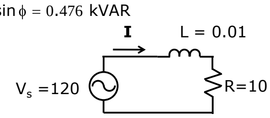
- Then

  $\mathbf{S} = (120\angle 0^\circ)(11.22\angle 20.7^\circ) = 1346\angle 20.7^\circ$

  $|\mathbf{S}| = 1346$VA

  $P = |\mathbf{S}|\cos\phi = 1346\cos(20.7^\circ) = 1.26$ kW

  $Q = |\mathbf{S}|\sin\phi = 0.476$ kVAR



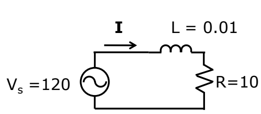
Power Factor Example

- Note: current lags the voltage (by 20.7 degrees)

  $\mathbf{V} = 120\angle 0^\circ$ V

  $\mathbf{I} = 11.22\angle -20.7^\circ$ A

- Power factor is lagging
- PF = cos(20.7°) = 0.94 (lagging)



Leading/Lagging Power Factor

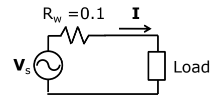
- Inductive circuits have a lagging power factor
  - Consumes reactive power
  - Q is positive
- Capacitive circuits have a leading power factor
  - Supplies reactive power
  - Q is negative

Power Factor Correction

- Low power factor requires more current to do the same amount of work
- Higher current increases losses in transmission of the electricity ($|\mathbf{I}|^2R$)

Power Factor Correction

- Consider a load that consumes 1kW of power at a power factor of 1.0 and operates at 115 V (rms). The load is connected to a voltage source by a wire with a resistance of 0.1.
- We want to know the power supplied by the source



Power Factor Correction

- First compute I

$\mathbf{S}_{load} = 1000 + 0j$

$\mathbf{S}_{load} = \mathbf{V}_{load}\mathbf{I}^*$

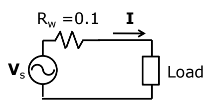
$\mathbf{I}^* = \frac{1000\angle 0}{115\angle 0} = 8.695\angle 0\text{A} = \mathbf{I}$

- Now find $\mathbf{V}_s$

$\mathbf{V}_s = 115\angle 0 + 0.1(8.695\angle 0) = 115.8695\angle 0$ V

- Now find P_s

$P_s = |115.8695||8.695|\cos(0) = 1007.5$ W



Power Factor Correction

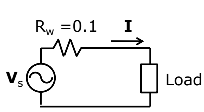
- Load consumed 1kW, source supplied 1.0075 kW
- 7.5W of loss, caused by the power consumed by the wire resistance
- By conservation of energy, we could have computed the power loss in the wire and added it to the power consumed in the load to arrive at 1.0075 kW

$P_{Loss} = |I|^2 R_w = 7.5$ W

$P_s = 1000 + 7.5 = 1007.5$ W

Power Factor Correction

- Consider a load that consumes 1kW of power at a power factor of 0.8 lagging and operates at 115 V (rms). The load is connected to a voltage source by a wire with a resistance of 0.1.
- What is the power supplied by the source?



Power Factor Correction

- First compute $\mathbf{I}$

$\phi = \cos^{-1}(0.8) = 36.87^\circ$

$|\mathbf{S}_{load}|\,0.8 = P_{load} = 1000$

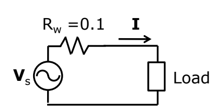
$|\mathbf{S}_{load}| = 1250\text{VA}$

$\mathbf{S}_{load} = 1250\angle 36.87^\circ$ VA

$\mathbf{S}_{load} = \mathbf{V}_{load}\mathbf{I}^*$

$\mathbf{I}^* = \frac{1250\angle 36.87^\circ}{115\angle 0} = 10.869\angle 36.87\text{A}$

$\mathbf{I} = 10.869\angle -36.87\text{A}$



Power Factor Correction

Completing the calculation

$\mathbf{I} = 10.869\angle -36.87^\circ \text{ A} = 8.695 - j6.521 \text{ A}$

$\mathbf{V}_s = 115 + 0.1(8.695 - j6.521) = 115.8695 - j0.652 = 115.87\angle -0.32^\circ \text{ V}$

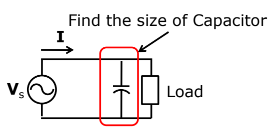
$P_s = (115.87)(10.869)\cos(36.55^\circ) = 1011.7 \text{ W}$

Power Factor

- Losses increased to 11.7W
- To operate efficiently, unity PF (PF=1) is desired
- Typically, PF ranges from 0.8 to 0.95
- Most loads are inductive (reactive power consumers)
- To increase PF, capacitors (reactive power suppliers) can be placed near the loads
  - Known as power factor correction

Power Factor Correction Example

A load draws 10 A of current at a power factor of 0.5 lagging. What size capacitor should be added to the load in parallel to increase the power factor to 0.8 lagging? Assume the voltage at the load is 120 V at 60 Hz.



Power Factor Correction Example

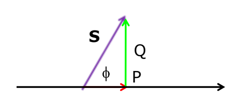
- First compute **S** at the load

  $\phi = \cos^{-1}(0.5) = 60^\circ$

  $\mathbf{S} = \mathbf{VI}^* = 120\angle 0^\circ \times (10\angle -60^\circ)^* = 1200\angle 60^\circ \text{ VA}$

  Lagging (ϕ is positive), so θ_i is negative

- What are P and Q?



Power Factor Correction Example

- What are P and Q?

  $\mathbf{S} = 1200\angle 60^\circ = P + jQ = 600 + j1039.2 \text{ VA}$

Power Factor Correction Example

- We need to reduce the amount of Q consumed in order to make the PF equal to 0.8
- The real power will remain at 600 W (since the capacitor is placed in parallel)
- At a PF of 0.8:

  $\phi = \cos^{-1}(0.8) = 36.9^\circ$ (target ϕ)

  $\phi = \tan^{-1}(Q/P)$

  $\tan\phi = Q/P$

  $Q = P\tan\phi = 450.5 \text{ VAR}$ (target Q)

Power Factor Correction Example

- We then need to find a capacitor that supplies 1039.2-450.5 = 588.7 VAR when placed in parallel with the load

$$Q = \frac{|V|^2}{jX_c}$$

$$jX_c = \frac{|V|^2}{Q} = -j24.44\Omega$$

$$C = \frac{1}{2\pi f X_C} = 108.53\ \mu F$$

- See text Example 1.3 for another way of solving this problem

Summary

- Power factor does not distinguish between inductive and capacitive circuits
- ELI the ICE man
- Low power factor increases loss in the system, and can mitigated by adding inductors/capacitors to the load