04-Boolean Algebra Part 2

Text: Unit 2

ECEGR/ISSC 201
Digital Operations and Computations
Winter 2011

Overview

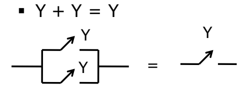
- Basic Theorems
- Simplification Theorems
- Multiplying and Factoring

Basic Theorems

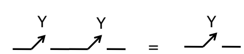
- Basic laws and theorems of Boolean algebra
  - Y + 0 = Y
  - Y + 1 = 1
  - Y x 1 = Y
  - Y x 0 = 0

Basic Theorems

- Idempotent laws
  - Y + Y = Y



  - Y x Y = Y

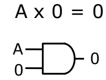


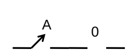
Basic Theorems

- Involution Law
  - (Y')' = Y
- Laws of complementarity
  - Y + Y' = 1
  - Y x Y' = 0

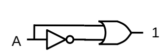
Examples

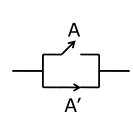
- A x 0 = 0





- A + A' = 1





Commutative, Associative and Distributive Laws

- Commutative laws
  - XY = YX
  - X + Y = Y + X
- Associative laws
  - XYZ = (XY)Z = X(YZ)
  - X +Y +Z = (X + Y) + Z = X + (Y + Z)

Commutative, Associative and Distributive Laws

- Distributive laws
  - X(Y + Z) = XY + XZ
  - X + YZ = (X +Y)(X +Z)
- Second law is not valid in ordinary algebra

$5+3\ 2\ =11$

$5+3\quad 5+2\ =56$

$5+3\ 2\ \neq\ 5+3\quad 5+2$

Commutative, Associative and Distributive Laws

- Proof: X + YZ = (X +Y)(X +Z)
- (X +Y)(X +Z) = XX + XZ + YX + YZ

  = X + XZ + YX + YZ

  = X1+ XZ + YX + YZ

  =X(1 + Z + Y) + YZ

  =X + YZ

Simplification Theorems

- XY + XY′ = X
- (X + Y)(X + Y′) = X
- X +XY = X
- X(X+Y) = X
- (X+Y′)Y = XY
- XY′ + Y = X +Y

Multiplying and Factoring Out

- Another simplifying theorem
  - (X+Y)(X′+Z) = XZ+X′Y

Simplification Theorems

- Proof:
  - (X+Y)(X′+Z) = XZ+X′Y
- Consider X = 1
  - (1+Y)(0+Z) = Z
- Consider X = 0
  - (0+Y)(1+Z) = Y
- Therefore: (X+Y)(X′+Z) = XZ+X′Y

Simplification Theorems

- Recall the statement
  - *The light is to turn on if button A and button B are pushed or if button A is pushed and button B is not pushed or if button C is pushed*
  - L = AB +AB' +C
- can be reduced to:
  - *The light turns on if button A or button C are pushed*
  - L = A +C
- Why?

Simplification Theorems

- L = AB +AB' +C
- Using XY+XY' = X
  - => L = A + C

Simplification Theorems

- Simplification theorems can be proved using truth tables or the basic laws
- Example:
  - Use a truth table and Boolean algebra to show that X +XY = X

X	Y	XY	X +XY
0	0		
0	1		
1	0		
1	1		

Example

- Use a truth table to show X +XY = X

X	Y	XY	X +XY
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

- X +XY = (X+X)(X+Y) = X(X +Y)
- Considering either value for Y: X +0 = X, X + 1 =1
  - XX = X
  - X(1) = X
  - => X(X +Y) = X

Simplification Theorems

- See page 42 for proofs of other simplification theorems
- Example: Simplify Z = A'BC +A'
  - Z = A'
  - Set A'=X; Y=BC
  - => Z = XY + X = X = A'

Simplification Theorems

- Simplify
  - Z = [A + B'C + D + EF][A + B'C + (D + EF)']
  - Set: X = A +B'C; Y = D +EF
  - Z = [X + Y][X + Y']
  - By second distributive law:
    - Z =[X + Y][X + Y'] = X+YY' = X = A+B'C

Multiplying and Factoring Out

- There are many ways of writing equivalent expressions using the simplification theorems
- We are interested in two particular standard forms

Multiplying and Factoring Out

- Sum-of-products (SoP): an expression that is the sum of products of single variables
  - Examples:
    - AB′ + CD′E + AC′E′
    - AB′ + CDE′ + F
    - Not in SoP form: (A+B)CD + EF
- Multiply out to get into SoP form

Multiplying and Factoring Out

- Product-of-sums (PoS): an expression in which the sums are sums of single variables
  - Examples
    - (A+B)(C+D′+E)(A+C′+E′)
    - AB′C(D′+E)
    - But not: (A+B)(C+D)+EF
- Factor to get into PoS form

Example

- Identify the following as being in SoP, PoS or an unspecified form
  - F = AB + AB′
  - F = (ABC)+(ABD)′
  - F= AB+C+D+E
  - F = (A+C)(E)

