03-Real, Imaginary and Complex Power

ECEGR 450
Electromechanical Energy Conversion

Overview

- Complex Power
- Power Triangle
- Examples

Questions

- How are Real, Reactive and Apparent Power different? How are they related?
- What is "Imaginary" power?

Complex Power

P is also known as Real Power, Active Power, Average Power

$P = |\mathbf{V}_s||\mathbf{I}|\cos(\phi)$

$$P = \frac{v_{max}}{\sqrt{2}}\frac{i_{max}}{\sqrt{2}}\cos(\theta_v - \theta_i)$$

$$P = \mathrm{Re}\left\{\frac{v_{max}}{\sqrt{2}}\frac{i_{max}}{\sqrt{2}}e^{j(\theta_v - \theta_i)}\right\} \quad \text{using } e^{\pm jx} = \cos(x) \pm j\sin(x)$$

$$P = \mathrm{Re}\left\{\frac{v_{max}}{\sqrt{2}}\frac{i_{max}}{\sqrt{2}}e^{j(\theta_v)}e^{-j(\theta_i)}\right\}$$

$$P = \mathrm{Re}\left\{\frac{v_{max}}{\sqrt{2}}e^{j(\theta_v)}\frac{i_{max}}{\sqrt{2}}e^{-j(\theta_i)}\right\}$$

$$P = \mathrm{Re}\{\mathbf{VI}^*\}$$

* is the complex conjugate operator, it denotes a change in sign of the imaginary part

Complex Power

- Let **S** be the complex power defined as
  $\mathbf{S} = \mathbf{IV}^*$
  then
  $P = \mathrm{Re}\{\mathbf{IV}^*\} = \mathrm{Re}\{\mathbf{S}\}$
- Let Q be the reactive power defined as
  $Q = \mathrm{Im}\{\mathbf{VI}^*\}$
- Then
  $Q = \mathrm{Im}\{\mathbf{S}\}$

  Q is also known as imaginary power
- And therefore:
  $\mathbf{S} = P + jQ$

  S is also known as apparent power

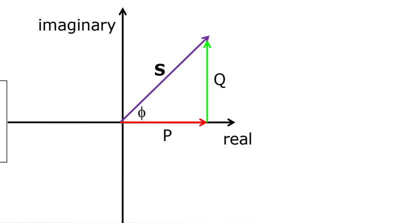
Complex Power

- Technically, units of **S**, Q and P are watts*
- To avoid confusion, alternate units are used in practice
  - S: Volt-Amps (VA)
  - Q: Volt-Amps Reactive (VAR)
- Inductors, capacitors consume/supply reactive power, Q
- S and Q are defined values
  - a meaningful physical interpretation is elusive

*See C. Gross "On VA's, VAR's, and Other Traditions in Electric Power Engineering"

Power Triangle

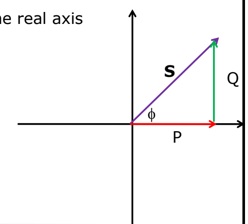
- Relationships between **S**, P and Q can be shown graphically
- **S** = P + jQ



Note:
P, Q are scalars representing the real and imaginary parts of **S**

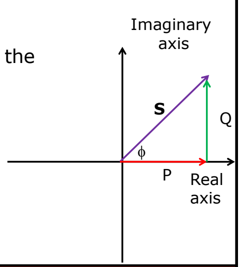
Power Triangle

- Consider P = |**V**||**I**|cos(ϕ)
- Since |**S**| = |**VI***| = |**VI**| = |**V**||**I**|
- Then P = |**S**|cos(ϕ)
  - P is the projection of S onto the real axis



Complex Power

- A similar result can be found for Q
  - P = |**S**|cos(ϕ)
  - Q = |**S**|sin(ϕ)
- Q is the projection of **S** onto the imaginary axis



