04-Review of Phasors and Complex Power

Text: 2.1 – 2.3

ECEGR 451
Power Systems

Overview

- Notation
- Phasor Transformation
- Complex Power
- Power Factor
- Leading/Lagging Circuits

Notation

- Lecture slides use bold uppercase characters (e.g. **V**, **I**) for phasors and other vectors
- Italicized bold characters used for vectors and matrices (e.g. ***Y***)
- Capital letters (e.g. V, I) or absolute values of phasors (|**V**|, |**I**|) are used to indicate the magnitude of the phasor
  $\mathbf{V} = |\mathbf{V}|\angle\theta = V\angle\theta$
- Lowercase variables (e.g. v, i) are preferred to represent scalars not associated with phasors and vectors
  - Notable exceptions P, Q for real and imaginary power

Review of Electric Circuits

- Voltage in AC circuits:
  - $v(t) = V_{max}\cos(\omega t + \theta_V)$
    - V_{max}: voltage amplitude (Volts)
    - ω: frequency (rad/sec)
    - θ_V: phase angle of the voltage (rad)
- Current in AC circuits:
  - $i(t) = I_{max}\cos(\omega t + \theta_i)$
    - I_{max}: amplitude (Amperes)
    - θ_i: phase angle of the voltage (rad)

Review of Electric Circuits

- Conversion of radians (θ_{rad}) to degrees (θ_{deg})

$$\theta_{deg} = \theta_{rad}\frac{180°}{\pi}$$

- Conversion of degrees to radians

$$\theta_{rad} = \theta_{deg}\frac{\pi}{180°}$$

Review of Electric Circuits

- Frequency in North American power systems is 60 Hz
  - f: frequency (Hertz)
  - $\omega = 2\pi f \sim 377$ rad/sec
- Other parts of the world 50 Hz is common
- We assume 60 Hz unless otherwise noted
- Voltage waveform is set as a reference, so $\theta_V = 0°$

Phasor Transform

- Shorthand for writing sinusoidal functions
- Used for steady-state calculations
- Contains amplitude and phase angle information
  - Assumed that frequency is known
- Relies on Euler's Identity

$v(t) = v_{max}\cos\,\omega t + \theta_v$ ⟶ Phasor Transform ⟶ $\frac{V_{max}}{\sqrt{2}}\angle\theta_v$

Note: division by square root of 2
is used in power system analysis ("effective phasor")

Phasor Transform

- Different expressions of a voltage phasor

$$\frac{V_{max}}{\sqrt{2}}\angle\theta_v = V_{rms}\angle\theta_v = |\mathbf{V}|\angle\theta_v = V\angle\theta_v = Ve^{j\theta_v}$$

- Current:

$$\frac{I_{max}}{\sqrt{2}}\angle\theta_I = I_{rms}\angle\theta_I = |\mathbf{I}|\angle\theta_I = I\angle\theta_I = Ie^{j\theta_I}$$

- Impedance:

$$|\mathbf{Z}|\angle\theta_Z = Z\angle\theta_Z = Ze^{j\theta_Z}$$

- Define: $\phi \triangleq \theta_v - \theta_i$ (remember this!)

Phasors in Matlab

- Consider: $\mathbf{V}_{an} = 120\angle 45°$
- Input into Matlab as:
  >> Van = 120*exp(j*45*pi/180)
  Van = 84.8528 +84.8528i %automatic conversion to rectangular coordinates
- Also useful:
  >> angle(Van)*pi/180%angle of Van
  >> abs(Van) %magnitude of Van
  >>real(Van) %real part of Van
  >>imag(Van) %imaginary part of Van
  >> [] = pol2cart() %conversion from polar to rect. See Matlab "Help"
  >>[] = cart2pol() %conversion from rect to polar. See Matlab "Help"

Complex Power

P is also known as Real Power, Active Power, Average Power

$P = |\mathbf{V}_s||\mathbf{I}|\cos(\phi)$

$P = \text{Re}\{\mathbf{VI}^*\}$

* is the complex conjugate operator,
it denotes a change in sign of the imaginary part

Complex Power

- Let **S** be the complex power defined as
  $\mathbf{S} = \mathbf{VI}^*$
  then
  $P = \text{Re}\{\mathbf{VI}^*\} = \text{Re}\{\mathbf{S}\}$
- Let Q be the reactive power defined as
  $Q = \text{Im}\{\mathbf{VI}^*\}$
- Then
  $Q = \text{Im}\{\mathbf{S}\}$
- And therefore:
  $\mathbf{S} = P + jQ$