Example

- Identify the following as being in SoP, PoS or an unspecified form
  - F = AB + AB′ SoP
  - F = (ABC)+(ABD)′ SoP
  - F= AB+C+D+E SoP
  - F = (A+C)(E) PoS

Multiplying and Factoring Out

- To convert an expression to SoP form, multiply it out
- Multiplying out usually involves one or more of these laws and simplifications:
  - (X+Y)(X+Z) = X+YZ
  - (X+Y)(X′+Z) = XZ + X′Y
  - X(Y+Z) = XY + XZ

Multiplying and Factoring Out

- In general, when multiplying out avoid unnecessary terms by:
  - First using:
    - (X+Y)(X+Z) = X+YZ
    - and/or (X+Y)(X'+Z) = XZ+X'Y
  - then use:
    - X(Y+Z) = XY + XZ

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Multiplying and Factoring Out

- F=(A+BC)(A+D+E)
  - Setting X = A; Y = BC; Z = D+E and using (X+Y)(X+Z) = X+YZ
  - F = A + BC(D+E)
  - F= A + BCD + BCE (using X(Y+Z) = XY + XZ)

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Multiplying and Factoring Out

- The alternative is multiplying out the original expression and cancelling redundant terms
- F=(A+BC)(A+D+E)
  - F = AA + AD + AE + BCA + BCD + BCE
  - F = A + A(D + E + BC) + BCD + BCE
  - Using X + XY = X
  - F = A + BCD + BCE

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Multiplying and Factoring Out

- To convert an expression to PoS form, factor it
- Factoring involves the same equations, but working right to left
  - (X+Y)(X+Z) = X+YZ
  - (X+Y)(X'+Z) = XZ + X'Y
  - X(Y+Z) = XY + XZ

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Multiplying and Factoring Out

- Factor F=C'D +C'E' + G'H
  - F=C'(D+E')+G'H (using X(Y+Z) = XY + XZ)
  - F=(C'(D+E') + G')(C'(D+E')+H) (using (X+Y)(X+Z) = X+YZ)
  - F=((G'+C')(G'+D+E'))((H+C')(H+D+E')) (using (X+Y)(X+Z) = X+YZ)

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Multiplying and Factoring Out

- Write the following in Sum of Product form
- F=(A+B+C')(A+B+D)(A+B+E)(A+D'+E)(A'+C)

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Multiplying and Factoring Out

- Want in SoP form, so we need to multiply out
- First use: (X+Y)(X+Z) = X+YZ

(A+B+C')(A+B+D)(A+B+E)(A+D'+E)(A'+C)

(A+B+C')(A+B+D)(A+B+E)(A+D'+E)(A'+C)

(A+B+C'D)(A+B+E)(A+D'+E)(A'+C)

Multiplying and Factoring Out

- Can we use it anywhere else?
  (X+Y)(X+Z) = X+YZ

(A+B+C'D)(A+B+E)(A+D'+E)(A'+C)

(A+B+C'D)(A+B+E)(A+D'+E)(A'+C)

(A+B+C'DE)(A+D'+E)(A'+C)

Multiplying and Factoring Out

- We could use it again, but it does not help much
- Instead look to use:
  (X+Y)(X'+Z) = XZ+X'Y

(A+B+C'DE)(A+ D'+E)(A' +C)

(A+B+C'DE)(A+ D'+E)(A' +C)

(A+B+C'DE)(AC + A'(D'+E))

Multiplying and Factoring Out

- Not much more we can do here, so multiply out

(A+B+C'DE)(AC + A'(D'+E))

AAC +AA'(D'+E) + BAC + BA'(D'+E) + C'DEAC + C'DEA'(D'+E)

- Simplifying further:

AC + 0 + BAC + BA'(D'+E) + 0 + C'DEA'(D'+E)

Multiplying and Factoring Out

- multiplying out again:

AC + BAC + BA'(D'+E) + C'DEA'(D'+E)

AC + BAC + BA'D'+ BA'E + C'DEA'D'+ C'DEA'E

- simplifying further

AC + BAC + BA'D'+ BA'E + 0 + C'DA'E

Multiplying and Factoring Out

- Are we done?
  - AC + BAC + BA'D'+ BA'E + C'DA'E
- It is in SoP form, but we can minimize it further
  - AC + BAC + BA'D'+ BA'E + C'DA'E
  - AC + BA'D'+ BA'E + C'DA'E
  - using X + XY = X

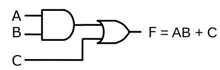
Multiplying and Factoring Out

- If we would have multiplied out the original expression, we would have 3 x 3 x 3 x 3 x 2 = 162 terms!
- (A+B+C')(A+B+D)(A+B+E)(A+D'+E)(A'+C)
- See text, page 60 for an example of factoring

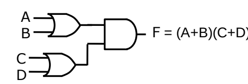
Multiplying and Factoring Out

- SoP: usually several AND gates input into a single OR gate
- PoS: usually OR gates input into a single AND gate

SoP



PoS



Simplifications

- See page 52 for a table of laws and theorems of Boolean Algebra