02-Phasors

ECEGR 450
Electromechanical Energy Conversion

Overview

- Phasors Transform
- Phasor Mathematics
- Circuit Analysis Using Phasors

Questions

Why are AC circuits solved in the phasor domain?

Phasors

- Analysis of circuits with R, L and/or C components requires solving differential equations
- We will consider only sinusoidal steady-state voltages and currents of the same frequency
- We can then analyze the circuits in the <u>phasor domain</u> much easier than in the time domain

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Phasor Transform

- Shorthand for writing sinusoidal functions
- Used for steady-state calculations
- Contains amplitude and phase angle information
  - Assumed that frequency is known
- Relies on Euler's Identity

$v(t) = v_{max}\cos(\omega t + \theta_v)$ → Phasor Transform → $V_{max}\angle\theta_v$

Phasor Transform

$v(t) = v_{max}\cos(\omega t + \theta_v)$ time-domain representation

$e^{\pm jx} = \cos(x) \pm j\sin(x)$ Euler's Identity

$v(t) = v_{max}\,\mathrm{Re}\left\{e^{j(\omega t + \theta_v)}\right\}$ using Euler's Identity

$v(t) = v_{max}\,\mathrm{Re}\left\{e^{j\omega t}e^{j\theta_v}\right\}$ regrouped

implied, so we suppress this

$v(t) = v_{max}e^{j\theta_v} = v_{max}\angle\theta_v$ transformed

$\mathbf{V} = \frac{v_{max}}{\sqrt{2}}e^{j\theta_v} = V_{rms}e^{j\theta_v}$ <u>in power we divide by $\sqrt{2}$, also known as the effective phasor</u>

Phasor Transform

- We use the effective phasor because
  $P = v_{rms} i_{rms} \cos(\phi)$
  - So we can then write $P = VI\cos(\phi)$
- Unless otherwise specified, <u>assume that voltages and currents are given in RMS and all phasors are "effective phasors"</u>
- Also note:
  $e^{-j90^\circ} = \cos(-90^\circ) + j\sin(-90^\circ) = j\sin(-90^\circ) = -j$

Notation

- A bug in the notation
- Book uses a tilde to indicate that a variable is a phasor, as in $\tilde{V}$, $\tilde{I}$
  - In other words, $\tilde{V}$, $\tilde{I}$ are understood to have a magnitude and phase component
- Book uses V, I to indicate the magnitude of the phasor

Notation

- Lecture slides use bold uppercase variables (e.g. **V**, **I**) for phasors and other vectors
- Capital letters (e.g. V, I) or absolute values of phasors (|**V**|, |**I**|) are used to indicate the magnitude of the phasor
  $\mathbf{V} = V\angle\theta$
- Lowercase variables (e.g. v, i) are preferred to represent scalars not associated with phasors and vectors
  - Notable exceptions P, Q for real and imaginary power

Phasors

- write the phasor representation of
  $v_1(t) = 1.41\cos(377t + 0)$
- write the phasor representation of
  $v_2(t) = 2.12\cos(377t + 45)$
- write the phasor representation of
  $v_3(t) = 1.41\cos(t + 0)$

Phasors

- write the phasor representation of
  $v_1(t) = 1.41\cos(377t + 0)$

  solution
  $\mathbf{V}_1 = 1\angle 0^\circ$
- write the phasor representation of
  $v_2(t) = 2.12\cos(377t + 45)$

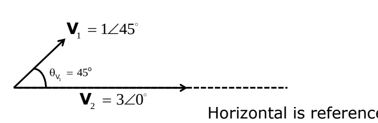
  solution
  $\mathbf{V}_2 = 1.5\angle 45^\circ$
- write the phasor representation of
  $v_3(t) = 1.41\cos(t + 0)$

  solution
  $\mathbf{V}_3 = 1\angle 0^\circ$

Phasors

Phasors have a direct geometric interpretation

- Polar form

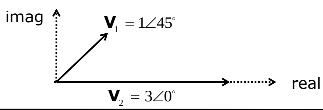


Phasors

- Another way of specifying phasors is in rectangular form
  - Let the Y-axis be the imaginary (j) axis
  - Let the X-axis be the real axis
- Resolving into real and imaginary components