Q is also known as imaginary power

S is also known as apparent power

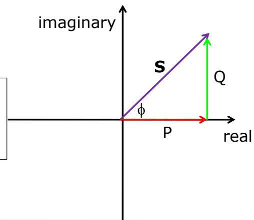
Complex Power

- Technically, units of **S**, Q and P are watts*
- To avoid confusion, alternate units are used in practice
  - S: Volt-Amps (VA)
  - Q: Volt-Amps Reactive (VAR)
- Inductors, capacitors consume/supply reactive power, Q
- S and Q are defined values
  - a meaningful physical interpretation is elusive

*See C. Gross "On VA's, VAR's, and Other Traditions in Electric Power Engineering"

Power Triangle

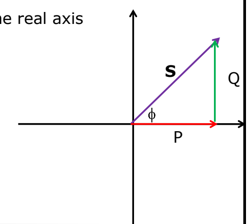
- Relationships between **S**, P and Q can be shown graphically
- **S** = P + jQ



Note:
P, Q are scalars representing the real and imaginary parts of **S**

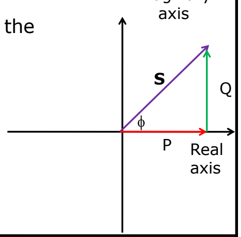
Power Triangle

- Consider P = |**V**||**I**|cos(ϕ)
- Since |**S**| = |**VI***| = |**VI**| = |**V**||**I**|
- Then P = |**S**|cos(ϕ)
  - P is the projection of S onto the real axis



Complex Power

- A similar result can be found for Q
  - P = |**S**|cos(ϕ)
  - Q = |**S**|sin(ϕ)
- Q is the projection of **S** onto the imaginary axis



Complex Power Cheat Sheet

$P = \mathrm{Re}\{\mathbf{VI}^*\}$	$Q = \mathrm{Im}\{\mathbf{VI}^*\}$
$= \mathrm{Re}\{\mathbf{IZI}^*\} = \lvert \mathbf{I} \rvert^2 \mathrm{Re}\{\mathbf{Z}\}$	$Q = \lvert \mathbf{I} \rvert^2 X$
$P = \lvert \mathbf{I} \rvert^2 R$	$Q = \lvert \mathbf{V} \rvert \lvert \mathbf{I} \rvert \sin\phi$
$P = \lvert \mathbf{V} \rvert \lvert \mathbf{I} \rvert \cos\phi$	$Q = \lvert \mathbf{I} \rvert^2 \lvert \mathbf{Z} \rvert \sin\phi$
$P = \lvert \mathbf{I} \rvert^2 \lvert \mathbf{Z} \rvert \cos\phi$	

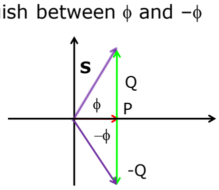
$$\mathbf{S} = P + jQ$$

$$\phi = \tan^{-1}(Q/P)$$

$$\cos\phi = \frac{P}{\sqrt{P^2 + Q^2}}$$

Power Factor

- Power factor is non-negative
- cos(ϕ) = cos($-\phi$)
- Need to distinguish between ϕ and $-\phi$



Power Factor

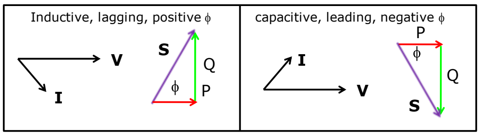
- For example let θ_v = 0°
- Case 1: θ_i = 30°
  - Capacitive circuit
  - PF = 0.866
- Case 2: θ_i = -30°
  - Inductive circuit
  - PF = 0.866

Same power factor

Leading/Lagging Power Factor

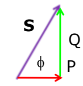
Must describe the PF value along with whether the current leads or lags voltage

- Lagging: current lags voltage (inductive)
- Leading: current leads voltage (capacitive)
- Useful mnemonic: ELI the ICE man



Summary

- P, Q, **S** related by power triangle



- P has a physical interpretation, Q and **S** do not
- **S** is a vector, Q and P are scalars
- Resistors associated with P; inductors/capacitors associated with Q