Complex Power Cheat Sheet

$P = \mathrm{Re}\{\mathbf{VI}^*\}$

$= \mathrm{Re}\{\mathbf{IZI}^*\} = |\mathbf{I}|^2 \mathrm{Re}\{\mathbf{Z}\}$

$P = |\mathbf{I}|^2 R$

$P = |\mathbf{V}||\mathbf{I}|\cos\phi$

$P = |\mathbf{I}|^2|\mathbf{Z}|\cos\phi$

$Q = \mathrm{Im}\{\mathbf{VI}^*\}$

$Q = |\mathbf{I}|^2 X$

$Q = |\mathbf{V}||\mathbf{I}|\sin\phi$

$Q = |\mathbf{I}|^2|\mathbf{Z}|\sin\phi$

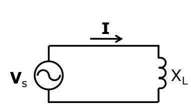
$$\mathbf{S} = P + jQ$$

$$\phi = \tan^{-1}(Q/P)$$

$$\cos\phi = \frac{P}{\sqrt{P^2 + Q^2}}$$

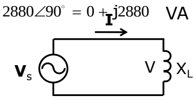
Example

- Find **S**, P and Q for the shown circuit if X_L is 5 Ohms and V_s = 120

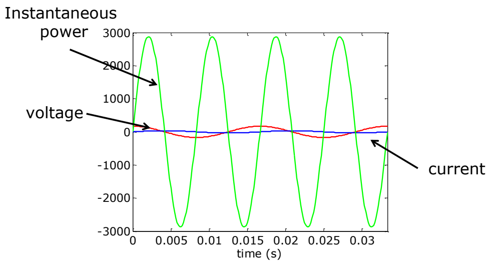


Example

- Find **S**, P and Q for the shown circuit if X_L is 5 Ohms and V_s = 120
  - **V** = $\mathbf{V}_s$
  - **Z** = jX_L
  - **I** = $\mathbf{V}_s/\mathbf{Z}$ $= 24\angle -90^\circ$
  - Φ = 0º - -90º= 90º
  - P = |**V**||**I**|cos(90) = 0 W
  - Q = $|\mathbf{I}|^2X$ = 2880 VAR
  - **S** = $\mathbf{VI}^*$ = $2880\angle 90^\circ = 0 + j2880$ VA

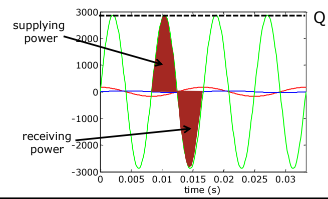


Example

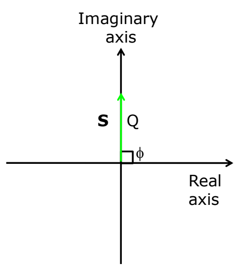


Example

- Observations:
  - Instantaneous power has a zero average
    - P = 0
  - Maximum instantaneous power is equal to Q

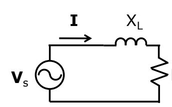


Example



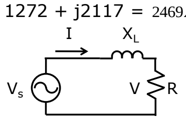
Example

- Now consider the same circuit with load R = 3 Ohms



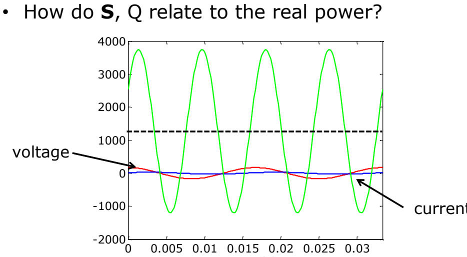
Example

- Now consider the same circuit with load R = 3 Ohms
  - $\mathbf{V} = \mathbf{V}_s$
  - $\mathbf{Z} = R + jX_L = 3 + 5j$
  - $\mathbf{I} = \mathbf{V}_s/\mathbf{Z} = 10.58 - 17.65j = 20.58\angle -59^\circ$
  - $P = |\mathbf{V}||\mathbf{I}|\cos(59^o) = 1272$ W
  - $Q = |\mathbf{V}||\mathbf{I}|\sin(59^o) = 2117$ VAR
  - $S = \mathbf{VI}^* = 1272 + j2117 \quad 2469\angle 59^\circ$ VA

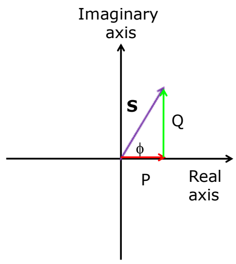


Example

- Average instantaneous power: 1272 W
- How do **S**, Q relate to the real power?



Example

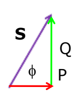


Significance of Reactive Power

- Only real power, P, can be used for work
- A circuit that consumes large amounts of reactive power requires more current to do the same work
  - increased heating losses
- Q is related to voltage profile
- Q is also the maximum instantaneous value of power in a purely inductive or capacitive circuit
- Interested in reducing the amount of Q needed for a given amount of P

Summary

- P, Q, **S** related by power triangle



- P has a physical interpretation, Q and **S** do not
- **S** is a vector, Q and P are scalars
- Resistors associated with P; inductors/capacitors associated with Q