$\mathbf{V}_1 = 1\angle 45^\circ = 0.707 + j0.707$

$\mathbf{V}_2 = 3\angle 0^\circ = 3 + j0$

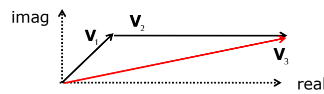


Addition of Phasors

- Addition and subtraction of phasors are simple using rectangular form
  - Simply add/subtract the real values and add/subtract the imaginary values

$\mathbf{V}_3 = \mathbf{V}_1 + \mathbf{V}_2 = 3.707 + j0.707$

Addition is "tip to tail"
Subtraction is "tail to tip"



Phasors

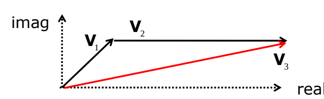
- To convert from rectangular to polar:
  - **V** = a + jb

$\mathbf{V} = V\angle\theta = \sqrt{a^2 + b^2}\angle \tan^{-1}\left(\frac{b}{a}\right)$

Phasors

- What is $\mathbf{V}_3$ in Phasor form?

$\mathbf{V}_3 = 3.707 + j.707$

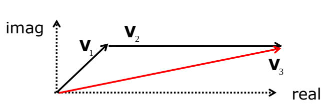


Phasors

- What is $\mathbf{V}_3$ in Phasor form?

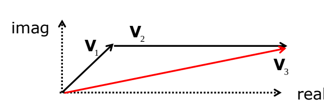
$\mathbf{V}_3 = 3.707 + j.707$

$\mathbf{V} = V\angle\theta = \sqrt{3.707^2 + 0.707^2}\angle \tan^{-1}\left(\frac{0.707}{3.707}\right) = 3.77\angle 10.8^\circ$



Phasors

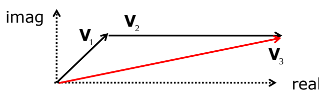
- What is $\mathbf{V}_3$ in the time domain?



Phasors

- What is $\mathbf{V}_3$ in the time domain?

$v(t) = 3.77\sqrt{2}\cos(\omega t + 10.8^\circ)$



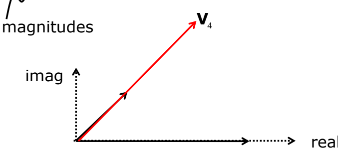
Multiplication of Phasors

- Multiplication and division are easier in polar form
  - For multiplication: multiply magnitudes, add angles
  - For division: divide magnitudes, subtract angles

$\mathbf{V}_4 = \mathbf{V}_1\mathbf{V}_2 = (1\angle 45^\circ)(3\angle 0^\circ)$

$\mathbf{V}_4 = 3\angle 45^\circ$ ← add angles

multiply magnitudes



Multiplication of Phasors

Using Matlab

```
>> V1=1*exp(j*45*pi/180)
V1 =
  0.7071 + 0.7071i
>> V2=3*exp(j*0*pi/180)
V2 =
    3
>> V3=V1+V2
V3 =
  3.7071 + 0.7071i
>> Vmag=abs(V3)
Vmag =
   3.7739
>> Vanglerad=angle(V3)
Vanglerad =
   0.1885
>> Vangledeg=Vanglerad*180/pi
Vangledeg =
  10.7991
```


Phasor Analysis of Inductors

- For inductors

$i(t) = i_{max}\cos(\omega t + \theta_i)$

$v(t) = L\dfrac{di}{dt}$

$v(t) = -L\omega i_{max}\sin(\omega t + \theta_i) = -L\omega i_{max}\cos(\omega t + \theta_i - 90^\circ)$

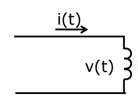
- Transforming into phasor form:

$v(t) = -L\omega i_{max}\cos(\omega t + \theta_i - 90^\circ)$

$\Rightarrow \mathbf{V} = -L\omega I e^{j\theta_i} e^{-j90^\circ}$

$\mathbf{V} = jL\omega I e^{j\theta_i} = jL\omega \mathbf{I}$

using $e^{j-90^\circ} = -j$



Phasor Analysis

$\mathbf{V} = j\omega L\mathbf{I}$

- Define $X_L = \omega L$ (inductive reactance)
- Therefore
  - $\mathbf{V} = jX_L\mathbf{I}$
- A similar derivation for capacitors yields
  - $X_C = -1/(\omega C)$ (capacitive reactance)
  - $\mathbf{V} = jX_C\mathbf{I}$

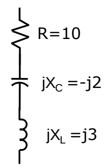
Phasor Analysis

- We can rewrite Ohm's Law to include complex impedances
- $\mathbf{V} = \mathbf{IZ}$
  - $\mathbf{Z}$: complex impedance (Ohms)
    - $\mathbf{Z} = R + jX_L + jX_C$ (if in series)
    - $1/\mathbf{Z} = 1/R + j/X_L + j/X_C$ (if in parallel)
- $\mathbf{Z}$ will have a magnitude and phase associated with it $\mathbf{Z} = Z\angle\theta_z$

Phasor Analysis

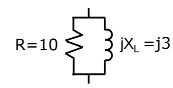
- **Z** can be found by adding the R, X_L and X_C if in series
  - **Z** = R + jX_C + jX_L
- Example
  - **Z** = 10 + -j2 + j3
  - **Z** = 10 + j (rectangular coordinates)

Note:
Re{**Z**} = R
Im{**Z**} = X_c + X_L



Phasor Analysis

- Parallel:
  - 1/**Z** = 1/R + 1/(-jX_C) + 1/(jX_L)
- Example
  - 1/**Z** = 1/10 + 1/(j3) = 0.1 -0.333j
  - **Z** = 2.87∠73.3°

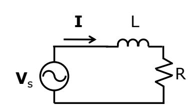


Phasor Analysis

- Note:
  - |**V**| = V_{rms} (magnitude of the phasor)
  - |**I**| = I_{rms}
- Using phasors
  - **V** = **IZ**
  - P = |**V**||**I**|cos(ϕ)
  - P = $|\mathbf{I}|^2$R = $|\mathbf{V}|^2$/R

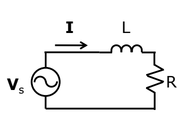
Phasor Analysis

- Find the current out of the source, the power out of the source, and power consumed by the resistor assuming:
  - V_s = 120 Volts at 60 Hz
  - L = 0.01 Henry
  - R = 10 Ohms



Phasor Analysis

- V_s = 120 Volts (RMS) at 60 Hz
- L = 0.01 Henry
  - jX_L = jωL = j(60 x 2π)x.01 = j3.77
- R = 10 Ohms



Phasor Analysis

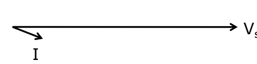
$$\mathbf{Z} = (3.77j + 10) = \sqrt{3.77^2 + 10^2}\angle \tan^{-1}\left(\frac{3.77}{10}\right)$$

$$\mathbf{Z} = 10.69\angle 20.7^\circ\,\Omega$$

$$\mathbf{V}_s = \mathbf{IZ}$$

$$\mathbf{I} = \frac{120\angle 0^\circ}{10.69\angle 20.7^\circ} = 11.22\angle -20.7^\circ\,\text{A}$$

phasor diagram



Phasor Analysis

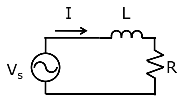
- Power from the source

$P = |\mathbf{V}_s||\mathbf{I}|\cos(\phi) = (120)(11.22)\cos(20.7^\circ) = 1.26\text{kW}$

- Power consumed by the load resistor

$P = |\mathbf{I}|^2 R = 11.22^2 \times 10 = 1.26\text{kW}$

- The inductor does not consume any power P



Summary

- Phasor Domain: allows steady state AC circuits to be easily analyzed (also graphical interpretation)
- Phasors: magnitude and phase info; freq. is assumed

$\mathbf{V} = \frac{V_{max}}{\sqrt{2}} e^{j\theta_v} = V_{rms} e^{j\theta_v}$

- Effective phasors are used in power systems
- Ohm's Law in phasor domain: $\mathbf{V}_s = \mathbf{IZ}$